Power dissipation

Recall that power dissipated by element is

\[ P = VI \]

where \( V \) and \( I \) are real.

Since we use a substitute

\[ V \cos(\omega t) \rightarrow Ve^{j\omega t} \]
\[ I \cos(\omega t) \rightarrow Ie^{j\omega t} \]

we need to write

\[ P = \text{Re}(V)\text{Re}(I) \]

Recall the Ohm's law

\[ V = ZI \]

Power dissipation by a reactive element

**Theorem**

Average power dissipated by a reactive element (C or L) is 0

Let's use as example an inductor.

\[ Z_L = j\omega L = e^{j\omega L} I_L = I_pe^{j\omega t} \]
\[ V_L = Z_L I_L = e^{j\omega L} I_L \omega L e^{j(\omega t + \frac{\pi}{2})} \]
\[ \text{Re}(I_L) = I_p \cos(\omega t), \text{Re}(V_L) = -\omega I_p L \sin(\omega t) \]

Thus average power dissipated by the inductor

\[ P = \int_0^T \text{Re}(I_L)\text{Re}(V_L)dt = -\int_0^T I_p \cos(\omega t)\omega I_p L \sin(\omega t)dt \]
\[ P = -\omega I_p^2 L \int_0^T \cos(\omega t) \sin(\omega t) dt = \frac{\omega I_p^2 L}{2} \int_0^T \frac{1}{2} \sin(2\omega t) dt = 0 \]

Fourier transform

If function \( f(t) \) goes to zero at \( \pm \infty \) then \( \tilde{f}(\omega) \) exists such as

\[ f(t) = \int_{-\infty}^{\infty} \tilde{f}(\omega)e^{j\omega t}d\omega \]
Transfer function

Time domain

\[ V_{in}(t) \rightarrow H(t) \rightarrow V_{out}(t) \]

Frequency domain

\[ V_{in}(\omega) \rightarrow G(\omega) \rightarrow V_{out}(\omega) \]

Definition

\[ G(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = |G(\omega)|e^{i\phi(\omega)} \]

Often used values of \( G \) in dB

\[ dB = 20 \log_{10}(|G(\omega)|) \]

Simple example: RC low-pass filter

\[ G(\omega) = \frac{1}{1 + i\frac{\omega}{\omega_{3dB}}} \]

Defining \( \omega_{3dB} = \frac{1}{RC} \)

\[ G(\omega) = \frac{1}{1 + i\frac{\omega}{\omega_{3dB}}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_{3dB}}\right)^2}} e^{i\phi}, \phi = \arctan\left(-\frac{\omega}{\omega_{3dB}}\right) \]

Note

\[ |G(\omega = \omega_{3dB})| = 20 \log_{10} \left(\frac{1}{\sqrt{1 + 1}}\right) = 20 \log_{10} \left(\frac{1}{\sqrt{2}}\right) = -3dB \]

RC low-pass filter at \( \omega = 1/RC \)

Signal vs time

Lissajous figure

RC low-pass filter at \( \omega = 1/RC \)

Signal vs time

Lissajous figure
RC low-pass filter at $\omega = 10/RC$

Signal vs time

Lissajous figure

Bode plots

Definition
Bode plot: plots of magnitude and phase of the transfer function, where $|G|$ is often plotted in dB

$$G(\omega) = \frac{1}{1 + i\omega RC}$$

RC high-pass filter

$$G(\omega) = \frac{\omega_3 dB}{1 + i\omega RC}$$

with $\omega_3 dB = \frac{1}{RC}$

RL filters

RL low-pass filter

$$G(\omega) = \frac{R}{R + i\omega L} \approx \frac{R}{\omega L}$$

RL high-pass filter

$$G(\omega) = \frac{i\omega L}{R + i\omega L}$$
Filters chain

Technically next stage loads the previous and it is quite hard to calculate total transfer function. However if we use rule of 10 to avoid overloading the previous filter. Every next stage resistor \( R_{i+1} > 10 R_i \) we can approximate

\[
G_t(\omega) \approx G_1(\omega) G_2(\omega) G_3(\omega) \cdots G_n(\omega)
\]

Example band pass filter

RLC band pass filter

Notch filter - Band stop filter