Lecture 23

Proto star formation

The Jeans Criterion

Gas cloud with random velocity distribution so we will use $\tau$ as a measure of typical speed/energy.

Assuming $\tau = \text{const}$ and spherical:

$$ U = - \int_{0}^{R_c} G \frac{M \rho \pi r^2 dr}{r} $$

$$ = - \int_{0}^{R_c} G \frac{\rho \pi r^3}{2} \cdot \frac{4 \pi r^2 dr}{r} $$

$$ = - G \rho \frac{4 \pi}{3} \int_{0}^{R} r^4 dr = - G \rho \frac{4 \pi}{3} \frac{4 \pi}{5} R_c^5 $$

$$ U = - G \frac{\rho}{5} \frac{M_c^2}{R_c} $$

Kinetic energy $K = \frac{3}{2} NkT = \frac{3}{2} \left( \frac{M_c}{M_{\text{MW}}} \right) NkT$

Viral theorem $2K + U = 0$

If $2K + U > 0$ cloud will expand (hot cloud)
So for cloud collapse we need

\[
2k < - U \\
\geq \frac{3}{5} \frac{M_c}{\mu_m m} K T < \frac{3}{5} \frac{G M_c^2}{R_c} \\
R_c = \left( \frac{M_c}{\xi_0} \right)^{1/3} \text{ initial density}
\]

\[
M_c > M_j = \frac{5K T}{G M M_H} \\
\frac{K T}{\mu_m m} < \frac{G M_c}{c^2 M_c^{1/3}} \left( \frac{4\pi}{3} \phi_0 \right)^{1/3} = M_c^{2/3} \left( \frac{4\pi}{3} \phi_0 \right)^{1/3}
\]

\[
M_c > \left( \frac{5K T}{G M M_H} \right)^{3/2} \left( \frac{3}{4\pi} \phi_0 \right)^{1/2} \equiv M_j
\]

Collapse condition

Seems like a paradox $M_j \sim \frac{1}{\sqrt{5}}$
while we might think the dense the cloud the more attraction but it also means larger number of particles $M$ increasing $K$

Using eq. 1 we can state

\[
R_j = \left( \frac{3}{4\pi} \left( \frac{5K T}{G M M_H} \right)^{3/2} \left( \frac{3}{4\pi} \phi_0 \right) \right)^{1/3} = \frac{1}{\sqrt{5}}
\]

\[
R_c > R_j = \left( \frac{1}{4\pi} \frac{15K T}{G M M_H} \phi_0 \right)^{1/2}
\]
Cloud with pressure

\( M_{\text{BE}} = \frac{118}{64} \times \sqrt{\frac{f_b}{g}} \)

More elaborate derivation claim

\( \text{BE} = \frac{118}{64} \times \sqrt{\frac{f_b}{g}} \)

Benner - Eboll mass

\( M_j = \left( \frac{5}{2} \right)^{3/2} \left( \frac{g^{-1}}{4 \mu m} \right)^{1/2} \)

\( P_0 = \left( \frac{8}{3} \right) \left( \frac{3}{4} \right) \left( \frac{f_b}{g} \right) \left( \frac{g^{-1}}{4 \mu m} \right)^{1/2} \)

\( M_j = \left( \frac{5}{2} \right)^{3/2} \left( \frac{g^{-1}}{4 \mu m} \right)^{1/2} \)

\( \frac{P_0}{T} \)

\( \frac{U_2}{\text{Kepler}} = \frac{f_b}{g} \)

\( \frac{f_b}{g} \left( \frac{g^{-1}}{4 \mu m} \right)^{1/2} \)}

Lecture stops here

since we discussed variable stars, driving and higher harmonics mechanism