Tunneling probability \( \sim e^{-2\pi^2 \frac{U_c}{E}} \)

\[
U_c = \frac{\hbar^2 Z_1 Z_2 e^2}{4E} = \frac{\hbar^2 Z_1 Z_2 e^2}{\left(\frac{3}{2}\right) \sqrt{2mE}}
\]

\[
= \frac{\hbar^2 Z_1 Z_2 e^2}{\frac{\sqrt{2}}{2} \sqrt{2mE}}
\]

\[
= \frac{\hbar^2 Z_1 Z_2 e^2}{\sqrt{2mE}}
\]

\[
= \frac{8\pi e}{\sqrt{2mE}}
\]

\[
\Rightarrow \quad \psi(E) = \frac{8\pi e}{\sqrt{2mE}} e^{-\frac{E}{kT}}
\]

Maxwell-Boltzmann distribution

\[
\rho(u) du \sim \frac{n}{(kT)^{3/2}} u^2 e^{-\frac{m}{2kT} u^2} du
\]

\[
\sim \frac{1}{(kT)^{3/2}} \sqrt{2E} e^{-\frac{E}{kT}} dE = n_d dE
\]

\[
\nu = \sqrt{\frac{2E}{m}}
\]

\[
d\nu = \sqrt{\frac{2E}{m}} \frac{dE}{2E}
\]
\[ r = \sum_{i} \int \frac{h_i n_i \sigma(E) S(E) \frac{hE}{E}}{(kT)^{3/2}} dE \]

\[ \approx n_i \frac{1}{(kT)^{3/2}} \int \frac{S(E) e^{-E/kT}}{E^{3/2}} dE \]

\[ \approx n_i \frac{1}{(kT)^{3/2}} \int S(E) e^{-E/kT} dE \]

\[ MB \text{ distribution contributions} \]

\[ \text{penetration probability} \]

\[ \text{Gamov peak} \]

\[ \max \text{ at } E_0 = \left( \frac{6kT}{2} \right)^{2/3} \text{ assuming } S(E) = \text{const} \]

\[ \text{The higher the temperature the faster the reaction goes} \]

\[ \text{For same } B \]
So far we considered collision of 2 different elements.

But if we looking in fusion of H to He, the reaction is

\[ 4 \cdot \text{H} \rightarrow \text{He} + 2 \text{e}^+ + 2 \nu_e + 2 \gamma \]

four particle collision required
this is highly improbable

Most likely it realised in a chain
when first 2 particles strucked,
then one more, and yet one more

We will use notation

\[ ^{A}_{Z}X \]

protons + neutrons

number of protons

element

this uniquely identify chemical properties and thus this
Proton-Proton chain (PP I)

\[ ^1H + ^1H \rightarrow \frac{2}{1}H + e^++\bar{\nu}_e \text{ (requires } p^+ \rightarrow n + e^++\bar{\nu}_e) \]

\[ ^2H + ^1H \rightarrow \frac{3}{2}He + Y \]

\[ \frac{3}{2}He + \frac{3}{2}He \rightarrow \frac{4}{2}He + 2^1H \]

\[ \text{High binding energy!! very stable} \]

\[ \text{PP II} \]

\[ \frac{3}{2}He + \frac{4}{2}He \rightarrow \frac{6}{2}Be + Y \]

\[ ^4Be + e^- \rightarrow \frac{4}{3}Li + \bar{\nu}_e \]

\[ \frac{3}{2}Li + ^1H \rightarrow 2^2He \]

\[ \text{PP III} \]

\[ ^4Be + ^1H \rightarrow \frac{8}{5}B + Y \]

\[ \frac{8}{5}B \rightarrow ^8Be + e^+ + \bar{\nu}_e \]

\[ ^8Be \rightarrow 2^4He \]

Energy generation of all this chains

Energy generation of all this chains (concentration of H)

\[ E_0 \approx E_0 \cdot T_6^{\alpha} \cdot X^2 \text{ where } T_6 = \frac{T}{10^6K} \]

\[ \frac{W}{kg} \sim 1.08 \cdot 10^{-11} \frac{W \cdot m^2}{kg^2} \text{ density} \]
CNO Cycle

1st Branch

\[ _{6}^{12}C + _{1}^{1}H \rightarrow _{7}^{13}N + \gamma \]

\[ _{7}^{13}N \rightarrow _{6}^{12}C + e^{+} + \bar{\nu}_{e} \]

\[ _{6}^{12}C + _{1}^{1}H \rightarrow _{7}^{13}N + \gamma \]

\[ _{8}^{14}N + _{1}^{1}H \rightarrow _{8}^{15}O + \gamma \]

\[ _{8}^{15}O \rightarrow _{7}^{14}N + e^{+} + \bar{\nu}_{e} \]

\[ _{7}^{15}N + _{1}^{1}H \rightarrow _{6}^{12}C + _{2}^{4}He \quad \text{(Carbon is catalyst)} \]

2nd Branch (0.04% of time)

\[ _{8}^{15}N + _{1}^{1}H \rightarrow _{8}^{16}O + \gamma \]

\[ _{8}^{16}O + _{1}^{1}H \rightarrow _{9}^{17}F + \gamma \]

\[ _{9}^{17}F \rightarrow _{8}^{16}O + e^{+} + \bar{\nu}_{e} \]

\[ _{8}^{16}O + _{1}^{1}H \rightarrow _{7}^{14}N + _{2}^{4}He \]

\[ E_{\text{CNO}} \sim E_{0}e^{5XX_{\text{CNO}}T_{6}} \quad \text{(9.9)} \]

\[ E_{0} = 8.24 \times 10^{-37} \text{ Wm}^{-3} \text{kg}^{-2} \]

- sharp dependence on temperature
The triple alpha process — burning of He

\[ _2^4 \text{He} + _2^4 \text{He} \rightarrow _4^8 \text{Be} \]

\[ _4^8 \text{Be} + _2^4 \text{He} \rightarrow _6^{12} \text{C} + \gamma \]

(recall that \( _1^2 \text{He}^+ \) is a particle)

Above looks like

\[ 3 \cdot _2^4 \text{He} 
\rightarrow _6^{12} \text{C} + \gamma \]

\[ E_{3\alpha} \leq E_{0,3\alpha} \frac{A^2}{V_3} \]

\( T_8 \) = super sharp dependence on \( T \)

\[ T_8 = T/10^8 \]

3α - process kicks in at high temperatures
Star evolution

Notice that we convert $H \rightarrow He$ but it requires 4 $H$ to make one $He$, so we reducing number of “free” particles recall $p = n k T$, $n = \frac{N}{V}$ so

if $N/4$ than $p < p_{eq}$ which mean that hydrostatic equilibrium is not maintained and star collapses. This will lead to increase of temperature $\Rightarrow$ fusion ignites for reactions which go at higher temperatures (3d for example) and a star stabilizes yet again

Q: what would happen if somehow we replace $H$ with something else in 1 to 1 fashion? Pressure will stay constant since $p = \frac{N}{V} k T$
Q: Let's say we want to settle around a star and we are looking for one which should last the longest. Should we choose a lighter or more massive one?

Naively more fuel (mass) the longer it lasts.

But Recall:

\[ L \text{ grows with mass} \]

\[ L(120M_\odot) = 10^6 L(M_\odot) \]

"Burn time" = \[ \frac{K \cdot M c^2}{L_\odot} \]

\[ \approx \frac{K \cdot 120M_\odot c^2}{10^6 L_\odot} \]

Conversion coef. \( 0.007 \) M → He

Recall

\[ \frac{120}{10^6} T_{\text{Sun}} = T_{\text{Sun}} \frac{1}{10^4} \]