

# Multi-D optimization problem

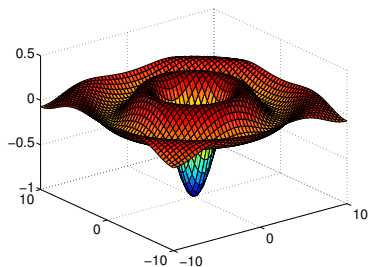
Eugeniy E. Mikhailov

The College of William & Mary



Lecture 16

# Multi-D optimization



Find  $\vec{x}$  that minimizes  $E(\vec{x})$  subject to  $g(\vec{x}) = 0, h(\vec{x}) \leq 0$

$\vec{x}$  design variables

$E(\vec{x})$  merit or objective or fitness or energy function

$g(\vec{x})$  and  $h(\vec{x})$  constrains

It is easy to see that the maximization problem is the same as minimization once  $E(\vec{x}) \rightarrow -E(\vec{x})$ .

# Solution with Matlab built in Multi-D minimization - fminsearch

```
[x, fval] = fminsearch(fun, x0)
```

**fun** handle to the multi-variable function

**x0** initial 'guess' (vector)

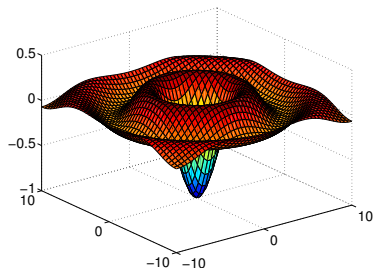
**x** optimum position vector

**fval** value of the function at the optimum

# fminsearch - usage example

## Example

```
function ret=fsample_sinc(v)
    x=v(1); y=v(2);
    r=sqrt(x^2+y^2);
    ret= -sin(r)/r;
end
```

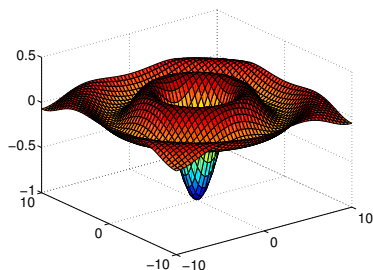


```
x0vec=[0.5, 0.5];
[xResVec,zopt]=fminsearch(@fsample_sinc, x0vec)
xResVec = [0.2852e-4, 0.1043e-4]
zopt = -1.0000
```

# It is easy to miss global minimum

## Example

```
function ret=fsample_sinc(v)
    x=v(1); y=v(2);
    r=sqrt(x^2+y^2);
    ret= -sin(r)/r;
end
```



## Example

```
x0vec=[5, 5];
[xResVec,zopt]=fminsearch(@fsample_sinc, x0vec)
xResVec = [ 5.6560    5.2621 ]
zopt = -0.1284
```

# Sample problem 1

Find the minimum of the function

$$F(x, y, z) = 2x^2 + y^2 + 2z^2 + 2xy + 1 - 2z + 2xz$$

# Sample problem 1

Find the minimum of the function

$$F(x, y, z) = 2x^2 + y^2 + 2z^2 + 2xy + 1 - 2z + 2xz$$

$$F(x, y, z) = (x + y)^2 + (x + z)^2 + (z - 1)^2$$

Minimum is  $[x, y, z] = [-1, 1, 1]$

## Sample problem 2: Potential well

Consider a 1D potential well with the following potential

$$U(x) = \begin{cases} \infty & : x < 0 \\ 0 & : x \leq L \\ U_0 & : x > L \end{cases}$$

The wave function for this problem

$$\psi(x) = \begin{cases} 0 & : x < 0 \\ \sin(kx) & : x \leq L \\ Be^{-\alpha x} & : x > L \end{cases}$$

Quantum Mechanics requires that  $k = \frac{\sqrt{2m(E-U_0)}}{\hbar}$  and  $\alpha = \frac{\sqrt{2m(U_0-E)}}{\hbar}$

We know that  $\psi$  function must be continuous and differentiable

$$\psi_{in}(L) = \psi_{out}(L)$$

$$\psi'_{in}(L) = \psi'_{out}(L)$$

Suppose that we somehow know  $k$ . What are the values for  $\alpha$  and  $B$ ?



## Sample problem 2: Potential well (cont)

Instead of solving the system of linear equations

$$\Psi_{in}(L) = \Psi_{out}(L)$$

$$\Psi'_{in}(L) = \Psi'_{out}(L)$$

Let's construct merit function

$$M(\alpha, B) = (\Psi_{in}(L) - \Psi_{out}(L))^2 + (\Psi'_{in}(L) - \Psi'_{out}(L))^2$$

## Sample problem 2: Potential well (cont)

Instead of solving the system of linear equations

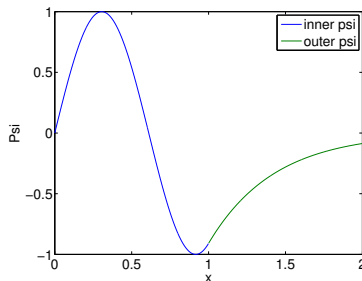
$$\Psi_{in}(L) = \Psi_{out}(L)$$

$$\Psi'_{in}(L) = \Psi'_{out}(L)$$

Let's construct merit function

$$M(\alpha, B) = (\Psi_{in}(L) - \Psi_{out}(L))^2 + (\Psi'_{in}(L) - \Psi'_{out}(L))^2$$

```
k=5.1416; L=1;
v0=fminsearch(...
    @merit_psi, [.11, 1])
v0 = 2.3531    -9.5640
%    alpha    B
```



## Sample problem 3: hanging weights

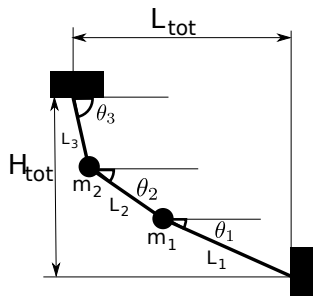
Consider masses  $m_1$  and  $m_2$  suspended by strings with length  $L_1$ ,  $L_2$ , and  $L_3$ .

Find the angles  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ .

We need to minimize potential energy subject to the length constraints. See merit function in the file 'EconstrainedSuspendedWeights.m'

For the following initial conditions

```
m1=2; m2=2;  
L1=3; L2=2; L3=3;  
Ltot=4; Htot=0;
```



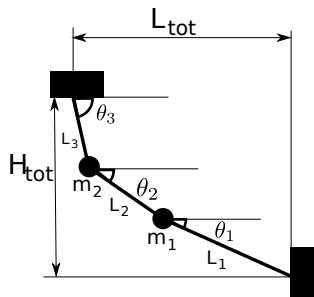
## Sample problem 3: hanging weights

Consider masses  $m_1$  and  $m_2$  suspended by strings with length  $L_1$ ,  $L_2$ , and  $L_3$ . Find the angles  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ .

We need to minimize potential energy subject to the length constraints. See merit function in the file 'EconstrainedSuspendedWeights.m'

For the following initial conditions

```
m1=2; m2=2;  
L1=3; L2=2; L3=3;  
Ltot=4; Htot=0;
```



The answer should be close to  $\theta_1 = -1.231$ ;  $\theta_2 = 0$ ;  $\theta_3 = 1.231$ ;

```
theta = fminsearch( @EconstrainedSuspendedWeights,  
    [-1, 0, -1], optimset('TolX', 1e-6))  
theta = -1.2321    -0.0044    1.2311
```