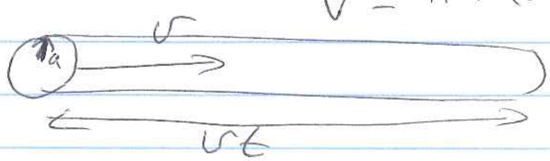
lecture 14

Light Absorption coefficient

$$V = \pi a^2 (v t)$$



N of atoms in this column

$N = n_0 V \Rightarrow$ thus ~~a~~ atoms spaced roughly by dist $\ell = \frac{vt}{N}$ (mean free path)

$$\boxed{\ell = \frac{\cancel{vt}}{n_0 \underbrace{(\pi a^2)}_{\sigma} \cancel{vt}} = \frac{1}{n\sigma}}$$

Since other atoms are moving too there is a correction

to $\boxed{\sigma = 4\pi a^2}$ for Hydrogen $\sigma \simeq 3.52 \cdot 10^{-20}\ m^2$

$$n = \frac{\rho}{m_H} \approx 1.25 \cdot 10^{23}\ m^{-3}$$

thus $\ell = \frac{1}{n\sigma} \approx 2.27 \cdot 10^{-4}\ m$

and within a small distance there are plenty collisions to bring system in the local temperature equilibrium (LTE)

Probability to deflect atom / photon should be

$\sim \dfrac{ds}{\ell_{\text{free path}}}$ ← path length

For Light $\quad dI_\lambda = -\, n\sigma_\lambda I_\lambda \, ds$

↑ photon

coef. of absorption or opacity

$= -\, \kappa_\lambda \rho \, I_\lambda \, ds$

at 500 nm for light in sun

$\rho \simeq 2.1 \cdot 10^{-4}\ \mathrm{kg/m^3}$

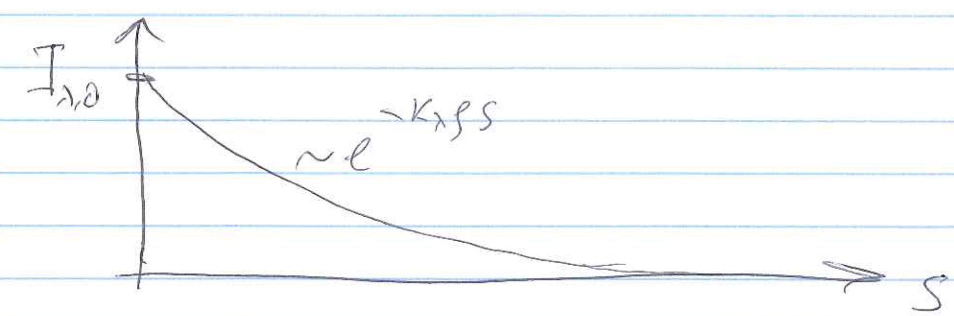
$\ell \simeq 160\ \mathrm{km}$

Optical depth (τ) $\quad \boxed{d\tau_\lambda = -\kappa_\lambda \rho \, ds}$

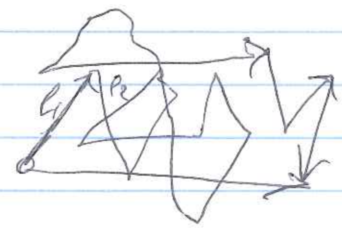
Note $\tau_\lambda = 1$ when $\boxed{s = \ell_{m.f.p.}}$ $\quad$ unit less $\quad \| \quad -\dfrac{ds}{\ell_{m.f.p.}}$

For constant $\kappa_\lambda \rho$

$$I_\lambda(s) = I_{\lambda,0}\, e^{-\kappa_\lambda \rho s}$$



Random walk.



$$\vec{d} = (\bar{l}_1 + \bar{l}_2 + \bar{l}_N)$$

$$|d|^2 = \left(\sum_i^N l_i\right)\left(\sum_j^N l_j\right)$$

$$= \sum_i \sum_j l_i l_j$$

$$= \sum_{i=N} l_{ii}^2 + \sum_{i=1}^{N} \sum_{j \neq i}^{N} l^2 \cdot \underbrace{\cos\theta_{ij}}_{0}$$

$$= N l^2$$

so $d = \sqrt{N}\, l$

$$\tau = \frac{d_0}{l_{mfp}} = \sqrt{N}$$

travel time $= \frac{l_{m.f.p}}{c} \cdot N =$

$$= \frac{l_{m.f.p}}{c} \cdot \left(\frac{d}{l_{m.f.p}}\right)^2$$

To travel the Radius of Sun

we need assuming $l_{m.f.p} = 160$ km = const

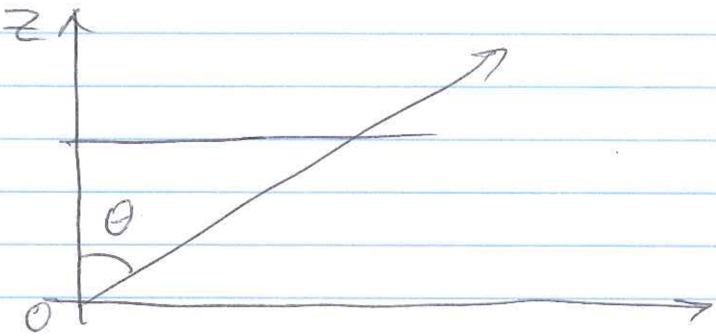
$$t = \left(\frac{160\cdot10^3}{3\cdot10^8}\right)\cdot\left(\frac{7\cdot10^8}{160\cdot10^3}\right)^2 = (0.5\cdot10^{-3}\,s)\cdot 19\cdot10^6$$

$= 10$ kS, it would be just $\frac{7\cdot10^8}{3\cdot10^8} \sim 2s$ if it were a straight path

effective speed $\frac{R_\odot}{t} = 68$ km/s

Recall for $k_\lambda = const$

$$I_\lambda(s) = I_\lambda(0)\, e^{-k_\lambda \rho s}$$



$$I(z) = I_\lambda(z=0)\, e^{-\frac{k_\lambda \rho}{\cos\theta} z}$$

$\frac{k_\lambda \rho}{\cos\theta} z = \tau = const$ = optical depth at which some light arive to us

$$z_{penetration} = \frac{\cos\theta\, \tau}{k_\lambda \rho}$$

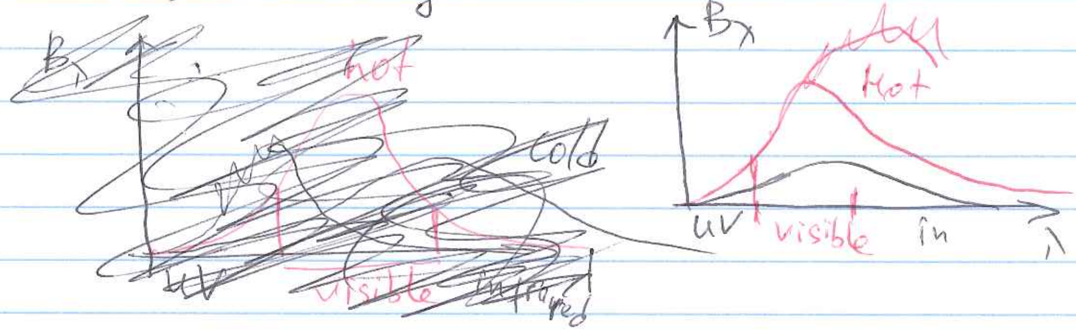
so at ~~different~~ increasing θ – we probe at smaller physical depth (z)

For a view along slab ~~@~~ (large θ)

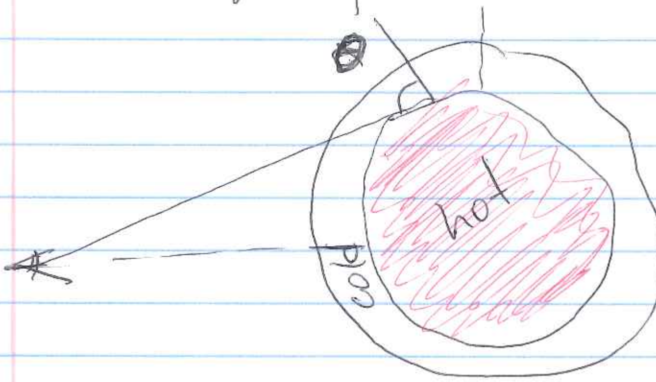
Output intensity replaced with S_λ of this slab $\sim B_\lambda(T_{outside})$

while for $\theta \simeq 0$ we see less attenuation and $I_{out} \sim S(T_{inside})$
$\sim B(T_{inside})$

Thus along slab we see "colder" radiation, which is less bright in visible range

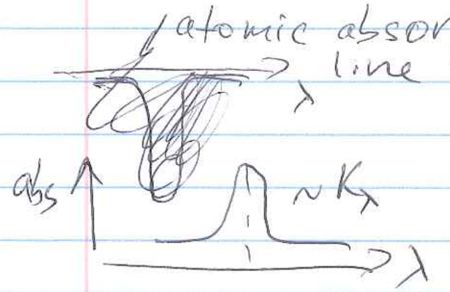


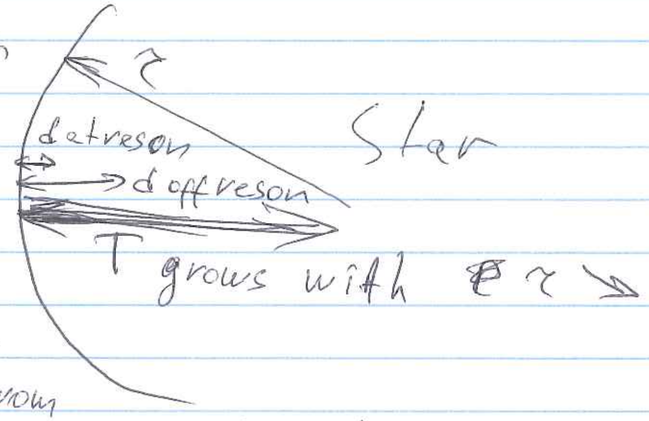
This effect responsible for sharp edges of the Sun.



~~Note this is wavelength dependent. If we were to observe in infrared we would see different situation (actually quite opposite)~~

This is also the reason for dark absorption line appearance in star spectras





thus at resonance we see light from smaller depth since star is more optically thick there, and off resonance we see deeper into the star and thus to hotter temperature

So resulting spectrum is underlying hot BB radiation which replaced with cold BB at resonance