Stellarium demo to show planets motion

* Did Greeks know distances to known planets?
  Yes! for inner planets, but relative to \( \oplus \leftrightarrow \odot \) distances

Idea:

\[
D_{SV} = L \cdot \sin \theta_{\text{max}} = 0.43 \text{ AU}
\]
Outer planets are tricky but doable too!

Measure periods of orbits with respect to stars. \( P_M \) and \( P_E \) not that easy.

** Wait for opposition, then wait for another one, let's call this time \( S \) (synodic).

Time \( S \) is the same to make \( 2\pi \) arc with respect to another planet.

Assuming circular orbits with respect to stars, each planet makes arc \( \frac{2\pi}{P} \) at.

So \( \frac{2\pi}{P_E} S - \frac{2\pi}{P_M} S = 2\pi \)

\[ \Rightarrow \frac{1}{S} = \frac{1}{P_E} - \frac{1}{P_M} \]

Flip sign if planets is an inner one. Mars is just an example true for any other planet.
Now we have $P_m$ and $P_E$.

So if we wait time $t_{\text{hr}}$ necessary to have arc difference of $\pi/2$ after an opposition we will get $\pi/2$ angle between a planet $\rightarrow$ sun directions.

Measure $\Theta$ and we will know distance to Mars.

\[
\frac{\pi}{2} = \frac{2\pi}{P_E} + \frac{\pi}{2} - \frac{2\pi}{P_m} \\
\frac{\pi}{2} = \frac{2\pi}{S} + \frac{\pi}{2} \\
\Rightarrow \quad t_{\text{hr}} = \frac{S}{4}
\]

$D_{SM} = \tan \cdot \tan \Theta$
How to measure distance to the Moon?

Idea from Aristarchus (310 - 230 BC) who by the way suggested (Helio)centric model

\[
\frac{DEm}{Des} = \cos \theta \\
\Rightarrow \text{DEM} = \cos \theta \times \text{1au}
\]

Aristarchus estimate of $\theta \approx 87^\circ$

Modern day values $= 89.50^\circ$

$\cos \theta = 0.003$
There is still a problem: what is 1 au?

How can we measure distance to the far object?

\[ L = \frac{B}{2} \cdot \frac{1}{\tan \theta} = \frac{1}{\theta} = \frac{B}{2\theta} \]

So why 1 au is hard?
the largest Base on Earth is its diameter

\[ P = \frac{6400 \text{ km}}{0.3'1.5'10''} = \frac{6400 \text{ km}}{8100 \text{ m}} = 10 \times 4 \times 10^{-6} \text{ rad} = \frac{180^\circ}{\pi} \cdot \frac{68'}{70'} = 0.49' \text{ arc. minutes} \]
Is it small or big?

Physiological limit

\[
\theta_{\text{min}} = \frac{2 \mu m}{2 \text{ cm}} = \frac{2 \times 10^{-6}}{2 \times 10^{-2}} \approx 10^{-4} \text{ rad} \\
\approx 0.34' \\
\]

Best possible condition

There is also diffraction limit (wave optics)

\[
\theta_{\text{min}} = \frac{\lambda}{d} = 1.22 \frac{500 \text{ nm}}{5 \text{ mm}} = 1.22 \cdot \frac{500 \cdot 10^{-9}}{5 \cdot 10^{-3}} = 1.22 \cdot 10^{-4} = 0.4' \\
\]

Amazingly close to physiological limit

All of above is for the best possible conditions.

So when Tycho Brahe reported 4' resolution, it was impressively good.

But not enough to measure distance to Venus, even with 1 RE base (unavailable at that time 1576-1601).