Midterm 03

Discuss relevant equations, describe your solution, show results. All Matlab code/scripts must be present in the carbon copy as well.

Make all you calculations in the S.I. units (m, kg, s).

Solar system celestial mechanics (100 points total)

We will model the motion of the relevant bodies of the solar system governed by the gravitational force. In total, we will consider 8 planets (Pluto is out), the Sun, and the Moon.

The model is simplified:

- assume that all bodies are moving in the same xy-plane
- disregard the influence of stars, asteroids and other objects

The data file with masses, initial positions, and velocities will be provided on the web. Download the file `'solar_system_data.mat'` and load it with `load 'solar_system_data.mat'`.
This will put the following variables to your workspace `body_names, xposition, yposition, vx, vy,` and `mass`. These are column vectors of corresponding data. To see which index corresponds to which celestial body refer to the `body_names` variable. For example, index 3 corresponds to Venus, since `body_names(3)` yields `'Venus'`.

Your job is to model the evolution of the many body system numerically (it is known that even a 3 body system dynamics is impossible to do analytically in a general case).

Do not hardcode the number of the celestial bodies, i.e. pull/deduce this information from the data file.

Important equations.

All you need to know is the Newton's second law (**Pay attention to the vector notation! If not sure consult with the class instructor**)

$$m_i\vec{r}_i{}'' = m_i\vec{a}_i = \vec{F}_i \tag{1}$$

here i is the index of the body, m_i is its mass, $\vec{r}_i$ is the radius vector pointing to the body, a_i is the ith body acceleration. The force ($\vec{F}_i$) acting on ith body is governed by the gravitational pull of **all** other bodies, we can write it as

$$\vec{F}_i = \sum_{j\neq i}\vec{F}_{ij} \tag{2}$$

$$\vec{F}_{ij} = G\frac{m_i m_j}{r_{ij}^3}\vec{r}_{ij} \tag{3}$$

$$\vec{r}_{ij} = \vec{r}_j - \vec{r}_i \tag{4}$$

here $G = 6.67428 \times 10^{-11}\ Nm^2/kg^2$ is the gravitational constant.

Since we consider only the xy-plane, all vectors have no z projection, i.e., $a_z = 0$, $F_z = 0, r_z = 0, v_z = 0$.

Task 1 (40 points): Calculate all bodies positions for the time span of at least one orbital period of Neptune ($\approx$ 165 years). Plot all $y_i(t)$ vs. $x_i(t)$ (i.e., orbit shapes) in the same graph for this time period.

Task 2 (10 points): Make a movie of the planets motion for the first 12 years. Mark a planet position with a circle proportional to its mass (Sun might be an exception, choose something reasonable for it) and leave a trace of previous positions with a line. Make sure that you have enough frames to show the dynamics of the system, but the movie size **must not exceed 2 MB**.

Task 3 (20 points): Have a closer look at the path of the Moon. Does it cross its own path or just wobble around the Earth's path? From a distant observer point of view, does the Moon circle around Earth? Show the representative plot leading to your conclusion.

Now, plot the Moon track (x vs. y) with respect to an observer on Earth. I.e., calculate and plot x and y with respect to the center of Earth location.

Make your final conclusion whether the Moon orbits around Earth or not.

Task 4 (20 points): Have a closer look at the Sun's orbit. Which planet has the most influence on the Sun's orbit? Try to remove the planet (assign its mass to zero) in question from the system and compare the Sun tracks. Show plots which support your conclusion. Note: you might need a quite long time span.

Note: removal of any celestial body will make the total momentum non zero, which results in a combined drift of the whole system in a certain direction. Pay attention to this drift and the wobble around it.

Task 5 (10 points):

It is well known that the presence of Neptune was first calculated by Urbain Le Verrier and then Neptune was observed very close to the predicted location in 1846. Le Verrier observed that he needed one more planet (later named Neptune) to explain the deviation of Uranus from the calculated track.

Can you show that removal of Neptune modifies the orbit of Uranus? Remember about the drift when a body is excluded from the system.

Make plots in Cartesian and polar coordinates (see a note at the end). Which one is more convincing?

Note: For some of these problems, it is more convenient to use the polar coordinate system where you convert x and y to $\rho = \sqrt{x^2 + y^2}$ and $\phi = \arctan(y/x)$. Use plots of ρ vs. ϕ, especially, for differences in one track with respect to another.