Lecture 19

(p1)

Tunneling probability $\sim e^{-2\pi^2 U_c/E}$

$$\frac{U_c}{E} = k_c \frac{Z_1 Z_2 e^2}{\lambda (mv^2/2)} = \frac{k Z_1 Z_2 e^2}{(h/p) \cdot p^2/2M} =$$

$\lambda = h/p$

$$= \frac{k Z_1 Z_2 e^2}{h \frac{v}{2}} \qquad v = \sqrt{2E/M} \qquad k = \boxed{\frac{k Z_1 Z_2 e^2 \sqrt{2M}}{h\sqrt{E}}}$$

$b =$

$$= b/\sqrt{E}$$

slow factor

$$\Rightarrow \sigma(E) \simeq \frac{S(E)}{E} \cdot e^{-bE^{-1/2}}$$

Maxwell - Boltzman distribution

$$n(v)\,dv \sim \frac{n}{(kT)^{3/2}} v^2 e^{-\frac{mv^2}{2kT}} dv$$

$\cancel{v = \sqrt{\frac{2}{m}E}}$

$$\sim \frac{1}{(kT)^{3/2}} \sqrt{E}\, e^{-E/kT} dE = n_E\, dE$$

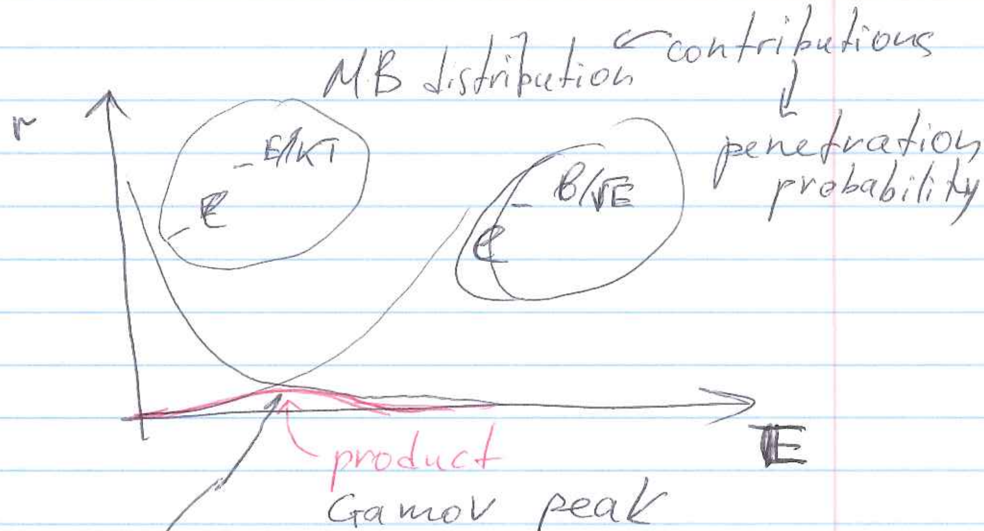
$$\boxed{\begin{aligned} v &= \sqrt{\frac{2E}{m}} \\ dv &= \sqrt{\frac{2}{m}} \frac{dE}{2\sqrt{E}} \end{aligned}}$$

$$r = \int_0^\infty n_i n_x \, \sigma(E) \, \overset{\sim\sqrt{E}}{v(E)} \, \frac{n_E}{n} \, dE \sim$$

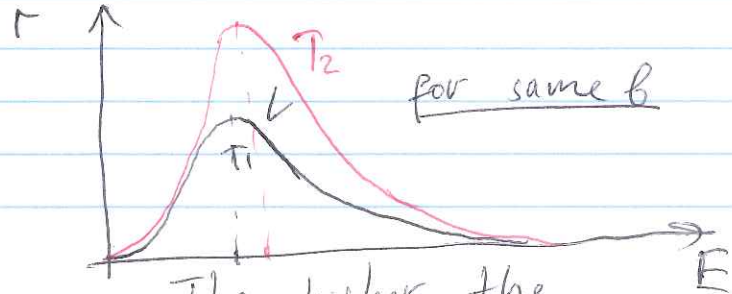
Will be gone since M.B ~ n

$$\sim n_i n_x \frac{1}{(kT)^{3/2}} \frac{S(E) e^{-b(E)^{-1/2}}}{\cancel{E}} \cdot \cancel{\sqrt{E}}\cancel{\sqrt{E}} e^{-E/kT} \, dE$$

$$\sim \frac{n_i n_x}{(kT)^{3/2}} \int_0^\infty S(E) \, e^{-b/\sqrt{E}} \, e^{-E/kT} \, dE$$



max at $E_0 = \left(\frac{bkT}{2}\right)^{2/3}$ assuming $S(E)$ = const



The higher the temperature the faster reaction goes

So far we considered collision of 2 different elements.

But if we looking in fusion of H to He the reaction is

$$4 \cdot {}^1_1H \rightarrow \underbrace{{}^4_2He} + \underbrace{2e^+}_{\text{positrons}} + \underbrace{2\nu_e}_{\text{electron neutrinos}} + \underbrace{2\gamma}_{\text{Photons}}$$

four particle collision required
this is highly unprobable

Most likely it realised in a chain when first 2 particles sticked, then one more, and yet one more

We will use notation

$${}^{A}_{Z}X$$

A — protons + neutrons

X — element

Z — number of protons

this unicly indentify chemical properties and thus this

Proton - Proton chain (PP I)

$${}^1_1H + {}^1_1H \longrightarrow {}^2_1H + e^+ + \nu_e \quad (\text{requires } p^+ \to n + e^+ + \nu_e)$$

$${}^2_1H + {}^1_1H \longrightarrow {}^3_2He + \gamma$$

$${}^3_2He + {}^3_2He \longrightarrow {}^4_2He + 2\,{}^1_1H \qquad \cancel{69\%}$$

↖ High binding energy!! very stable

PP II

$${}^3_2He + {}^4_2He \longrightarrow {}^7_4Be + \gamma \qquad \cancel{31\%}$$

$${}^7_4Be + e^- \longrightarrow {}^7_3Li + \nu_e$$

$${}^7_3Li + {}^1_1H \longrightarrow 2\,{}^4_2He$$

PP III

$${}^7_4Be + {}^1_1H \longrightarrow {}^8_5B + \gamma$$

$${}^8_5B \longrightarrow {}^8_4Be + e^+ + \nu_e$$

$${}^8_4Be \longrightarrow 2\,{}^4_2He$$

Energy generation of all this chains

concentration of H

$$\varepsilon_0 \sim \varepsilon_0' \, T_6^4 \, \rho \, X^2 \quad \text{where} \quad T_6 = \frac{T}{10^6\,K}$$

$\frac{W}{kg}$ $\quad 1.08 \cdot 10^{-12} \, \frac{W\,m^3}{kg^2}$ $\quad$ density

CNO Cycle

1st Branch

$$\underline{{}^{12}_{6}C} + {}^{1}_{1}H \rightarrow {}^{13}_{7}N + \gamma$$

$$ {}^{13}_{7}N \rightarrow {}^{13}_{6}C + e^{+} + \nu_e$$

$$ {}^{13}_{6}C + {}^{1}_{1}H \rightarrow {}^{14}_{7}N + \gamma$$

$$ {}^{14}_{7}N + {}^{1}_{1}H \rightarrow {}^{15}_{8}O + \gamma$$

$$ {}^{15}_{8}O \rightarrow {}^{15}_{7}N + e^{+} + \nu_e$$

$$ {}^{15}_{7}N + {}^{1}_{1}H \rightarrow \underline{{}^{12}_{6}C} + {}^{4}_{2}He$$

Carbon is catalyzer

2nd Branch (0.04% of time)

$$ {}^{15}_{7}N + {}^{1}_{1}H \rightarrow {}^{16}_{8}O + \gamma$$

$$ {}^{16}_{8}O + {}^{1}_{1}H \rightarrow {}^{17}_{9}F + \gamma$$

$$ {}^{17}_{9}F \rightarrow {}^{17}_{8}O + e^{+} + \nu_e$$

$$ {}^{17}_{8}O + {}^{1}_{1}H \rightarrow {}^{14}_{7}N + {}^{4}_{2}He$$

$$\varepsilon_{CNO} \approx \varepsilon'_{0_{CNO}}\, \rho\, X X_{CNO}\, T_6^{19.9}$$

— sharp dependence on Temperature

$$\varepsilon'_0 = 8.24 \cdot 10^{-31}\ \frac{W\,m^3}{kg^2}$$

The triple Alpha Process —
— Burning of He

$${}^4_2He + {}^4_2He \rightleftharpoons {}^8_4Be$$

(recall that α-particle is ${}^4_2He^+$)

$${}^8_4Be + {}^4_2He \rightarrow {}^{12}_6C + \gamma$$

Above looks like

$$3 \cdot {}^4_2He \rightarrow {}^{12}_6C + \gamma$$

$$\varepsilon_{3\alpha} \simeq \varepsilon'_{0,3\alpha} \rho^2 Y^3 \ldots T_8^{41.0}$$

← super sharp dependence on T

notice

$T_8 = T/10^8$

3α - process kicks in at high temperatures

Star evolution

Notice that we convert H → He but it requires 4H to make one He, so we reducing number of "free" particles

recall $P = nkT$, $n = \frac{N}{V}$ so

if $N \searrow$ than $P \searrow$ which mean that hydrostatic equilibrium is not maintained and star collapses. This will lead to increase of temperature ⟹ fusion ignites for reactions which go at higher temperatures (3α for example) and a star stabilizes yet again

Q: what would happen to the pressure if somehow we replace H with something else in 1 to 1 fashion? Pressure will stay constant since $P = \frac{N}{V} kT$

Q: let's say we want to settle around star and we are looking for one which should last the longest. Should we chose a lighter or more massive one?

Naively more fuel (mass) the longer it lasts.

But Recall:

L grows with mass

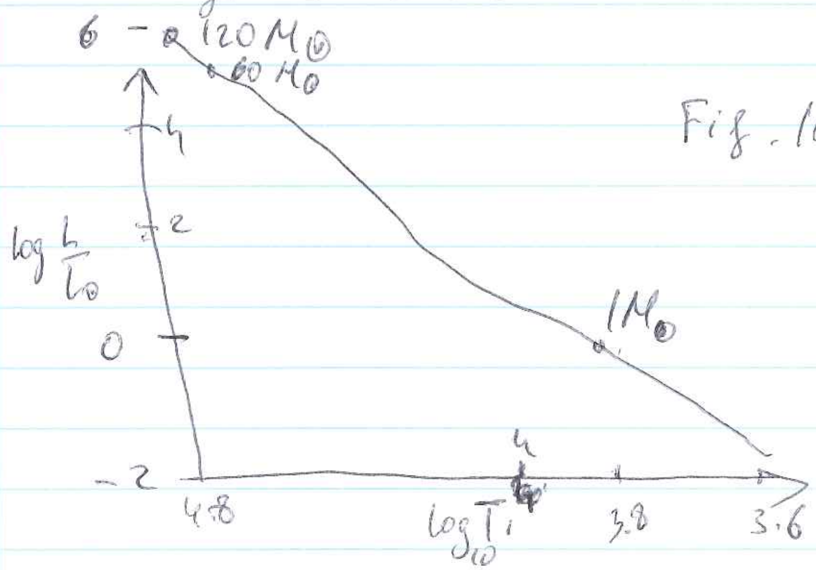


Fig. 10.13

$$L(120 M_\odot) = 10^6 L(M_\odot)$$

"Sun Time"

$$\text{"Burn time"} = \frac{K M c^2}{L_\odot} = \frac{K \cdot 120 M_\odot c^2}{10^6 L_\odot}$$

Conversion coef. 0.007 H → He recall

$$= \frac{120}{10^6} T_{sun} \approx \frac{T_{sun}}{10^4}$$