lecture 17

(p1)

How stars work

1st let's estimate how much energy per kg we get at the sun

$$\frac{L_\odot}{M_\odot} = \frac{3.8\cdot10^{26}}{2\cdot10^{30}} \approx 2\cdot10^{-4}\ \frac{W}{kg}$$

To power 100 W bulb we would need $\frac{100\,W}{2\cdot10^{-4}\ W/kg} = 5\cdot10^{5}\ kg = 500$ Tonn

On a bright side it will shine for 10^{11} years

So why power output is so slow?
(This is good in a long run but can it be more "Luminos"?)

From other hand if we convert 1kg of H to He we will get

$$\underbrace{(1kg\cdot0.007)}_{\text{mass converted}}\cdot c^2 = 7\cdot10^{-3}\cdot(3\cdot10^{8})^2 =$$

$$= 6.3\cdot10^{-2}\cdot10^{16} = 6.3\cdot10^{14}\ J$$

Compare with Diesel 1kg $\Rightarrow 4\cdot10^{7}$ J

~~For comparison estimated humans energy requirement/production is about~~ $\cancel{\simeq 142\cdot1000\cdot\frac{TW\,hour}{year}\cdot365\cdot24\frac{hours}{year}\ \frac{year}{hour}}$

$\cancel{= 1.24\cdot10^{24}\ J}$

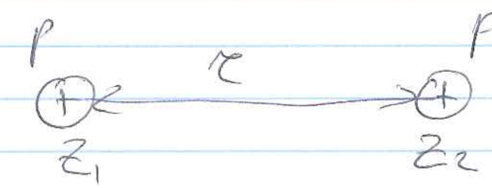
Let's think: what is required condition for fusion?
We need to fuse atoms (nuclei) i.e. bring them in contact.

Which means bring them to separations of about <u>nuclei size</u> $\approx 1\,fm = 10^{-15}\,m$

So here is the first problem probability of collision

$\sim n \sigma v$ where $\sigma \approx 10^{-15}$
this way smaller than atom collision cross section!

But we have a bigger problem:
Coulomb ~~Coulomb~~ repulsion



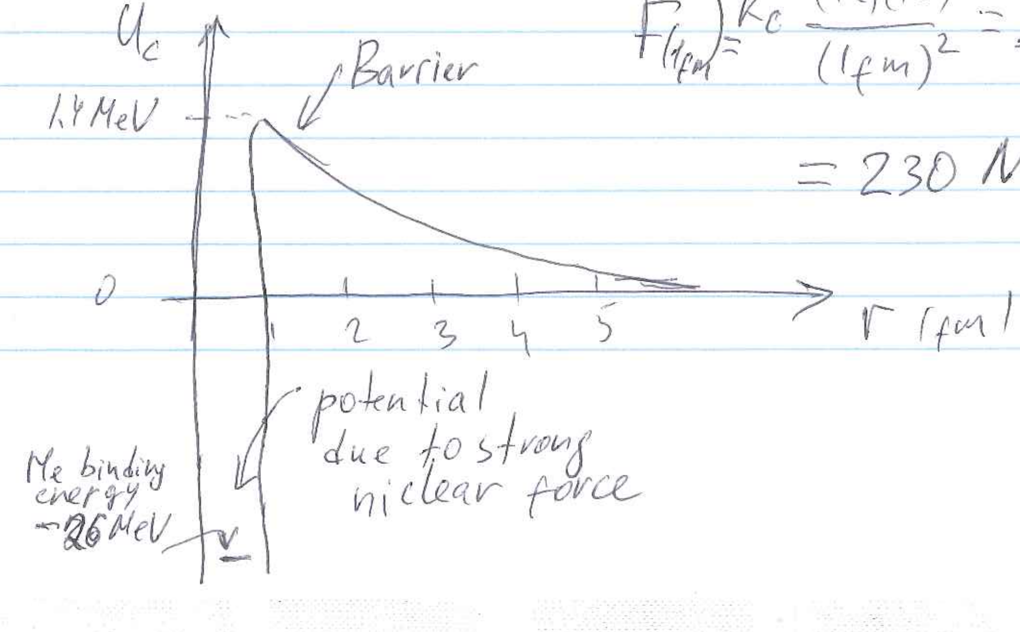
charges in 'e'

$$F = K_c \frac{Z_1 Z_2}{r^2}$$

$H \Rightarrow Z = 1$

$$F_{(1fm)} = K_c \frac{(1e)(1e)}{(1fm)^2} = \frac{9 \cdot 10^9 (1.6 \cdot 10^{-19})^2}{(10^{-15})^2}$$

$$= 230\ N$$



So it seems that our H atoms need to have at least 1.4 MeV to overcome the repulsion barrier

$$1.4\ MeV = \frac{1}{2} m \bar{v}^2 = \frac{3}{2} k T_{classical}$$

$$\Rightarrow \quad T = \frac{2}{3} \cdot \frac{1.4 \cdot 10^6 \, eV \cdot 1.6 \cdot 10^{-19} \frac{J}{eV}}{1.38 \cdot 10^{-23}} =$$

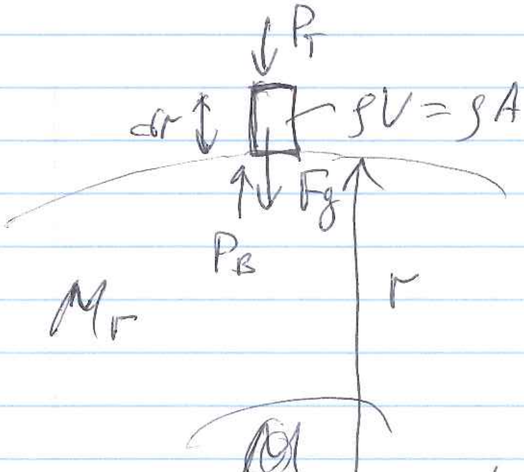
$$= 10^{10}\ K$$ It looks hot but should we be alert?

To answer this we need to step away and consider pressures in star.

$$\bar{v} = \sqrt{\frac{2\ MeV}{m_H}} = \sqrt{\frac{1.4 \cdot 10^6 \cdot 1.6 \cdot 10^{-19}}{1.67 \cdot 10^{-27}}} =$$

$$= 9 \cdot 10^6\ m/c$$

The pressure equation



$$m\vec{a} = \Sigma \vec{F}$$

$$dma = -F_g - P_T \cdot A + P_B \cdot A$$

local acceleration due to gravity

$$dma = -mg - dP \cdot A$$

$$\rho\, dr \cancel{A}\, \underset{\ddot{r}}{a} = -\rho g\, dr \cancel{A} - dP \cdot \cancel{A}$$

$$dr\, \rho \frac{d^2 r}{dt^2} = -\rho g\, dr - (dP)$$

$$\rho \frac{d^2 r}{dt^2} = -\rho g - \frac{dP}{dr}$$

$$\boxed{\rho \frac{d^2 r}{dt^2} = -\rho G \frac{M_r}{r^2} - \frac{dP}{dr}}$$

<u>in equilibrium</u> $\frac{d^2 r}{dt^2} = a = 0$

$$\Rightarrow \boxed{\frac{dP}{dr} = -\rho G \frac{M_r}{r^2}}$$

Q: which star with the same mass would have to have a larger pressure small or large

Assuming $\rho = \text{const}$

$\Rightarrow M_r = \frac{4\pi}{3} r^3 \rho$

$$\frac{dP}{dr} = -\rho G \frac{4\pi}{3} r \rho$$

Pressure at surface $= 0$

$$\left(\int_0^{R_\odot} r\, dr\right)\left(-\rho^2 G \frac{4\pi}{3}\right) = \underbrace{P(R_\odot)}_{=0 \text{ at surface}} - P(0)$$

$$= \underset{\uparrow \text{ the center of sun}}{P(0)} = +\rho^2 G \frac{4\pi}{3} \frac{R_\odot^2}{2} \cdot \frac{R_\odot}{R_\odot}$$

$$= \frac{G M_\odot}{2 R_\odot} \frac{\rho R_\odot \frac{4\pi}{3} R_\odot^2}{\frac{4\pi}{3} R_\odot^3} \; M_\odot =$$

$$\boxed{P_c = \frac{3 G M_\odot^2}{8\pi R_\odot^4}} \simeq \frac{3 \cdot 6.67 \cdot 10^{-11}}{8 \pi} \cdot \frac{(2 \cdot 10^{30})^2}{(7 \cdot 10^8)^4}$$

$= 4.16 \cdot 10^{14}$ Pa

there were assumption of $\rho = \text{const}$

a ~~more~~ better model it gives,

$\simeq 2.34 \cdot 10^{16}$ Pa

Lecture 17 ends here