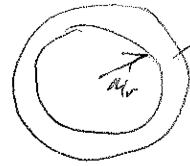
Lecture 23

Proto star formation

The Jeans Criterion

Gas cloud with random velocity distribution so we will use T as a measure of typical speed / energy

Assuming $\rho = \text{const}$ and spherical shape



$$U = -\int_0^{R_c} G \frac{M_r\, dm}{r} =$$

$$= -\int_0^{R_c} G \frac{\rho \frac{4\pi}{3} r^3 \cdot 4\pi r^2 dr \rho}{r} =$$

$$= -G\rho^2 \frac{4\pi}{3} \cdot 4\pi \int_0^{R} r^4 dr = -G\rho^2 \frac{4\pi}{3} \frac{4\pi}{5} R_c^5$$

$$\boxed{U = -G \frac{3}{5} \frac{M_c^2}{R_c}}$$

Kinetic energy $K = \frac{3}{2} N k T = \frac{3}{2} \frac{M_c}{\mu m_H} k T$

ideal monoatomic gas; N; mean molecular weight

Virial theorem $2K + U = 0$

if $2K + U > 0$ cloud will expand (hot cloud)

So for cloud collapse we need

$2K < -U$

$$\cancel{3}\frac{M_c}{\mu m_H} KT < \frac{\cancel{3}}{5}\frac{G M_c^{\cancel{2}}}{R_c}$$

$$R_c = \left(\frac{M_c}{\frac{4\pi}{3}\rho_0}\right)^{1/3}$$ eq 1

initial density

$$\cancel{M_c > M_J = \frac{5KT}{G\mu m_H}}$$

$$\frac{KT}{\mu m_H} < \frac{G}{5}\frac{M_c}{M_c^{1/3}} \cdot \left(\frac{4\pi}{3}\rho_0\right)^{1/3} = M_c^{2/3} \cdot \left(\frac{4\pi}{3}\rho_0\right)^{1/3}$$

$$\boxed{M_c > \left(\frac{5KT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2} \equiv M_J}$$ Collapse condition

Seems like a paradox $M_J \sim \frac{1}{\sqrt{\rho}}$
while we might think the dense the cloud the more atraction but it also means larger number of particles (N) increasing 'K'

Using eq. 1 we can state

$$R_J = \left(\frac{3}{4\pi}\left(\frac{5KT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi}\right)^{1/2}\frac{1}{\rho^{3/2}}\right)^{1/3} =$$

$$\boxed{R_c > R_J = \left(\frac{1}{4\pi}\frac{15KT}{G\mu m_H}\frac{1}{\rho_0}\right)^{1/2}}$$

P3

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

$$v_{Thermal} = \sqrt{\frac{kT}{m_{part}}} = \sqrt{\frac{kT}{\mu m_H}}$$

$$P_0 = \frac{N}{V}kT = \frac{\rho_0}{\mu m_H}kT$$

$$M_J = \left(\frac{5}{G}\right)^{3/2} v_T^3 \cdot \left(\frac{3}{4\pi}\cdot\frac{1}{\underbrace{\rho_0\frac{kT}{\mu m_H}}_{=P_0}}\cdot\underbrace{\frac{kT}{\mu m_H}}_{=v_T^2}\right)^{1/2}$$

$$M_J = \frac{v_T^4}{P_0^{1/2}G^{3/2}}\cdot\left(\frac{5^3\cdot 3}{4\pi}\right)^{1/2} =$$

$$\boxed{M_J = \underbrace{5.46}_{=C_J}\frac{v_T^4}{P_0^{1/2}G^{3/2}}}$$

More elaborative derivation claim that C_J must be replace by $\boxed{C_{BE} = 1.18 \quad \text{Bonnor-Ebert}}$ mass

Cloud with external pressure P_0 $\longrightarrow$ $M_{BE} = 1.18\cdot\frac{v_T^4}{P_0^{1/2}G^{3/2}}$

will contract/collapse when $\boxed{M_C > M_{BE}}$

Since we discussed variable stars. Driving and higher harmonics mechanism

lecture stops here