

Numerical integration

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Lecture 09

Integration problem statement

Suppose we are given function

$$f(x)$$

our goal is to find

$$\int_a^b f(x) dx$$

Not all function can be easily integrated analytically in the elementary enough form.

Example

$$\int_0^y \exp(-x^2) dx$$

So we must use numerical methods.

The Rectangle method

Recall the Riemann integral definition our goal is to find

$$\int_a^b f(x)dx = \lim_{N \rightarrow \infty} \sum_{i=1}^{N-1} f(x_i)h$$

where N is the number of points, $h = (b - a)/(N - 1)$ is the distance between points.

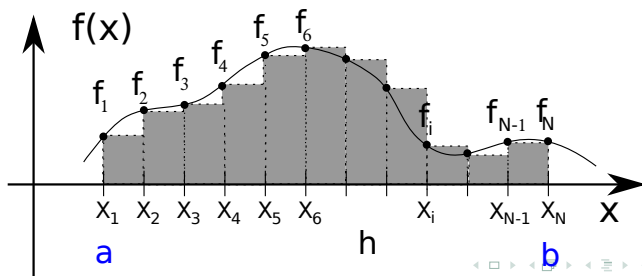
The Rectangle method continued

Riemann rule is almost direct recipe.

Rectangle method

$$\int_a^b f(x) dx \approx \sum_{i=1}^{N-1} f(x_i)h, \quad \text{where } h = \frac{b-a}{N-1} \text{ and } x_i = a + (i-1)h$$

We just need to remember about round off errors so h should not be too small or equivalently N should not be too big.

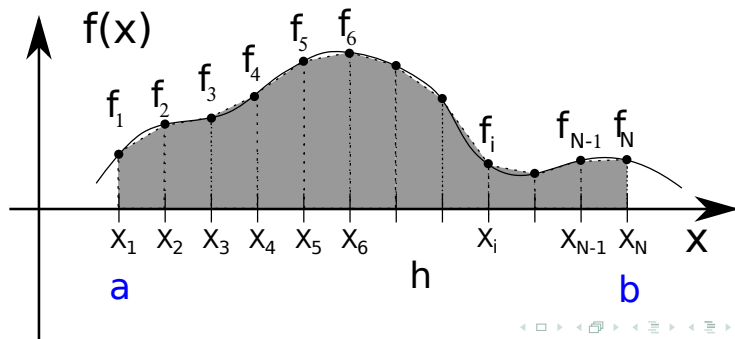


Trapezoidal method

Trapezoidal method

$$\int_a^b f(x) dx \approx h \times \left(\frac{1}{2} f_1 + f_2 + f_3 + \dots + f_{N-2} + f_{N-1} + \frac{1}{2} f_N \right) = h \sum_{i=1}^N f(x_i) w_i,$$

$$\text{where } h = \frac{b-a}{N-1} \text{ and } x_i = a + (i-1)h$$



Simpson method

Simpson method - approximation by parabolas

$$\int_a^b f(x) dx \approx h \frac{1}{3} \times (f_1 + 4f_2 + 2f_3 + 4f_4 + \dots + 2f_{N-2} + 4f_{N-1} + f_N)$$
$$= h \sum_{i=1}^N f(x_i) w_i, \text{ where } h = \frac{b-a}{N-1} \text{ and } x_i = a + (i-1)h$$

note that N must be in special form $N = 2k + 1$ i.e. odd

