Full network analysis.

Eugeniy E. Mikhailov

The College of William & Mary

Lecture 02

Kirchhoff's Current Law

Kirchhoff's Current Law

The algebraic sum of currents entering and exiting a node equals zero

Convention (quite arbitrary): currents going into the nodes are positive, the ones which go out of the node are negative.

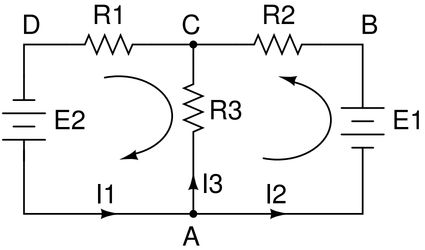
Kirchhoff's Voltage Law

The algebraic sum of all voltage changes (aka voltage drops) in a loop equals zero

Notes:

- chose a direction along which you travel a network. If you go over a resistor and current runs the same way then voltage change is negative, otherwise its positive.
- If you go over a voltage source from negative terminal to positive the voltage change is positive, otherwise negative.

Example



our goal is to find $I1$, $I2$, and $I3$

We chose $V_A = 0$

For node A:

$$I1 - I2 - I3 = 0 \qquad (1)$$

We need 2 more independent equations.

For this we will go over 2 small loops as indicated by arrows.

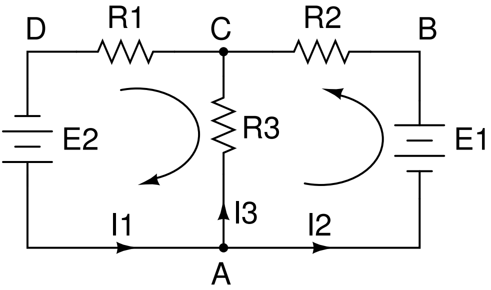
$$V_{DC} + V_{CA} + V_{AD} = 0 \qquad (2)$$

$$V_{AB} + V_{BC} + V_{CA} = 0 \qquad (3)$$

Notice:

$V_{AB} = +E1$, $V_{BC} = -R2 \times I2$, $V_{CA} = +R3 \times I3$,
$V_{DC} = +R1 \times I1$, $V_{AD} = -E2$.

Example (continued)



$$I1 - I2 - I3 = 0$$
$$V_{DC} + V_{CA} + V_{AD} = 0$$
$$V_{AB} + V_{BC} + V_{CA} = 0$$

$\rightarrow$

$$I1 - I2 - I3 = 0$$
$$R1 \times I1 + R3 \times I3 - E2 = 0$$
$$E1 - R2 \times I2 + R3 \times I3 = 0$$

Maple as the math aid

$$solve(\{I1 - I2 - I3 = 0, E1 - R2 \cdot I2 + R3 \cdot I3 = 0, R1 \cdot I1 + R3 \cdot I3 - E2 = 0\}, [I1, I2, I3])$$

$$\left[\left[I1 = \frac{R3\,E1 + R3\,E2 + R2\,E2}{R3\,R1 + R1\,R2 + R3\,R2}, I2 = \frac{R3\,E1 + R3\,E2 + R1\,E1}{R3\,R1 + R1\,R2 + R3\,R2}, I3 = -\frac{R1\,E1 - R2\,E2}{R3\,R1 + R1\,R2 + R3\,R2}\right]\right] \quad (1$$

Maple as the math aid (continued)

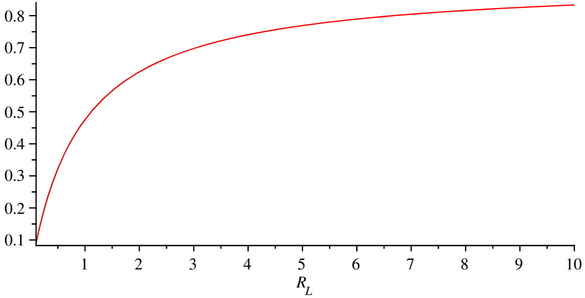
$$V_{out} := \frac{V_{\mathbf{in}} \cdot R2}{(R1 + R2)} \cdot \frac{R_L}{R_L + \frac{R1 \cdot R2}{R1 + R2}}$$

$$\frac{V_{\mathbf{in}}\, R2\, R_L}{(R1 + R2)\left(R_L + \frac{R1\, R2}{R1 + R2}\right)} \qquad \mathbf{(1)}$$

$V_{\mathbf{in}} := 10;\ R1 := 10;\ R2 := 1;$

$$10$$
$$10$$
$$1 \qquad \mathbf{(2)}$$

$plot\left(V_{out}, R_L = .1\,..10\right)$



Thévenin's and Norton's equivalent circuit theorems

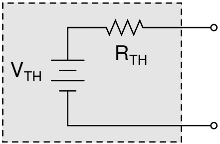
Any combination of voltage sources, current sources and resistors with two terminals is electrically equivalent

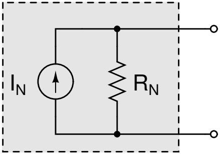
Thévenin's theorem

to a single voltage source V_{TH} and a single series resistor R_{TH} connected in series.

Norton's theorem

to a single current source I_N and a single series resistor R_N connected in parallel.





Note above circuits are equivalent to each other when

$$R_{TH} = R_N \text{ and } I_N = V_{TH}/R_{TH}$$