lecture 16

p1

Solar energy sources

Sun emits $L_{\odot} = 3.8 \cdot 10^{26}$ W

What could be the source of it.

Can it be due to oil burn?

Diesel gives ~ 43 MJ/kg = ρ_E once it burned. $= \rho_E$ – energy density

So per second ~~we~~ Sun need to use

$$\frac{m}{\Delta t} = \frac{L}{\rho_E} = \frac{3.8 \cdot 10^{26}}{43 \cdot 10^{6}} \frac{W}{J/kg} = \approx 10^{19} \frac{kg}{s}$$

The mass of the Sun $M_{\odot} \approx 2 \cdot 10^{30}$ kg

So we would used up our sun in

$$T = M_{\odot} / \frac{m}{\Delta t} = \frac{2 \cdot 10^{30} \text{ kg}}{10^{19} \frac{kg}{s}} = 2 \cdot 10^{11} \text{ sec}$$

One year $= \pi \cdot 10^{7}$ s so

$$T = \frac{2 \cdot 10^{11} s}{\pi \cdot 10^{7}} = 6000 \text{ years}$$

Wow! looks like Biblical time, ~~only~~ but it is actually time till the "End of the world" from the start of the cycle.

Well there is a problem: we have a lot of support from Archeology that at least Egypt is 6000 year old

$\boxed{\text{Oil is no good}}$

Ok may be it is a gravitational energy?

$$dU = -G\frac{dm_r M}{r}$$

$dm_r = 4\pi r^2 dr \cdot \rho$

$M = \frac{4\pi}{3} r^3 \cdot \rho$

if $\rho = \text{const}$

$$\int dU = -G\rho^2 \frac{16\pi^2}{3}\int_0^{R_{sun}} \frac{r^5 dr}{r}$$

$$U = -G\rho^2\frac{16\pi^2}{3}\int r^4 dr = -G\rho^2\frac{16\pi^2}{3}\,\frac{R^5}{5}$$

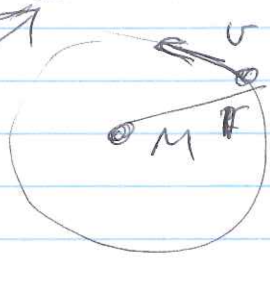
$$\frac{4\pi}{3}\rho R^3 = M_\odot$$

$$= U_g = -\frac{G\cdot 3 M_\odot^2}{R\cdot 5}$$

$$\boxed{U_g = -\frac{3}{5}\frac{GM_\odot^2}{R}}$$

Bonus Q when this guy was born and died

→ Virial theorem – particular case: orbiting body



$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\underbrace{\left(\frac{GMm}{r}\right)}_{-U_g} = \underbrace{\left(\frac{mv^2}{2}\right)}_{K}\cdot 2$$

in equilibrium → $\boxed{-\overline{U}_g = 2\overline{K}}$

more General: $\overline{U}$, $\overline{K}$ averaged in time and in C.M system of coordin.

$$E_{Total} = U_g + K =$$

$$= U_g - U_g/2 = -\frac{U_g}{2}$$

So $\boxed{E_{\odot} = -\frac{3}{10}\frac{G M_{\odot}^2}{R}}$

or any massive body

So as object shrinks ($R \downarrow$)
and it looses energy thus
lost energy (due to Energy conservation)
must take other forms.
For example radiation. (Think about
a meteorit in the Earth atmosphere.)
So suppose Sun was originaly
very lage $R_{init} \to \infty$ then during
it collapce

Full emitted energy

$$\boxed{E_{in} - E_{fin}} = 0 - \left(-\frac{3}{10}\frac{G M_{\odot}^2}{R_{\odot}}\right)$$

current radius

$$= \frac{3}{10} G \frac{M_{\odot}^2}{R_{\odot}} \approx \frac{3}{10} \cdot 6.67 \cdot 10^{-11} \cdot \frac{(2 \cdot 10^{30})^2}{7 \cdot 10^{8}}$$

$$\boxed{= 1.14 \cdot 10^{41}\ J}$$

Kelvin - Helmholtz time scale

Ok how ~~much~~ much time gravitational collapse ~~to~~ gives us?

$$\frac{E_{in} - E_{fin}}{L_\odot} = \frac{1.14 \cdot 10^{41} J}{3.8 \cdot 10^{26} W} =$$

$$= 3 \cdot 10^{14} s \approx 10^7 \text{ years} = 10 \text{ M year}$$

Yet again we have a problem:
Dinasours lived more than 70 M years ago. Not to mention even older creatures, lobsters $\approx$ 110 Myears

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Ok. What if we convert mass directly to energy $E = mc^2$

$$\frac{m}{\Delta t} = \frac{L}{c^2} = \frac{3.8 \cdot 10^{26} W}{(3 \cdot 10^8)^2} \approx 4.2 \cdot 10^9 \frac{Kg}{s}$$

Ok with this rate of conversion

$$T = \frac{M_\odot}{4.2 \cdot 10^9} = \frac{2 \cdot 10^{30}}{4.2 \cdot 10^9} = 0.5 \cdot 10^{21} \approx 5 \cdot 10^{20} \text{ sec}$$

$$\approx 1.7 \cdot 10^{13} \text{ years}$$

sounds quite plenty to "the end of the world."

Note here we assumed total mass to energy conversion which is quite unrealistic.

~~Stel~~ Most likely we converto only fraction of mass.

Stars have plenty of 'H', let's convert it to 'He'

$m_H = 1.0078\ u$

$M_{He} = 4.002603\ u$

$1u = 1.66 \cdot 10^{-27}\ kg$
$\Rightarrow mc^2 \Rightarrow 931.49\ \frac{MeV}{c^2}$
$1 MeV = 1.6 \cdot 10^{-13} J$

Note ~~the~~ that reaction is

$$4H \longrightarrow 1He + \Delta E$$

$$\frac{\Delta E}{c^2} = 4 \cdot (1.0078) - 4.002603 =$$

$$= 0.028\ u = 0.028 \cdot \left(\frac{931.49\ MeV/c^2}{1.6 \cdot 10^{-13}} \right)^{-1} =$$

$$~~Let~~ = 4.8 \cdot 10^{-18}\ \frac{J}{c^2}$$

also noticed that $\frac{4M_H - M_{He}}{4M_H} = \frac{0.028}{4 \cdot 1.0078} \approx$

$$= \frac{0.028}{4} = 0.007 \simeq 0.7\%$$

i.e we can convert only 0.7% of the sun mass in such ~~a~~ reaction

So instead of 10^{13} years we have factor of 100 less. Which is still perfectly fine.