Definition of par sec
parallax second. = pc

The largest base available to Earth based observer is the diameter of Earth orbit.

\[
\tan \alpha \cdot d = \frac{1 \text{au}}{\tan p} = \frac{1}{p} \text{ is very small,}
\]

Side note:

1. Closest star: Proxima Centauri has distance to us = 1.3 pc.

\[
d = \frac{1 \text{au}}{p} \left( \frac{\pi}{3600} \right) \left( \frac{1}{180} \right) = \frac{1 \text{au}}{12 \times 10^5 \text{ pc}''}
\]

\[
d (p = 1''') = 1 \text{ parsec} = \text{pc} = \frac{1.5 \times 10^{16}}{2.06 \times 10^5}
\]

Generally distance
\[
d = \frac{1 \text{ pc}}{\text{parallax}} \approx 3.02 \times 10^{16} \text{ m} = 3.26 \text{ lightyears} = 3.26 \text{ Ly}
\]
Beyond naked eye observation.

Telescopes:

One lens is not enough though it often refers to as a magnifying glass.

Why it make very small image of a distant objects. Recall burning ants adventures

Side note:

Negative II lenses form "imaginary" image so we need 2nd element

So we need two element system, one form image, 2nd magnifies
\[
\begin{align*}
\frac{h_o}{h_s} & = \frac{f_o}{D_s} \ll 1 \quad \text{but by itself it is not very useful} \\
he & = \frac{D_e}{h_o} \gg 1 \\
\text{Note though that} \\
\Theta_{\text{naked}} & = \frac{h_s}{D_s} \quad \text{was old (unaided) angular size of the star} \\
\Theta_{\text{telescope}} & = \frac{he}{D_e} \quad \text{is new angular size} \\
\text{So angular magnification} \\
M & = \frac{\Theta_T}{\Theta_{\text{naked}}} = \frac{h_s}{D_s} \cdot \frac{D_e}{he} = \frac{h_o}{f_o} \cdot \frac{fe}{f_e} = \frac{fe}{f_o}
\end{align*}
\]

\[M = \frac{fe}{f_o}\]

So we can boost angular resolution with a telescope.
A few practical limitations

\[ M \text{ can be large but we need to hold lens assembly together} \]

\[ \uparrow \text{ telescope} \]

So \( \text{fe + fo} \approx 10 \text{m} \)

\[ \text{fe} \approx \times \frac{1\text{m}}{1\text{m}} \text{ but this is hard to operate} \]

So max \( M \approx \frac{10\text{m}}{1\text{m}} = 10^7 \).

So theoretically we can do the smallest angular resolution of

\[ \Theta_{\text{detect}} / M = 0.5' / 10^7 \text{ sounds quite good but is it all to consider?} \]
Chromatic aberrations

\[ \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Recall that \( n \) is a function of wavelength \( n = n(\lambda) \); otherwise, we will see a set of green, red, and blue images on top of each other.

Solutions: * color filter \( \to \) less light to detect
* Reflector telescope suggested by Newton, i.e., replace lenses with mirror.

This technique was mastered by Herschel who build very large mirrors by literally polishing horse manure.

(1801 - find new planet Uranus
1803-1804 - binary star systems 120 years of observations)

Same guy "map" of our galaxy as brightness over distance estimate.
A larger issue is diffraction!

Recall single slit experiment:

On a screen we collect all light from the slit i.e. from every element $dx$

Electromagnetic field (light) will sum up on the screen

$$E_{\text{Screen}} = E_0 \frac{E'}{d} \left( \frac{1}{2\pi L + x \cdot \sin \theta} \right)$$

proportionality coef

field per "pixel" = $\frac{\theta \ll 1}{\text{but we will keep it}} = \frac{E'}{d} \int e^{i \frac{2\pi}{\lambda} x \cdot \sin \theta} \, dx$

boring common factor
\[ E_s = \frac{E'}{d} \int_0^d e^{i \frac{2\pi}{\lambda} \sin \Theta x} \, dx = \]

\[ = \frac{E'}{d} \left. \frac{1}{i \frac{2\pi}{\lambda} \sin \Theta \cdot \left( e^{i \frac{2\pi}{\lambda} \sin \theta d} - 1 \right) \right|_0^d - \frac{1}{2} i \frac{2\pi}{\lambda} \sin \Theta d \]

\[ = \frac{E'}{e^{i \frac{2\pi}{\lambda} \sin \Theta d}} \frac{e^{-\frac{1}{2} i \frac{2\pi}{\lambda} \sin \theta d} - e^{-\frac{1}{2} i \frac{2\pi}{\lambda} \sin \theta d}}{2i \sin \left( \frac{\pi}{\lambda} \sin \theta d \right)} \]

Another boring factor

\[ = 2\pi \left( \sin \left( \frac{\pi}{\lambda} \sin \theta d \right) \right) \frac{\sin \left( \frac{\pi}{\lambda} \sin \theta d \right)}{\frac{\pi}{\lambda} \sin \theta d} \]

\[ \text{const phase } \phi \]

\[ E_s = \frac{E'}{\sin \left( \frac{\pi}{\lambda} \sin \theta d \right)} \]

Minima condition \( E_s = 0 \)

\[ \frac{\pi}{\lambda} d \cdot \sin \Theta = m \pi \]

\[ \sin \Theta = \frac{m \lambda}{d} \]

Main maxima angle \( \Theta_m = \frac{\lambda}{d} \)
Well, round apertures are almost the same.

\[ E_s(\theta) = 2 \sqrt{J_1\left(\frac{\pi}{\lambda} D \sin \theta\right)} \]

Bessel function

Now if we have two objects close to each other, they form two overlapping Airy disks, which in cross section looks like.

\[ \Theta_{\text{min}} = 1.22 \frac{\lambda}{D} \]

Rayleigh criterion

So size of the objective is the actual limit.

\[ \Theta_m = 1.2 \frac{\lambda}{D} = 12 \times 10^{-6} \text{m} = 10^{-4} \text{rad} \]

Physical limit matches

\[ D \approx 10 \text{m} \]

\[ \lambda \approx 1/\mu\text{m} \]
Recall that aided eye limited by maximum $M$ - achievable by construction,

$$\theta_m = \frac{\theta_{eyec}}{M} = \frac{2\text{mm}}{2\text{cm}} \cdot \frac{10\text{m}}{1\text{m}} =$$

$$= 10^{-3} / 10^3 = 10^{-6} \text{rad}$$

is not achievable due to diffraction with $\theta_m \propto \frac{\lambda}{D} = \frac{10^{-6}}{10\text{m}} \approx 10^{-7} \text{rad}$

\[ 1 \text{ rad} = 2 \cdot 10^5 \text{ arc sec} \]