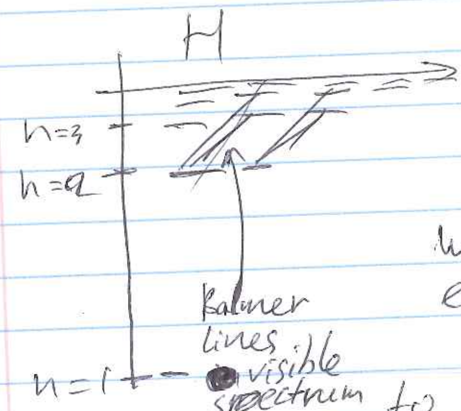
lecture 12 ~~Et~~ Disappearing spectral lines and Saha equation

it was noticed that many spectral lines appear and then dissappear as one sort the stars spectra in increasing temperature order.

~~At~~ Luck of line at small temperature corresponding to transitions from non ground level is not a big puzle:



For example for Balmer lines corresponding to absorption of a photon from $n=2$ to $n' > 2$

We need to have some electron population (i.e. probab. of state to be occupied) to be quite signipicant

~~P(2)~~ Boltzmann distribution dictates

$$\frac{P(n=2)}{P(n=1)} = \frac{\frac{1}{Z}\, g_{n=2}\, e^{-E_2/kT}}{\frac{1}{Z}\, g_{n=1}\, e^{-E_1/kT}} = \frac{2\cdot 4}{2\cdot 1}\, e^{-\frac{(E_2-E_1)}{kT}}$$

degeneracy of a state with $n=2,1$

$$g_n = \underbrace{2}_{\text{spin}} \cdot \underbrace{n^2}_{l\ \text{degeneracy}}$$

$$Z = \sum_{n=1}^{\infty} g_n\, e^{-E_n/kT}$$

(p2)

$$\frac{P(n=2)}{P(n=1)} = 4 \cdot e^{-\left(-\frac{13.6eV}{4} + \frac{13.6eV}{1}\right)\frac{1}{kT}}$$

$$= 4 \cdot e^{-\frac{3}{4}\frac{13.6eV}{kT}} =$$

$$/ \quad 1eV = 11'600\ K \quad /$$

$$= 4 \cdot e^{-\frac{3}{4}\cdot\frac{13.6\cdot 11600}{T}} = 4 e^{-118000/T}$$

In order to get any reasonable population at $n=2$ we need huge Temperatures $\left(\boxed{T \sim 10^5}\right.$, while ~~stars~~ even the hot one have $\left.\boxed{T \lesssim 40000\,K}\right)$.

Apperance of the line from $n=3 \to n>3$ is even less probable

Note: it seems that for high n

$$\frac{p(n \to \infty)}{p(n=1)} = \frac{2 \cdot n^2 e^{-E_\infty/kT}}{2 \cdot e^{-E_1/kT}} \sim \underbrace{n^2}_{\text{goes to } \infty}\ \underbrace{\boxed{e^{\frac{-13.6eV}{kT}}}}_{\text{fixed}}$$

so we might think that ~~we~~ high 'n' levels might be well populated compare to the ground levels but

$n = \infty$ is unrealistic

size of the electron orbit grows as $\boxed{r_n \sim a_0 n^2}$ for Hydrogen

~~et~~ so high 'n' is ~~ph~~ ~~ph~~ unphysical

So far so good, so we see that line should increase it strength (~~also~~ absorption grows) with higher temperature but why it disappear at high T ?

Idea 1, hot stars are made of not hydrogen. But what would be the mechanism ?

Idea 2: neutral Hydrogen replaced with its ion.

$H \rightarrow H^+ + e^-$, no electron around proton then no ~~problem~~ absorption ~~state~~ at the former neutral hydrogen transition.

if we call N_{i+1} - number of atoms with i electron removed i.e. $i \Leftrightarrow$ ionization level

Saha eq.

partition function

$$\boxed{\frac{N_{i+1}}{N_i} = \frac{2 Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/T}}$$

electron density

χ_i - ionization energy $i \rightarrow i+1$

$$Z_i = \sum_n g_n e^{-\frac{E_{n_i} - E_{g_i}}{kT}}$$

m_e - electron mass

n_e - electron density

ideal gas $P_e = n_e kT$

pressure

Note that now temperature dependence favor ions much more drasticly

$$\frac{N_{i+1}}{N_i} \sim (T)^{3/2} e^{-\chi/kT}$$

$\frac{1}{n_e}$ – probability of ions to recombine higher if there is abundance of free electrons

$\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}$ ~ to available space in velocity domain

$\# \sim v^3 \sim (kT)^{3/2}$

volume in velocity

$$E = \frac{3}{2}kTn = n\frac{mv^2}{2}$$

n_e – is hard to define from 1st principle, it deduced from observed line pressure broadening.

also note that for hydrogen

$$n_e = N_{II} \Longleftrightarrow \boxed{\underbrace{\text{ion} + e^-}_{1:1 \text{ ratio}} = M}$$

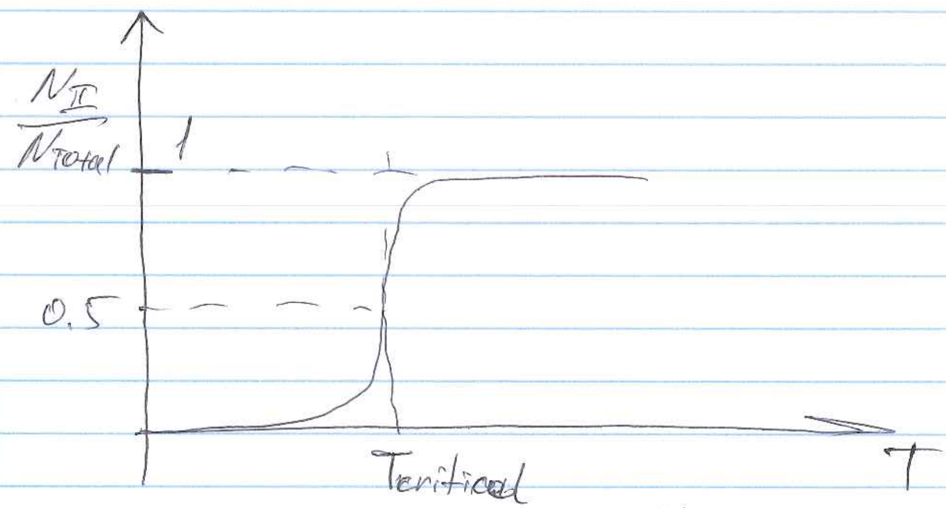
Ionization level – assume hydrogen only star then H_{II} is the hyghest

$Z_{II} = 1 \leftarrow$ only one way to make strip of Hydrogen

$$Z_I \sim 2 \cdot e^{-E_g/kT} + \text{small corrections}$$

$\uparrow$ population mostly at ground level.

Then $$\frac{N_{II}}{N_{Total}} = \frac{N_{II}}{N_I + N_{II}} = \frac{\frac{N_{II}}{N_I}}{1 + \frac{N_{II}}{N_I}}$$



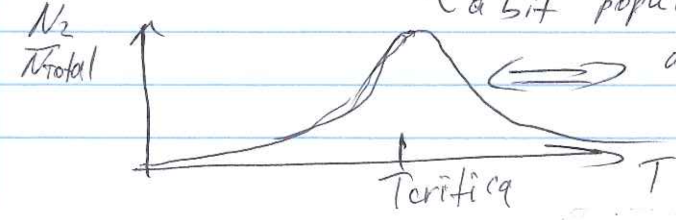
for $P_e = 20 \frac{N}{m^2} = 20\ Pa$

$T_{critical} \simeq 10'000$ "4"

Note that $N_2 \sim N_1 e^{-(E_2 - E_1)/kT} \cdot \left(\frac{g_2}{g_1}\right)$

$N_I = N_1 + N_2$

$\uparrow$ a bit population to exited level



$\Longleftrightarrow$ appearance and dissapearance of absorption line with $T \nearrow$