lecture 15

Strange paradox

"crazy" intensity $\frac{W}{m^3\,sr}$ – brightness per wavelength

if $dI_\lambda = -\kappa_\lambda \rho I_\lambda ds$

then for long enough pathes

$I_\lambda \sim I_\lambda(0)\, e^{-\kappa_\lambda \rho s} \longrightarrow 0$

So ~~th~~ absorbing object should be dark.

This is clearly not a case with stars, which absorb but shine

So we need a correction to above equation

absorption coeff — absorption — emission coeff

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds + j_\lambda \rho ds$$

with $d\tau = -\kappa_\lambda \rho ds$ optical depth diff

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - \frac{j_\lambda}{\kappa_\lambda \rho} = S_\lambda$$

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda$$

Intensity propagation (Note: this is crazy astrophysist intensity in $\left[\frac{W}{m^3 sr}\right]$ while normal I is $\left[\frac{W}{m^2}\right]$)

$$dI_\lambda = \underbrace{-\kappa_\lambda \rho I_\lambda \, ds}_{\text{absorption}} + \underbrace{j_\lambda \rho \, ds}_{\text{source or emission}}$$

net change

j_λ - is emission coef.

$$-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - \frac{j_\lambda}{\kappa_\lambda}$$

S_λ - source function

$$\boxed{-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - S_\lambda}$$

Note in equilibrium $\boxed{I_\lambda = S_\lambda}$

Look like feedback stabilization:

if $I_\lambda > S_L$ $\quad I_\lambda \searrow$

if $I_\lambda < S_L$ $\quad I_\lambda \nearrow$

But the problem is S_λ which might quickly change with 's'-distance

In equilibrium we know that steady state of "crazy" intensity $\left[\frac{W}{m^3 sr}\right]$ should be given by Black box radiation

thus $\boxed{I_\lambda = S_\lambda = B_\lambda}$

$$\frac{dI_\lambda}{d\tau_\lambda} e^{-\tau_\lambda} = e^{-\tau_\lambda} I_\lambda - e^{-\tau_\lambda} S_\lambda$$

$$\frac{d(I_\lambda e^{-\tau_\lambda})}{d\tau} = -e^{-\tau_\lambda} S_\lambda$$

$$I_\lambda e^{-\tau_\lambda} \Big|_{+\tau_{\lambda,0}}^{0} = -\int_{\tau_\lambda}^{0} e^{-\tau_\lambda} S_\lambda \, d\tau_\lambda$$

$$I_\lambda(0) = I_\lambda(\tau_{\lambda,0}) e^{-\tau_{\lambda,0}} - \int_{\tau_{\lambda,0}}^{0} e^{-\tau_\lambda} S_\lambda \, d\tau_\lambda$$

at surface

certain optical depth

if $S_\lambda = \text{const}$

$$\boxed{I_\lambda(0) = I_\lambda(\tau_{\lambda,0}) e^{-\tau_{\lambda,0}} + S_\lambda (1 - e^{-\tau_{\lambda,0}})}$$

$$\tau_{\lambda,0} \longrightarrow \infty \quad \Rightarrow \quad \boxed{I_{\lambda,0} = S_\lambda}$$

Also note that if there is no change in I_λ

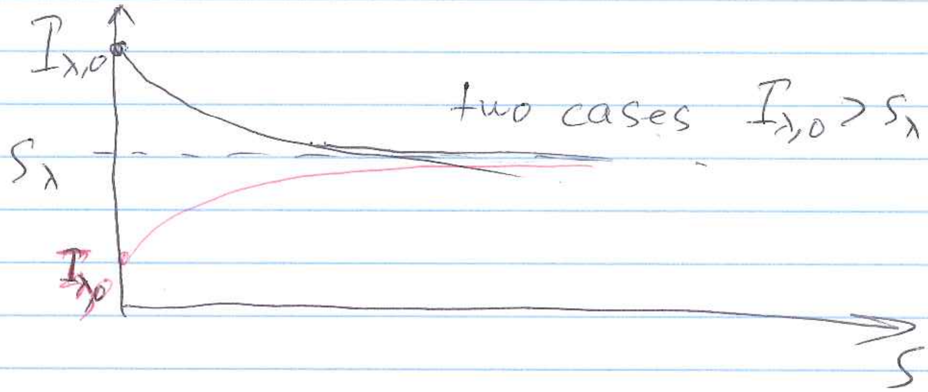
$$0 = \frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda \quad \Rightarrow \quad I_\lambda = S_\lambda = B_\lambda$$

Black Body radiation

in equilibrium

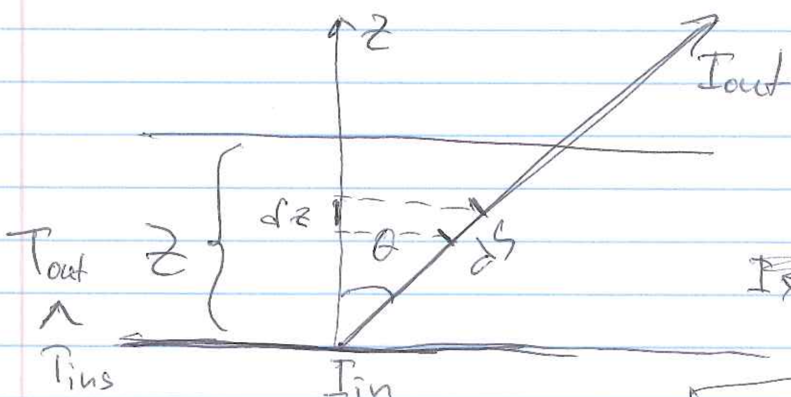
If $k_\lambda = \text{const}$, and $S_\lambda = \text{const}$

$$\Rightarrow \underline{I_\lambda = I_{\lambda,0} e^{-k_\lambda \rho s} + S_\lambda \left(1 - e^{-k_\lambda \rho s}\right)}$$



two cases $I_{\lambda,0} > S_\lambda > I_{\lambda,0}$

lets consider a beam propagating along angle θ with vertical

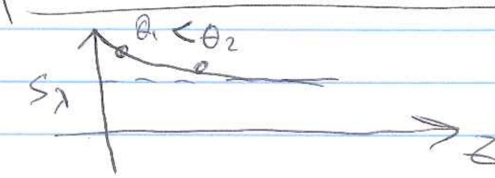


$$\frac{dz}{\cos\theta} = ds$$

~~$I_\lambda(s) =$~~

$$\frac{1}{k_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - S_\lambda \Rightarrow \boxed{\frac{\cos\theta}{k_\lambda \rho} \frac{dI_\lambda}{dz} = I_\lambda - S_\lambda}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

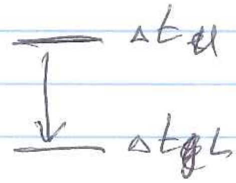


for constant S_λ, k_λ

$$I_\lambda = I_{\lambda,0} e^{-\sec\theta k_\lambda \rho z} - S_\lambda \left(1 - e^{-\sec\theta k_\lambda \rho z}\right)$$

line width:

natural linewidth



$$\boxed{\Delta f = \frac{1}{\pi}\left(\frac{1}{\Delta t_u} + \frac{1}{\Delta t_L}\right) = \frac{1}{\Delta t}\frac{1}{\cancel{2}\pi}}$$

$\Rightarrow$ $H\alpha$ line with $\lambda = 656.3\ nm$

$\Delta t = 10^{-8}\ s$

$$\Rightarrow \Delta f = \frac{1}{\cancel{2}\pi}\frac{1}{10^{-8}} = \frac{10^{8}}{\cancel{2}\pi}\ Hz = \left(\frac{100}{\cancel{2}\pi}\right) MHz \approx 30\ MHz$$

$$\Delta\lambda = \frac{c}{f_1} - \frac{c}{f_2} = \frac{c}{f_1} - \frac{c}{f_1 + \Delta f} =$$

$$= \frac{c}{f_1}\left(1 - \left(1 - \frac{\Delta f}{f_1}\right)\right) = \frac{c}{f_1}\,\frac{1}{f_1}\cdot\frac{c}{c}\,\Delta f$$

$$\boxed{\Delta\lambda = \frac{\lambda^2}{c}\Delta f = \frac{\lambda^2}{c}\,\frac{1}{\pi\Delta t}}$$

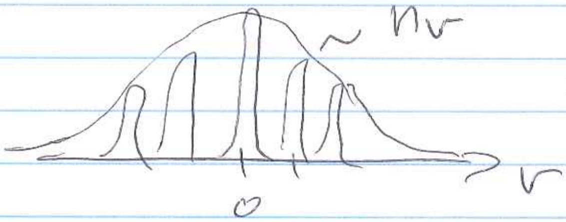
$\lambda = 656\ nm$

$\Delta t = 10^{-8}$

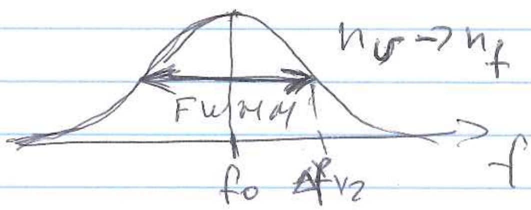
$\Delta\lambda = 4.5 \cdot 10^{-5}\ nm$

Doppler ~~the~~ Broadening

D. shift $= f_0 \cdot \frac{v}{c} = \Delta f$



$=$ if FWHM >> natural linewidth



$$n(v) = \sim e^{-\frac{mv^2}{2kT}} =$$

$$\Rightarrow n(\Delta f) = e^{-\frac{m\left(\frac{\Delta f}{f_0}c\right)^2}{2kT}}$$

$\Delta f_{1/2}$ when $n(\Delta f_{1/2}) = \frac{1}{2}$

$$e^{-\frac{m}{2kT}\left(\frac{\Delta f}{f_0}c\right)^2} = \frac{1}{2}$$

$$\frac{m}{2kT}\underbrace{\left(\frac{\Delta f\, c}{f_0}\right)}_{v_{1/2}}^2 = \ln 2$$

$$v_{1/2} = \sqrt{\frac{2kT}{m}\ln 2}$$

$$\Delta f_{1/2} = \underbrace{\left(\frac{f_0}{c}\right)}_{\lambda^{-1}}\sqrt{\frac{2kT}{m}\ln 2} \Rightarrow \text{FWHM} = 2\Delta f_{1/2}$$

Recall that

$$\Delta\lambda = \frac{\lambda^2 \Delta f}{c} = \frac{\lambda^2}{c} \underbrace{\left(\frac{f_0}{c}\right)}_{\lambda^{-1}} 2\sqrt{\frac{2kT}{m}\ln 2}$$

$$= \frac{\lambda}{c}\, 2\sqrt{\frac{2kT}{m}\ln 2}$$

$$\text{FWHM} = \frac{1}{\lambda}\, 2\sqrt{\frac{2kT}{m}\ln 2}$$

$\lambda = 656\,\text{nm}$

$$= \frac{1}{656\cdot 10^{-9}}\, 2\sqrt{\frac{2\cdot 1.38\cdot 10^{-23}\; 5800\text{K}}{\underbrace{1.67\cdot 10^{-27}\,\text{kg}}_{m_H}} \underbrace{\ln 2}_{\simeq 0.7}}$$

$$= 2.48\cdot 10^{10} \simeq 24\,\text{GHz}$$

way Broader than natural linewidth

$$\Delta\lambda = \frac{\lambda^2}{c}\Delta f = 3.56\cdot 10^{-11} \simeq 0.035\cdot 10^{-9}\,\text{m}$$

$$\simeq 0.035\,\text{nm}$$

Note for heavier atoms
Doppler broadening is not so massive

Pressure broadening

average

$$= \sqrt{\frac{2kT}{m}}$$

$$\Delta f = \frac{1}{\pi}\frac{1}{\Delta t} = \frac{1}{\pi}\left(\frac{v}{\ell_{m.f.p}}\right) = \frac{1}{\pi} n\sigma \cdot v$$

time between collisions

cross section

$$\Delta f = \frac{1}{\pi} n\sigma \cdot \sqrt{\frac{2kT}{m}}$$

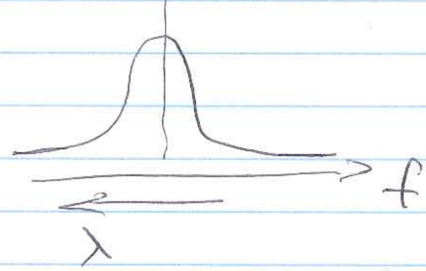
$$\frac{\rho}{m_{atom}} =$$

$$\Delta\lambda = \frac{\lambda^2}{c}\Delta f = \frac{\lambda^2}{c}\frac{n\sigma}{\pi}\sqrt{\frac{2kT}{m}}$$

Voight Profiles

Simle case $K_\lambda = \text{const}$

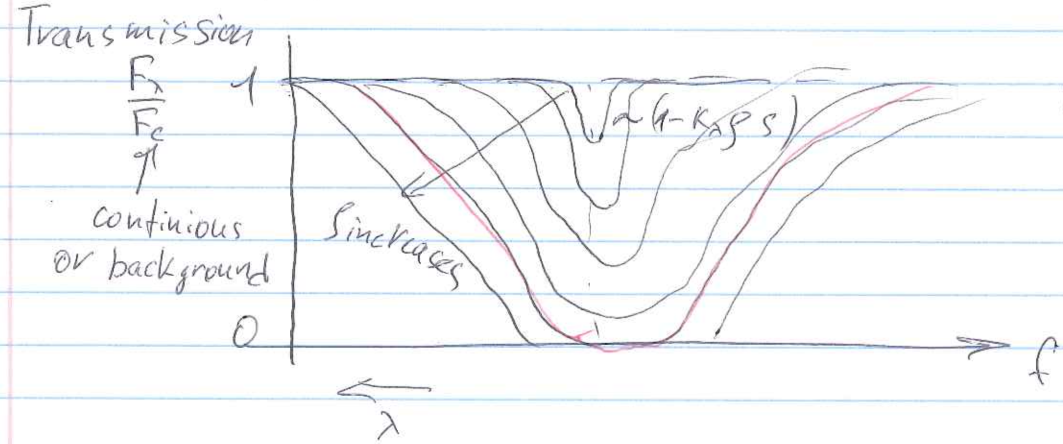
$K_\lambda \sim$ line shape



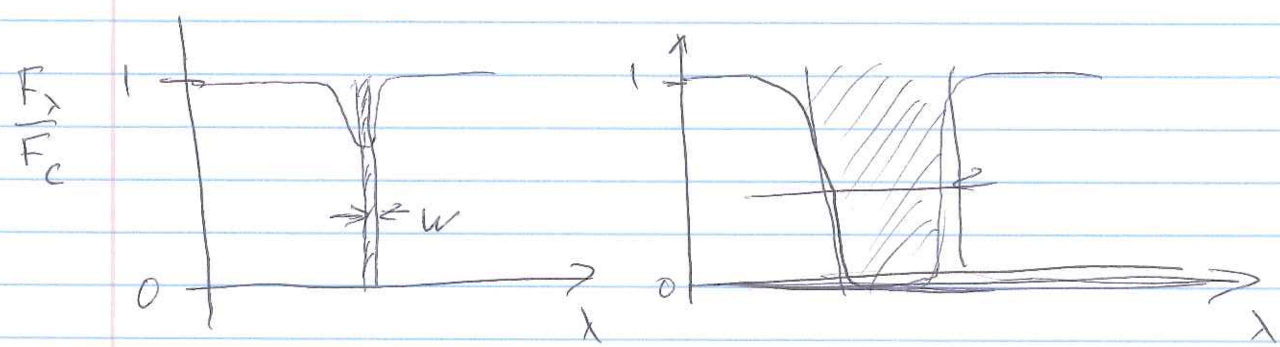
Recall that $I_\lambda(s) = I_\lambda(0)\, e^{-K_\lambda \rho s}$

$$\frac{I_\lambda(s)}{I_\lambda(0)} = e^{-K_\lambda \rho s} = \frac{F_\lambda(s)}{F_\lambda(0)}$$

ρ of particular atom contributing to absorption



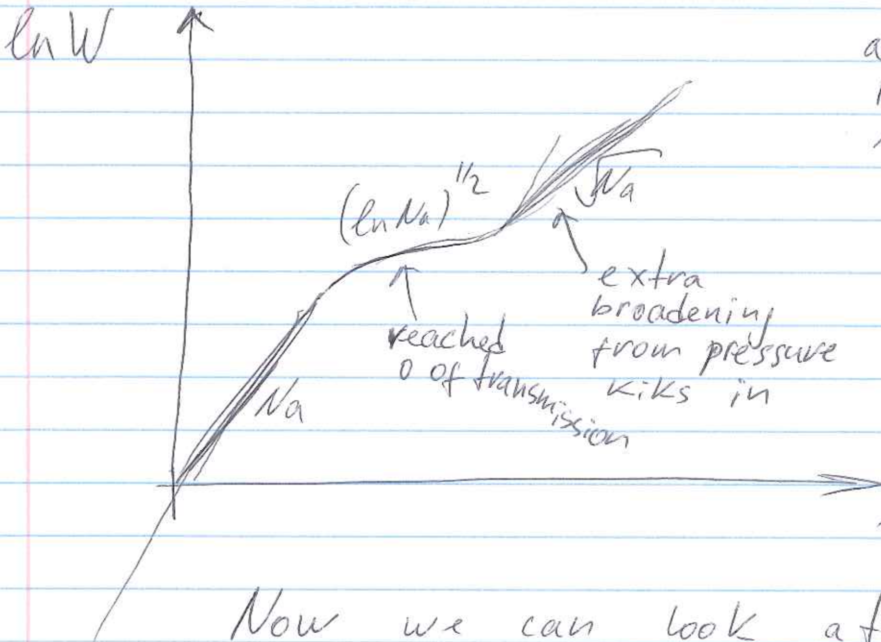
Equivalent width



$$W = \int \frac{F_c - F_\lambda}{F_c}\, d\lambda \quad \Rightarrow \quad W \cdot F_c = \int (F_c - F_\lambda)\, d\lambda$$

"Curve of growth"

W vs ρ or ~~de~~ atom # in the column of sight



at first line profile goes dipper $\sim \ln \rho$
so $\ln W \sim \ln \rho$
then we hit bottom

Now we can look at W and estimate ~~d~~density of particular atom