

Chapter 1

Optical Interferometry

Experiment objectives: Assemble and align Michelson and Fabry-Perot interferometers, calibrate them using a laser of known wavelength, and then use them characterize the bright yellow emission line in sodium (Na).

Introduction

Optical interferometers are the instruments that rely on interference of two or more superimposed reflections of the input laser beam. These are one of the most common optical tools, and are used for precision measurements, surface diagnostics, astrophysics, seismology, quantum information, etc. There are many configurations of optical interferometers, and in this lab you will become familiar with two of the more common setups.

The **Michelson interferometer**, shown in Fig. 1.1, is based on the interference of two beams: the initial light is split into two arms on a beam splitter, and then these resulting beams are reflected and recombined on the same beamsplitter again. The difference in optical paths in the two arms leads to a changing relative phase of two beams, so when overlapped the two light fields will interfere constructively or destructively. Such an interferometer was first used by Michelson and Morley in 1887 to determine that electromagnetic waves propagate in vacuum, giving the first strong evidence against the theory of a *luminiferous aether* (a fictitious medium for light wave propagation) and providing insight into the true nature of electromagnetic radiation. Michelson interferometers are widely used in many areas of physics and engineering. At the end of this writeup we describe LIGO, the world's largest Michelson interferometer, designed to measure the gravitational waves and thus test general relativity.

Figure 1.1 shows the traditional setting for a Michelson interferometer. A beamsplitter (a glass plate which is partially silver-coated on the front surface and angled at 45 degrees) splits the laser beam into two parts of equal amplitude. One beam (that was initially transmitted by the beamsplitter) travels to a fixed mirror M_1 and back again. One-half of this amplitude is then reflected from the partially-silvered surface and directed at 90 degrees toward the observer (you will use a viewing screen). At the same time the second beam (reflected by

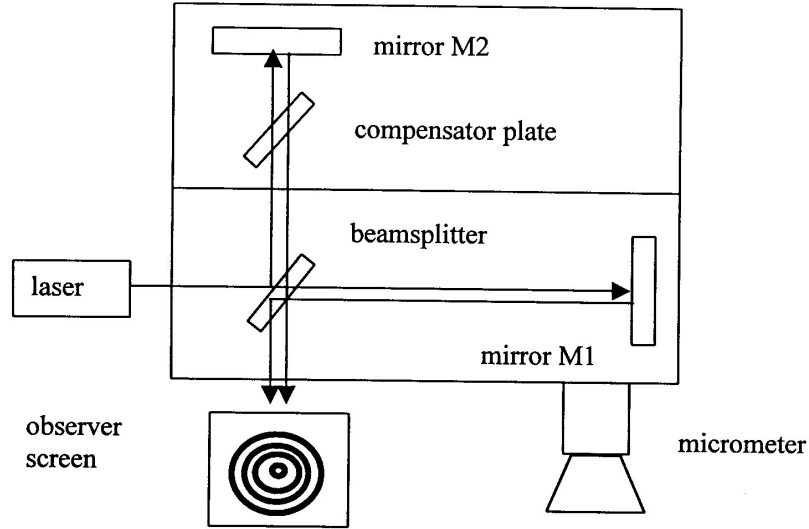


Figure 1.1: A Michelson Interferometer setup.

the beamsplitter) travels at 90 degrees toward mirror M_2 and back. Since this beam never travels through the glass beamsplitter plate, its optical path length is shorter than for the first beam. To compensate for that, it passes twice through a clear glass plate called the compensator plate, that has the same thickness. At the beamsplitter one-half of this light is transmitted to an observer, overlapping with the first beam, and the total amplitude of the light at the screen is a combination of amplitude of the two beams:

$$\mathbf{E}_{total} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{1}{2}\mathbf{E}_0 + \frac{1}{2}\mathbf{E}_0 \cos(k\Delta l) \quad (1.1)$$

Here $k\Delta l$ is a phase shift (“optical delay”) between the two wavefronts caused by the difference in optical path lengths for the two beams Δl , $k = 2\pi n/\lambda$ is the wave number, λ is the wavelength of light in vacuum, and n is the refractive index of the optical medium (in our case - air).

Mirror M_2 is mounted on a precision traveling platform. Imagine that we adjust its position (by turning the micrometer screw) such that the distance traversed on both arms is exactly identical. Because the thickness of the compensator plate and the beamsplitter are the same, both wavefronts pass through the same amount of glass and air, so the path length of the light beams in both interferometer arms will be exactly the same. Therefore, the two fields will arrive in phase to the observer, and their amplitudes will add up constructively, producing a bright spot on the viewing screen. If now you turn the micrometer to offset the length of one arm by a half of light wavelength, $\Delta l = \lambda/2$, they will acquire a relative phase of π , and total destructive interference will occur:

$$\mathbf{E}_1 + \mathbf{E}_2 = 0 \quad \text{or} \quad \mathbf{E}_1 = -\mathbf{E}_2.$$

It is easy to see that constructive interference happens when the difference between path lengths in the two interferometer arms is equal to the integer number of wavelengths $\Delta l = m\lambda$, and destructive interference corresponds to a half-integer number of wavelengths $\Delta l = (m+1/2)\lambda$ (here m is an integer number). Since the wavelength of light is small (600–700 nm for a red laser), Michelson interferometers are able to measure distance variation with very good precision.

In **Fabry-Perot configuration** the input light field bounces between two closely spaced partially reflecting surfaces, creating a large number of reflections. Interference of these multiple beams produces sharp spikes in the transmission for certain light frequencies. Thanks to the large number of interfering rays, this type of interferometer has extremely high resolution, much better than a Michelson interferometer. For that reason Fabry-Perot interferometers are widely used in telecommunications, lasers and spectroscopy to control and measure the wavelengths of light. In this experiment we will take advantage of the high spectral resolution of the Fabry-Perot interferometer to resolve two very closely-spaced emission lines in Na spectra by observing changes in overlapping interference fringes from the two lines.

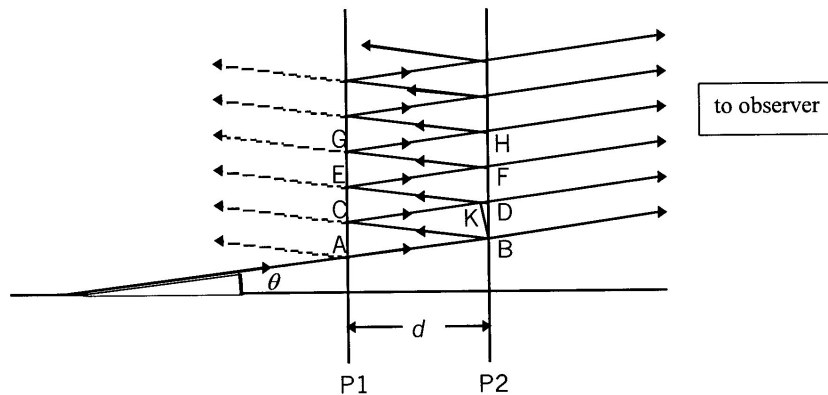


Figure 1.2: Sequence of Reflection and Transmission for a ray arriving at the treated inner surfaces P_1 & P_2 .

A Fabry-Perot interferometer consists of two parallel glass plates, flat to better than $1/4$ of an optical wavelength λ , and coated on the inner surfaces with a partially transmitting metallic layer. Such two-mirror arrangement is normally called an *optical cavity*. The light in a cavity by definition bounces back and forth many times before escaping; the idea of such a cavity is crucial for the construction of a laser. Any light transmitted through such cavity is a product of interference between beams transmitted at each bounce as diagrammed in Figure 1.2. When the incident ray arrives at interface point A, a fraction t is transmitted and the remaining fraction r is reflected, such that $t + r = 1$ (this assumes no light is lost inside the cavity). The same thing happens at each of the points A, B, C, D, E, F, G, H . . . ,

splitting the initial ray into parallel rays AB, CD, EF, GH , etc. Between adjacent ray pairs, say AB and CD , there is a path difference of :

$$\delta = BC + CK \quad (1.2)$$

where BK is normal to CD . In a development similar to that used for the Michelson interferometer, you can show that:

$$\delta = 2d \cos \theta \quad (1.3)$$

If this path difference produces constructive interference, then δ is some integer multiple of λ , namely,

$$m\lambda = 2d \cos \theta \quad (1.4)$$

This applies equally to ray pairs CD and EF , EF and GH , etc, so that all parallel rays to the right of $P2$ will constructively interfere with one another when brought together.

Issues of intensity of fringes & contrast between fringes and dark background are addressed in Melissinos, *Experiments in Modern Physics*, pp.309-312.

Laser Safety

Never look directly at the laser beam! Align the laser so that it is not at eye level. Even a weak laser beam can be dangerous for your eyes.

Alignment of Michelson interferometer

Equipment needed: Pasco precision interferometry kit, a laser, Na lamp, adjustable-height platform (or a few magazines or books).

To simplify the alignment of a Michelson interferometer, it is convenient to work with diverging optical beams. In this case an interference pattern will look like a set of concentric bright and dark circles, since the components of the diverging beam travel at slightly different angles, and therefore acquire different phase, as illustrated in Figure 1.3. Suppose that the actual path length difference between two arms is d . Then the path length difference for two off-axis rays arriving at the observer is $\Delta l = a + b$ where $a = d / \cos \theta$ and $b = a \cos 2\theta$:

$$\Delta l = \frac{d}{\cos \theta} + \frac{d}{\cos \theta} \cos 2\theta \quad (1.5)$$

Recalling that $\cos 2\theta = 2(\cos \theta)^2 - 1$, we obtain $\Delta l = 2d \cos \theta$. The two rays interfere constructively for any angle θ_c for which $\Delta l = 2d \cos \theta = m\lambda$ (m =integer); at the same time, two beams traveling at the angle θ_d interfere destructively when $\Delta l = 2d \cos \theta = (m + 1/2)\lambda$ (m =integer). Because of the symmetry about the normal direction to the mirrors, this will mean that interference (bright and dark fringes) appears in a circular shape. If the fringes are not circular, it means simply that the mirrors are not parallel, and additional alignment of the interferometer is required.

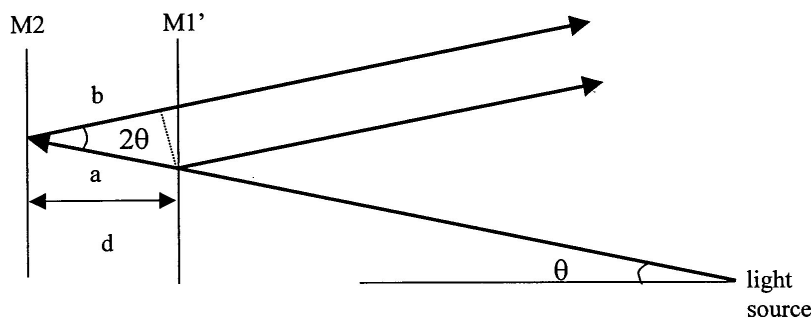


Figure 1.3: Explanation of circular fringes. Notice that to simplify the figure we have “unfold” the interferometer by neglecting the reflections on the beamsplitter.

When the path length difference Δl is varied by moving one of the mirrors using the micrometer, the fringes appear to “move”. As the micrometer is turned, the condition for constructive and destructive interference is alternately satisfied at any given angle. If we fix our eyes on one particular spot and count, for example, how many bright fringes pass that spot as we move mirror M_2 by a known distance, we can determine the wavelength of light in the media using the condition for constructive interference, $\Delta l = 2d \cos \theta = m\lambda$.

For simplicity, we might concentrate on the center of the fringe bull’s eye at $\theta = 0$. The equation above for constructive interference then reduces to $2\Delta l = m\lambda$ ($m = \text{integer}$). If X_1 is the initial position of the mirror M_2 (as measured on the micrometer) and X_2 is the final position after a number of fringes δm has been counted, we have $2(X_2 - X_1) = \lambda\Delta m$. Then the laser wavelength, λ , is then given as:

$$\lambda = 2(X_2 - X_1)/\Delta m. \quad (1.6)$$

Set up the interferometer as shown in Figure 1.1 using components from the PASCO interferometry kit. The mirrors $M_{1,2}$ are, correspondingly, a movable and an adjustable mirror from the kit. Align the interferometer with a laser beam. Adjust the beam so that it is impinging on the beamsplitter and on the viewing screen. Try to make the beams to hit near the center of all the optics, including both mirrors, the compensator plate and beam splitter. The interferometer has leveling legs which can be adjusted. Align the beams such that they overlap on the viewing screen, and so that the reflected beam is directed back into the laser. This can be tricky to get right the first time. Be patient, make small changes, think about what you are doing, and get some help from the instructor and TA.

Once the interferometer is aligned, insert a convex lens ($f = 18 \text{ mm}$ works well) after the laser to spread out the beam (ideally the laser beam should be pass through the center of the lens to preserve alignment). Adjust the adjustable mirror slightly until you see the interference fringes in the screen. Continue to make small adjustments until you see a clear bull’s eye circular pattern. *A word of caution: sometimes dust on a mirror or imperfections on optical surfaces may produce similar intensity patterns. True interference disappears if you block one arm of the interferometer. Try it!*

Note: before starting the measurements, make sure you understand how to read the micrometer properly!



(a) Reading = $211 \mu\text{m}$

(b) Reading = $345 \mu\text{m}$

(c) Reading = $166 \mu\text{m}$

Figure 1.4: Micrometer readings. The coarse division equals to $100 \mu\text{m}$, and smallest division on the rotary dial is $1 \mu\text{m}$ (same as 1 micron). The final measurements is the sum of two.

Wavelength measurements using Michelson interferometer

Calibration of the interferometer

Record the initial reading on the micrometer. Focus on the central fringe and begin turning the micrometer. You will see that the fringes move. For example, the central spot will change from bright to dark to bright again, that is counted as one fringe. A good method: pick out a reference line on the screen and then softly count fringes as they pass the point. Count a total of about $\Delta m = 50$ fringes and record the new reading on the micrometer.

Each lab partner should make at least two independent measurements, starting from different initial positions of the micrometer. For each trial, approximately 50 fringes should be accurately counted and tabulated with the initial X_1 and final X_2 micrometer settings. Do this at least five times (e.g., 5×50 fringes). Consider moving the mirror both forward and backward. Make sure that the difference $X_2 - X_1$ is consistent between all the measurements. Calculate the average value of the micrometer readings $\langle X_2 - X_1 \rangle$.

Experimental tips

1. Avoid touching the face of the front-surface mirrors, the beamsplitter, and any other optical elements!
2. The person turning the micrometer should also do the counting of fringes. It can be easier to count them in bunches of 5 or 10 (*i.e.* 100 fringes = 10 bunches of 10 fringes).
3. Use a reference point or line and count fringes as they pass.

4. Before the initial position X_1 is read make sure that the micrometer has engaged the drive screw (There can be a problem with “backlash”). Just turn it randomly before counting.
5. Avoid hitting the table which can cause a sudden jump in the number of fringes.

Measurement of the Na lamp wavelength

A calibrated Michelson interferometer can be used as a **wavemeter** to determine the wavelength of different light sources. In this experiment you will use it to measure the wavelength of strong yellow sodium fluorescent light, produced by the discharge lamp.¹

Without changing the alignment of the interferometer (i.e. without touching any mirrors), remove the focusing lens and carefully place the interferometer assembly on top of an adjustable-height platform such that it is at the same level as the output of the lamp. Since the light power in this case is much weaker than for a laser, you won't be able to use the viewing screen. You will have to observe the interference looking directly to the output beam - unlike laser radiation, the spontaneous emission of a discharge is not dangerous². However, your eyes will get tired quickly! Placing a diffuser plate in front of the lamp will make the observations easier. Since the interferometer is already aligned, you should see the interference picture. Make small adjustments to the adjustable mirror to make sure you see the center of the bull's eye.

Repeat the same measurements as in the previous part by moving the mirror and counting the number of fringes. Each lab partner should make at least two independent measurements, recording initial and final position of the micrometer, and you should do at least five trials. Calculate the wavelength of the Na light for each trial. Then calculate the average value and its experimental uncertainty. Compare with the expected value of 589 nm.

In reality, the Na discharge lamp produces a doublet - two spectral lines that are very close to each other: 589 nm and 589.59 nm. Do you think your Michelson interferometer can resolve this small difference? Hint: the answer is no - we will use a Fabry-Perot interferometer for that task.

Alignment of the Fabry-Perot interferometer

Disassemble the Michelson Interferometer, and assemble the Fabry-Perot interferometer as shown in Figure 1.5. First, place the viewing screen behind the two partially-reflecting mirrors ($P1$ and $P2$), and adjust the mirrors such that the multiple reflections on the screen

¹Actually, sodium might be a better calibration source than a HeNe laser, since it has well known lines, whereas a HeNe can lase at different wavelengths. Perhaps an even better calibration source might be a line from the Hydrogen Balmer series, which can be calculated from the Standard Model.

²In the “old days” beams in high energy physics were aligned using a similar technique. An experimenter would close his eyes and then put his head in a collimated particle beam. Cerenkov radiation caused by particles traversing the experimenter's eyeball is visible as a blue glow or flashes. This is dangerous but various people claim to have done it... when a radiation safety officer isn't around.

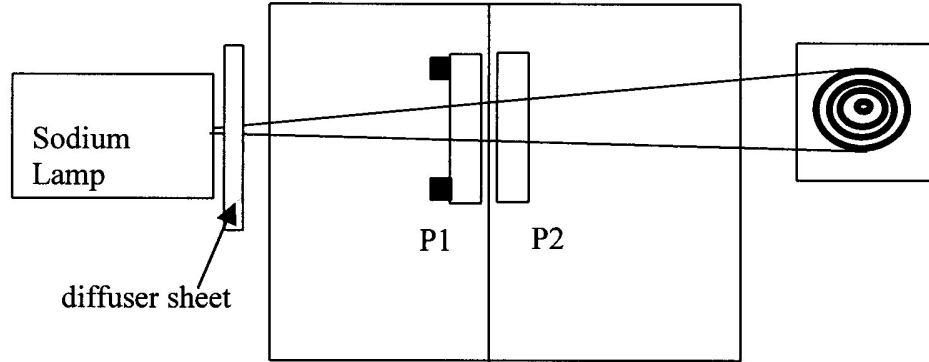


Figure 1.5: The Fabry-Perot Interferometer. For initial alignment the laser and the convex lens are used instead of the Na lamp.

overlap. Then place a convex lens after the laser to spread out the beam, and make small adjustments until you see the concentric circles. Is there any difference between the thickness of the bright lines for the two different interferometers? Why?

Loosen the screw that mounts the movable mirror and change the distance between the mirrors. Realign the interferometer again, and comment on the difference in the interference picture. Can you explain it?

Align the interferometer one more time such that the distance between the two mirrors is 1.0 – 1.5 mm, but make sure the mirrors do not touch!

Sodium doublet measurements

1. Turn off the laser, remove the viewing screen and the lens, and place the interferometer on the adjustable-height platform, or alternatively place the Na lamp on its side and plan to adjust its height with books or magazines. With the diffuser sheet in front of the lamp, check that you see the interference fringes when you look directly at the lamp through the interferometer. If necessary, adjust the knobs on the adjustable mirror to get the best fringe pattern.
2. Because the Na emission consists of two lights at two close wavelengths, the interference picture consists of two sets of rings, one corresponding to fringes of λ_1 , the other to those for λ_2 . Move the mirror back and forth (by rotating the micrometer) to identify two sets of rings. Notice that they move at slightly different rates (due to the wavelength difference).
3. Seek the START condition illustrated in Fig.(1.6), such that all bright fringes are evenly spaced. Note that alternate fringes may be of somewhat different intensities. Practice

going through the fringe conditions as shown in Fig.(1.6) by turning the micrometer and viewing the relative movement of fringes. Do not be surprised if you have to move the micrometer quite a bit to return to the original condition again.

4. Turn the micrometer close to zero reading, and then find a place on the micrometer (d_1) where you have the “START” condition for fringes shown in Fig.(1.6). Now advance the micrometer rapidly while viewing the fringe pattern (NO COUNTING OF FRINGES IS REQUIRED). Note how the fringes of one intensity are moving to overtake those of the other intensity (in the manner of Fig.(1.6)). Keep turning until the “STOP” pattern is achieved (the same condition you started with). Record the micrometer reading as d_2 .
5. Each lab partner should repeat this measurement at least one time, and each group should have at least three independent measurements.
6. Make sure to tabulate all the data taken by your group.

We chose the “START” condition (the equally spaced two sets of rings) such that for the given distance between the two mirrors, d_1 , the bright fringes of λ_1 occur at the points of destructive interference for λ_2 . Thus, the bull’s eye center ($\theta = 0$) we can write this down as:

$$2d_1 = m_1\lambda_1 = \left(m_1 + n + \frac{1}{2}\right) \lambda_2 \quad (1.7)$$

Here the integer n accounts for the fact that $\lambda_1 > \lambda_2$, and the $1/2$ for the condition of destructive interference for λ_2 at the center. The “STOP” condition corresponds to the similar situation, but the net action of advancing by many fringes has been to increment the fringe count of λ_2 by one more than that of λ_1 :

$$2d_2 = m_2\lambda_1 = \left(m_2 + n + \frac{3}{2}\right) \lambda_2 \quad (1.8)$$

Try to estimate your uncertainty in identifying the START and STOP positions by turning the micrometer back and forth, identifying the points at which you can begin to see doublet, rather than equally spaced, lines. The variance from multiple measurements should agree, at least approximately, with this estimate.

Subtracting the two interference equations, and solving for the distance traveled by the mirror $d_2 - d_1$ we obtain:

$$2(d_2 - d_1) = \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)} \quad (1.9)$$

Solving this for $\Delta\lambda = \lambda_1 - \lambda_2$, and accepting as valid the approximation that $\lambda_1\lambda_2 \approx \lambda^2$ (where λ is the average of λ_1 and $\lambda_2 \approx 589.26$ nm), we obtain:

$$\Delta\lambda \approx \frac{\lambda^2}{2(d_2 - d_1)} \quad (1.10)$$

Use this equation and your experimental measurements to calculate average value of Na doublet splitting and its standard deviation. Compare your result with the established value of $\Delta\lambda_{Na} = 0.598 \text{ nm}$.

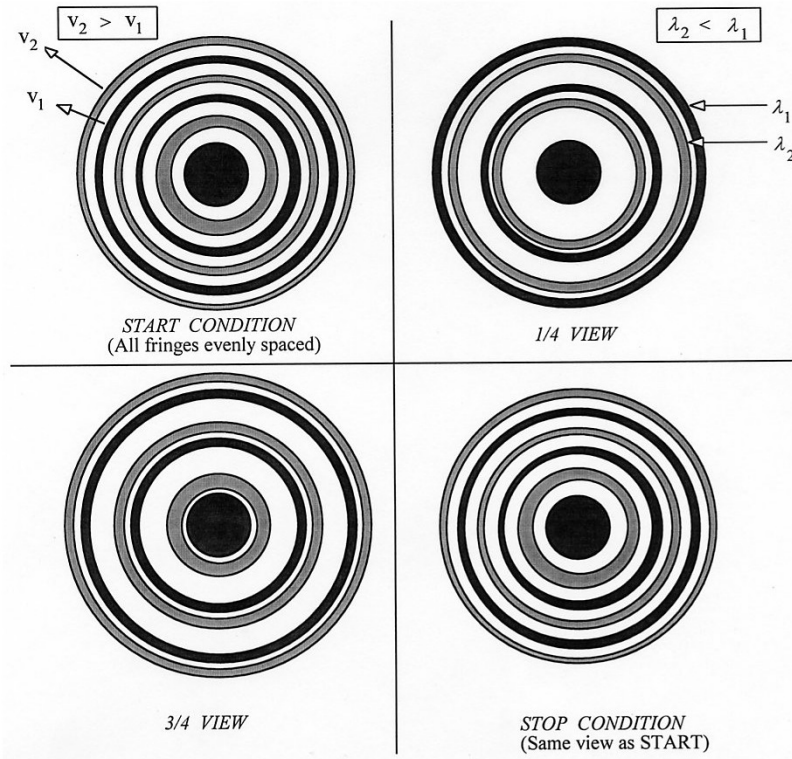
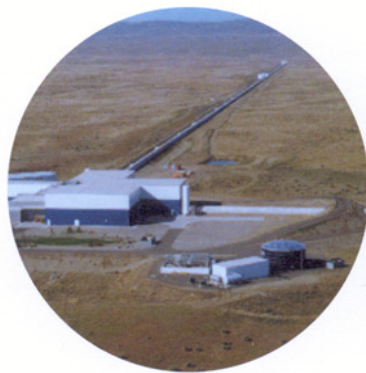


Figure 1.6: The sequence of fringe patterns encountered in the course of the measurements. Note false colors: in your experiment the background is black, and both sets of rings are bright yellow.

Detection of Gravitational Waves

A Michelson interferometer can help to test the theory of relativity! Gravity waves, predicted by the theory of relativity, are ripples in the fabric of space and time produced by violent events in the distant universe, such as the collision of two black holes. Gravitational waves are emitted by accelerating masses much as electromagnetic waves are produced by accelerating charges, and often travel to Earth. The only indirect evidence for these waves has been in the observation of the rotation of a binary pulsar (for which the 1993 Nobel Prize was awarded). Laser Interferometry Gravitational-wave Observatory (LIGO) sets the



Hanford, Washington



Livingston, Louisiana

Figure 1.7: For more details see <http://www.ligo.caltech.edu/>

ambitious goal of the direct detection of a gravitational wave. The measuring tool in this project is a giant Michelson interferometer. Two mirrors hang 2.5 mi apart, forming one “arm” of the interferometer, and two more mirrors make a second arm perpendicular to the first. Laser light enters the arms through a beam splitter located at the corner of the L, dividing the light between the arms. The light is allowed to bounce between the mirrors repeatedly before it returns to the beam splitter. If the two arms have identical lengths, then interference between the light beams returning to the beam splitter will direct all of the light back toward the laser. But if there is any difference between the lengths of the two arms, some light will travel to where it can be recorded by a photodetector.

The space-time ripples cause the distance measured by a light beam to change as the gravitational wave passes by. These changes are minute: just 10^{-16} centimeters, or one-hundred-millionth the diameter of a hydrogen atom over the 2.5 mile length of the arm. Yet, they are enough to change the amount of light falling on the photodetector, which produces a signal defining how the light falling on changes over time. LIGO requires at least two widely separated detectors, operated in unison, to rule out false signals and confirm that a gravitational wave has passed through the earth. Three interferometers were built for LIGO – two near Richland, Washington, and the other near Baton Rouge, Louisiana.