Homework 07

General comments:

- Do not forget to run some test cases.

Problem 1 (2 points)
Prove (analytically) that the golden section algorithm R is still given by the same expression even if we need to choose $a' = x_1$ and $b' = b$.

Problem 2 (3 points)
Assume that the initial spacing between initial bracket points is h. Estimate (analytically) how many iterations it requires to narrow the bracket to the $10^{-9} \times h$ space.

Problem 3 (5 points)
Implement the golden section algorithm. Do not forget to check your code with simple test cases. Find where the function $E1(x) = x^2 - 100 * (1 - \exp(-x))$ has a minimum.

Problem 4 (5 points)
For the coin flipping game described on lecture 14, find the optimal (maximizing your gain) betting fraction using the golden section algorithm and Monte Carlo simulation. Feel free to reuse provided complimentary codes.

Note: you need a lot of game runs (at least a 1000) to have reasonably small uncertainty for the merit function evaluations. I would suggest to average at least 1000 runs with length of 100 coin flips each.

Problem 5 (5 points)
Find the point where function

$$F(x, y, z, w, u) = (x - 3)^2 + (y - 1)^4 + (u - z)^2 + (u - 2 * w)^2 + (u - 6)^2 + 12$$

has a minimum. What is the value of $F(x, y, z, w, u)$ at this point?