lecture 13

H-R Diagram.

From previous lecture we see that a particular element spectral line will have characteristic temperature dependence.

\[ \frac{N_{i+1}}{N_i} \sim (T)^{3/2} e^{-\frac{\Delta E}{kT}} \]

Different ionization energy \( \Rightarrow \) different position along temperature.

Smaller \( \Delta E \) earlier appearance at low \( T \).

This nice but what can we say about relative abundance?

Above is a profile per particular element.

Overall line strength (observed)

\[ \sim N_{\text{partic}} \times \text{(line strength per element)} \]

Depends on element dipole absorption cross section.
Hertzsprung-Russel diagram (H-R) diagram

- Absolute magnitude
- Super Giants
- Main sequence
- Giant &`.
- White dwarfs

Peculiar observations dependence

How do we know size?

Recall that position along B-V related to temperature,

\[ M \sim \log L = \log R^{2.5} T^4 \]

so for the same temperature large star is more luminous => smaller M.

Note also that temperature places a star in particular O, B, A, F, G, K, M class right away.

But so far no way to say something about luminosity => or size
Fortunately there is one more parameter in spectra — line width, do not mix it with strength.

The same type let's say 'A' will have a narrower lines as we go to a brighter star.

So now just looking at spectra we will now placement of a star at H-R diagram i.e. its T and M notice absolute magnitude M, now if one measure observed 'm' we will know the distance as well.

\[
d = 10 \frac{(m-M)+5}{5}
\]

\[ \text{in pc} \]
Let's draw L-T diagram closely related to H-R

\[ L \sim R^2 T^4 \]

\[ \log L/\text{L}_\odot = k + \log (T/\text{T}_\odot) + 2 \log \frac{R}{R_\odot} \]

Recall experimental observation that \[ M \propto L^\alpha \]

For stars around \( M_\odot \)

\[ M \approx 10^{-4} M_\odot \]

\[ L \approx 10^{-4} \text{L}_\odot \]

\[ R \approx 10^{-4} R_\odot \]

\[ T \approx 10^{-4} \text{T}_\odot \]
Finally, why lines a narrower for giant stars.

Line width related to pressure broadening (spectra show linewidth to large to be explained by Doppler broadening $\propto U_{\text{motional}} \propto \sqrt{T}$)

Pressure broadening $\propto \frac{1}{\text{density}} \sim \frac{1}{\text{average velocity}}$

$\Rightarrow \Rightarrow \text{density} \sim \frac{1}{\sqrt{T}}$

So for the same $T$, giants must have a less dense structure

$\rho_\odot = \frac{M_\odot}{\frac{4\pi}{3} R_\odot^3} = 1410 \frac{\text{kg}}{\text{m}^3}$ a bit more than water (1000 kg/m$^3$)

Betelgeuse

$\rho_B = \frac{10 M_\odot}{\frac{4\pi}{3} (1000 R_\odot)^3} \approx \frac{10^5}{10^3} = \frac{10^2}{10^3}$

$= 1410 \frac{1}{10^8} \approx 1.4 \times 10^{-5}$

$\approx 0.14 \times 10^{-6} \frac{\text{kg}}{\text{m}^3}$

Compare to the air density $1 \frac{\text{kg}}{\text{m}^3}$