Lecture 22 ~~std~~ gravitational mechanism of stellar pulsation

g-mode — buble slaushing
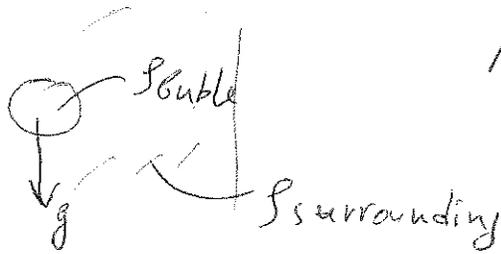
Note that $p^{(b)} = p^{(s)}$ otherwise buble would compess or expand atill this condition is met

Buoyant force

$$f_{net} = (-\rho^{(b)}g + \rho^{(b)}_s g)V = (\rho^{(s)} - \rho^{(b)})gV$$

gravity pull down

once buble displaced ~~&~~ dr
we write Taylor expansion

$$\rho^{(s)} = \rho^{(s)}_{initial} + \frac{d\rho^{(s)}}{dr}\delta r$$

$$\rho^{(b)} = \rho^{(b)}_{initial} + \frac{d\rho^{(b)}}{dr}\delta r$$

assuming ~~that~~ $\rho^{(s)}_{initial} = \rho^{(b)}_{initial}$ i.e. they have
the same starting point
from which buble emerge

$$f_{net} = g\left(\frac{d\rho^{(s)}}{dr}\delta r - \frac{d\rho^{(b)}}{dr}\delta r\right)V =$$

$$= \left(\frac{d\rho^{(s)}}{dr}\delta r - \frac{d\rho^{(b)}}{d\rho^{(b)}} \cdot \frac{dP^{(b)}}{dr}\delta r\right)gV$$

We require <u>adiabatic</u> process for buble

$PV^\gamma = const$

. since m of the buble is a const,
. i.e. we have mass conservation

$P\left(\dfrac{V}{m}\right)^\gamma_{=V\rho} = const_2$

$$d\left(P\rho^{-\gamma}\right) = dP\,\rho^{-\gamma} + (-\gamma)P\rho^{-\gamma-1}d\rho = 0$$

$$\dfrac{d\rho}{dP} = \dfrac{\rho^{-\gamma}}{+\gamma\rho^{-\gamma-1}P} = \dfrac{\rho_i^{(b)}}{\gamma P_i^{(b)}}$$

recall that this is about buble

start cond.

now we can plug it to the f net eq.

$$f_{net} = V_g\left(\dfrac{d\rho^{(s)}}{dr} - \dfrac{\rho_i^{(b)}}{\gamma P_i^{(b)}}\dfrac{dP^{(b)}}{dr}\right)dr$$

since $P^{(b)} = P^{(s)}$ to maintain the buble
and $\rho_i^{(b)} = \rho_i^{(s)}$ we can drop
'b' and 's' subscript, keeping in mind
that it is actually 's' everywhere now

$$f_{net} = V_g\left(\dfrac{\rho}{\rho}\dfrac{d\rho}{dr} - \dfrac{\rho}{\gamma P}\dfrac{dP}{dr}\right)dr = V\left(\dfrac{1}{\rho}\dfrac{d\rho}{dr} - \dfrac{1}{\gamma P}\dfrac{dP}{dr}\right)\rho g\,dr$$

some coef.   A

$$\boxed{f_{net} = V_g A \rho\, dr}$$

$$\boxed{A = \dfrac{1}{\rho}\dfrac{d\rho}{d\rho} - \dfrac{1}{\gamma P}\dfrac{dP}{dr}}$$

if $A < 0$ we have a restoring force

think about $\smile$ g . So our
buble will go ~~back~~ back to equilibrium
and thus oscillate around this point.

since $\rho V = m$ of the buble

$$f_{net} = ma = mg A \delta r$$
$$a = (\ddot{\delta r})$$

get again equation of harmonic oscillator

$$\boxed{\omega_{g_{mode}} = \sqrt{-gA} = \sqrt{\left(\frac{1}{\gamma P}\frac{dP}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right)g}}$$

$$\pi = \frac{2\pi}{\omega}$$

Note that if $\underline{A > 0}$ then there is
no restoring force. Once the buble
start to move up it will keep
going to do it $\Rightarrow$ ~~convection~~
convection condition

It is not enough to have conditions for oscillations, some mechanism must drive this oscillation since otherwise oscillation energy will dissipate and oscillation will seize,

On top of it whatever the source of energy (fusion) it must be applied right to do a positive work on a slice of a star.
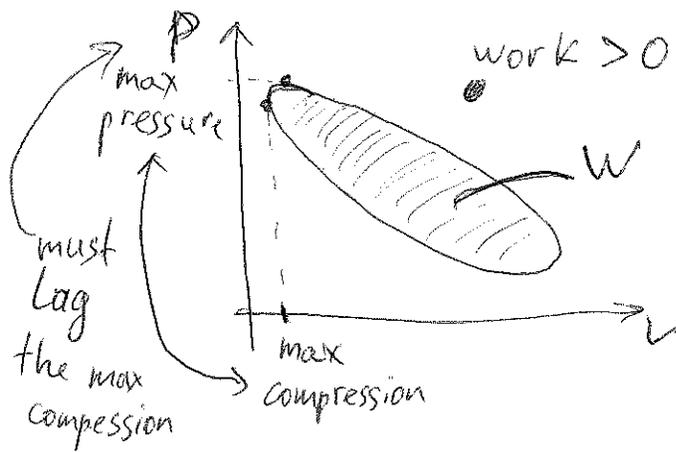
work > 0 if we go ↻

and negative if we do ↺ direction

$$W = \oint P \, dV$$

max pressure

must Lag the max Compession

max compression

Opacity effects, κ and γ - Mechanism

absorption

Generally $\kappa \sim \frac{g}{T^{3.5}}$ coef.

We need κ - to be increased with T
can be achieved in partially ionization
zone if ~~increase of T~~ injection of
heat goes no to kinetic energy of atoms
(and thus increase of T) but to
increased ionization and thus $\nearrow \kappa$

This will introduce delay:
Max pressure will happen after maximum
compression

Additionally if some region is heated
less during compression (i.e. T is smaller
than ~~to~~ surrounding)
then heat additionally will go to such
region : γ - mechanism

Partial ionization zones thus acts like a
pistons. and their location in star
defines what mode will oscillate if any

This in turn depends on star temperatures
see fig. 14.14

p-modes. Going back to pressure governed mode. We discussed motion of the spherical layer as a whole, but we can also observe 'riples' on the surface or in dipper part of the star
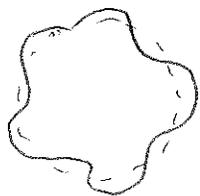
this described by spherical harmonic function $Y_\ell^m(\theta, \varphi)$
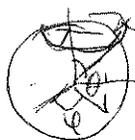
$\ell = 0, 1, 2, 3 \ldots$
$m = -\ell, \ldots 0, \ldots, \ell$ overall $2\ell + 1$ values

$Y_0^0(\theta, \varphi) = K_0^0$ (const)

$Y_1^0(\theta, \varphi) = K_1^0 \cos\theta$

$Y_1^{\pm 1}(\theta, \varphi) = K_1^{\pm 1} \sin\theta\, e^{i\varphi}$

wavelenglt of ostillation $\lambda = \dfrac{2\pi R}{\sqrt{\ell(\ell+1)}}$
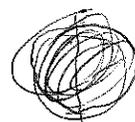
$f_\ell = \dfrac{v_{sound}}{\lambda} = \sqrt{\dfrac{\delta P}{\rho}} \dfrac{\sqrt{\ell(\ell+1)}}{2\pi R}$
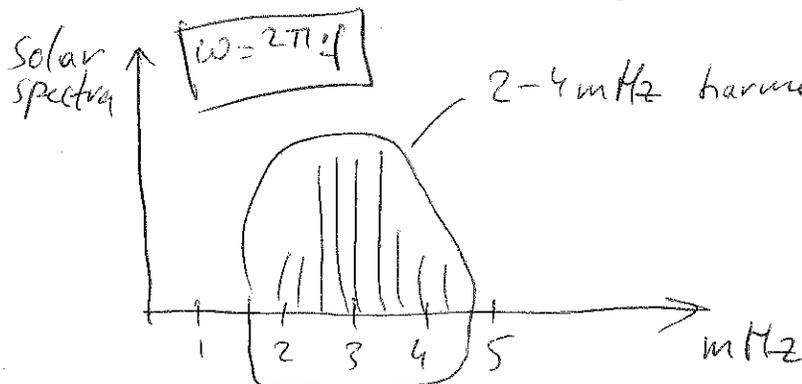
distance from center not the star radius!

$\boxed{\omega = 2\pi f}$

Solar spectra

2-4 mHz harmonics

mHz

$3 mHz \approx 300 \sec = 5 min$ oscillation