Homework 10

Problem 1 (5 points)

Have a look at the particular realization of the N point forward DFT

$$C_n = \sum_{k=1}^{N} y_k exp(-i2\pi(k-1)n/N)$$

The normalization coefficient is omitted.

Analytically prove that the forward discrete Fourier transform is periodic, i.e., $c_{n+N} = c_n$. Note: recall that $exp(\pm i2\pi) = 1$.

Does this also prove that $c_{-n} = c_{N-n}$?

Problem 2 (5 points)

Use proof for the previous problem relationships and show that the following relationship holds for any sample set which has only real values (i.e., no complex part)

$$c_n = c^*_{N-n}$$

Where * depicts the complex conjugation.

Problem 3 (15 points)

Load the data from the file 'hw_data_for_filter.dat' provided at the class web page. It contains a table with y vs t data points (the first column holds the time, the second holds y). These data points are taken with the same sampling rate.

Subproblem: 3a (2 points)

What is the sampling rate?

Subproblem: 3b (3 points)

Calculate forward DFT of the data (use Matlab built-ins) and find which 2 frequency components of the spectrum (measured in Hz not rad$^-$1) are the largest. Note, I refer to the real frequency of the sin or cos component, i.e., only positive frequencies.

Subproblem: 3c (2 points)

What is the largest possible frequency (in Hz) in this data set which we can scientifically discuss?

Subproblem: 3d (5 points)

Consider everything else but above 2 components of the DFT as noise. Construct a low-pass filter which will pass these two components. Plot the filter frequency representation (positive and negative frequency). Explain your choice of the filter and its parameters.

Subproblem: 3e (3 points)

Apply the filter to the data Fourier representation and calculate the inverse DFT. Plot the resulting filtered data representation and raw data points in the same plot. Does your filter completely get rid of noise? If not why is it so?