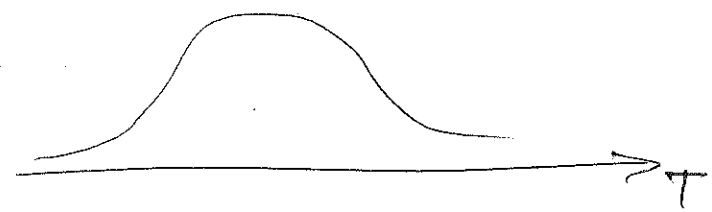


finished 10 minutes earlier

P1

lecture 13 H-R diagram.

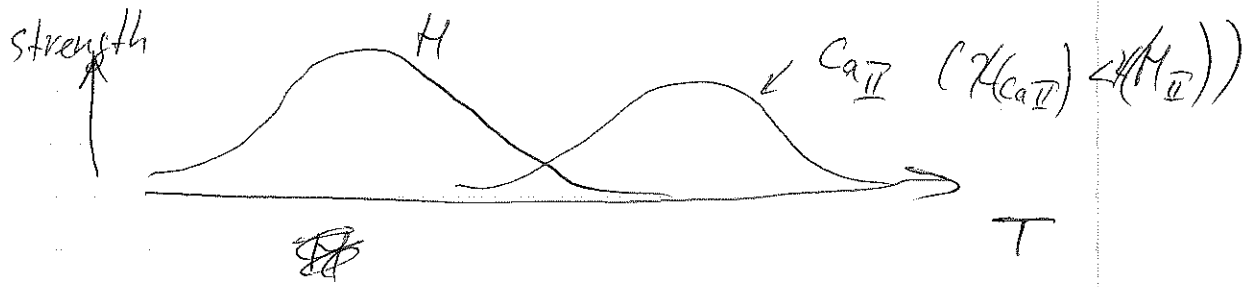
From previous lecture we see that a particular element spectral line will have characteristic temperature dependence



Different ionization energy \Rightarrow different position along T

$$\frac{N_{i+1}}{N_i} \sim (T)^{3/2} e^{-\chi_i/T}$$

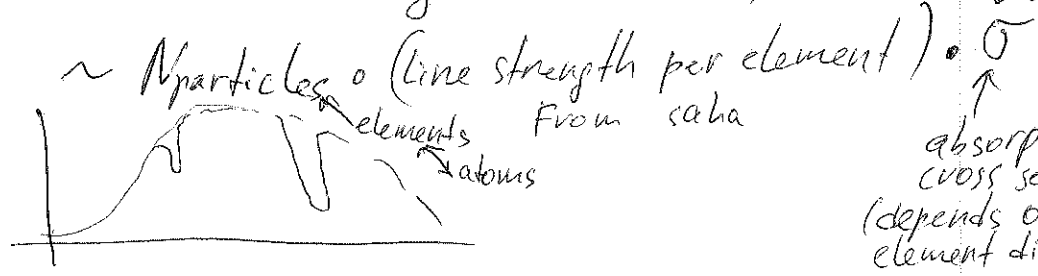
smaller χ earlier appearance at low T



This nice but what can we say about relative abundance?

Above is a profile per particular element

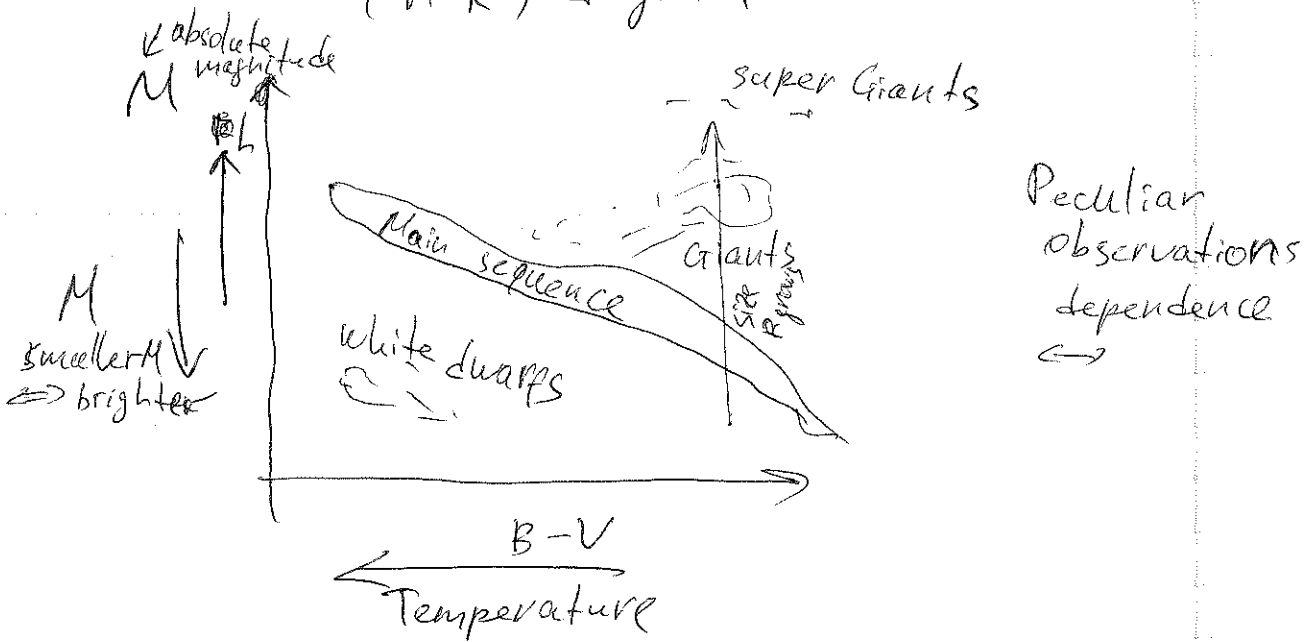
Overall line strength (observed)



measured in labs \downarrow at Earth
 \uparrow absorption cross section (depends on element dipole)

Hertzsprung - Russell Diagram (M-R) Diagram

(p2)



How do we know size?

Recall that position along $B-V$ related to temperature,

$$M \sim \log L \approx \log R^2 \sigma T^4$$

so for the same temperature
 large star is more luminous
 \Rightarrow smaller M .

Note also that temperature places a star in particular O, B, A, F, G, K, M class right away.

But so far no way to say something about luminosity \Leftrightarrow or size

Fortunately there is one more parameter in spectra \rightarrow line width, do not mix it with strength!

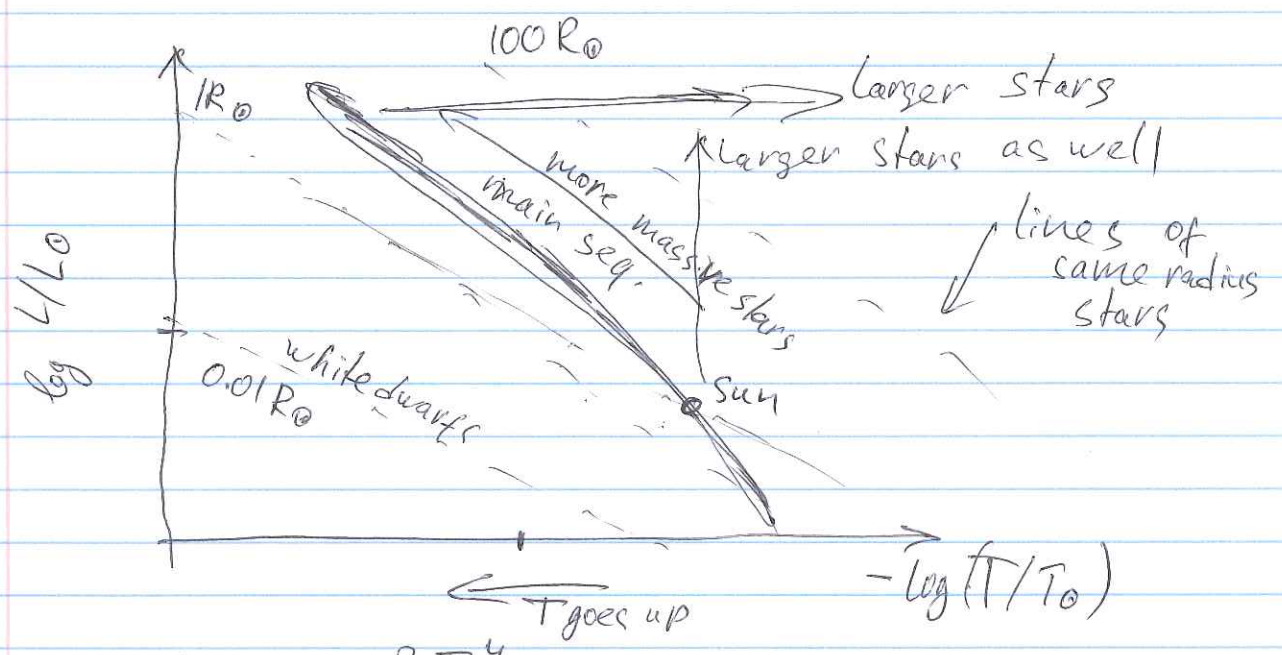
The same ~~type~~ type let's say 'A' will have narrower lines as we go to a brighter star.

So now just looking at spectra we will now placement of a star at M-R ~~fig~~ diagram i.e. its T and M notice absolute magnitude M , now if one measure observed 'm' we will know the distance as well.

$$d = 10^{(m-M+5)/5}$$

$$d \text{ in pc}$$

Let's draw L-T diagram closely related to H-R



$$L \sim R^2 T^4$$

~~$$\log L/L_{\odot} \sim 4 \log(T/T_{\odot}) + 2 \log R/R_{\odot}$$~~

Recall experimental observation

That ~~$M \uparrow \Rightarrow L \uparrow$~~

For stars around M_{\odot}

~~$L/L_{\odot} \approx (M/M_{\odot})^4$~~

not always true

~~$$\frac{L}{L_{\odot}} \sim \frac{R^2 T^4}{R_{\odot}^2 T_{\odot}^4} \approx \left(\frac{M}{M_{\odot}}\right)^4$$~~

~~$$\Rightarrow \frac{(M_{\odot})^4}{R_{\odot}^2 T_{\odot}^4} \approx \frac{M^4}{R^2 T^4}$$~~

(PS)

Finally, why lines are narrower for giant stars.

Line width related to pressure broadening (spectras show linewidth too large to be explained by Doppler broadening $\sim v_{\text{thermal average}} \sim \sqrt{T}$)

Pressure broadening $\sim \frac{1}{t_{\text{between collision}}} \sim \frac{\text{density}}{\text{average velocity}} \sim \sqrt{T}$
 $\Rightarrow \Rightarrow \frac{\text{density}}{\sqrt{T}}$

So for the same T giants must have a less dense structure

$$\rho_{\odot} = \frac{M_{\odot}}{\frac{4\pi R_{\odot}^3}{3}} = 1410 \frac{\text{kg}}{\text{m}^3} \quad \text{a bit more than water (1000 kg/m}^3\text{)}$$

Betelgeuse

$$\begin{aligned} \rho_B &= \frac{10 M_{\odot}}{\frac{4\pi (1000 R_{\odot})^3}{3}} \approx \frac{10 \rho_{\odot}}{10^9} = \frac{\rho_{\odot}}{10^8} \\ &= \frac{1410}{10^8} \approx 1.4 \cdot 10^{-5} \approx 0.14 \cdot 10^{-6} \frac{\text{kg}}{\text{m}^3} \end{aligned}$$

Compare to the air density ~~1 kg~~ $1 \frac{\text{kg}}{\text{m}^3}$