Lecture 11

Star Composition via Spectroscopy

① let's recall quantum mechanic and in particular states of Hydrogen atoms.

for non Hydrogen atom and one electron $E_0 \propto Z^2$ ← nuclear charg

$$E_n = -\underbrace{13.6}_{E_0}\, eV \frac{1}{n^2} \quad , \quad n = 1, 2, 3 \ldots$$

Strictly speaking 'n' by itself is not enough to describe H state, we need extra 'quantum properties'

ℓ - related to angular momentum
m_ℓ - its projection
m_s - spin of electron $m_s = \pm 1/2$

$\ell = 0, 2, 3, \ldots n-1 \Rightarrow n$ combinations
$m = -\ell, -\ell+1, 0, 1, 2, \ldots \ell \Rightarrow (2\ell+1)$ ~~states~~ combinations
$m_s = \pm 1/2 \Rightarrow 2$ combinations

So for atom with energy E_n there are

$$\# \text{combinations} = \sum_{\ell=0}^{n-1} (2\ell+1) \cdot 2 = \cancel{2 \cdot (n-1) + 2 \cdot \sum_{\ell=0}^{n-1} \ell}$$

$$\cancel{= 2(n-1) + 2\frac{0 + n-1}{2} \cdot (n)} =$$

$$= \cancel{2(n-1) +} (n-1)(n) =$$

$$\cancel{= (n-1)^2 - 1}$$

$$= 2 \cdot \frac{1 + [2(n-1)+1]}{2} \cdot n = 2 \cdot n^2$$ ← degeneracy

$E_n \Rightarrow E_{(n, e, m_l, m_s)}$

$\Updownarrow$

State description

$\Downarrow$

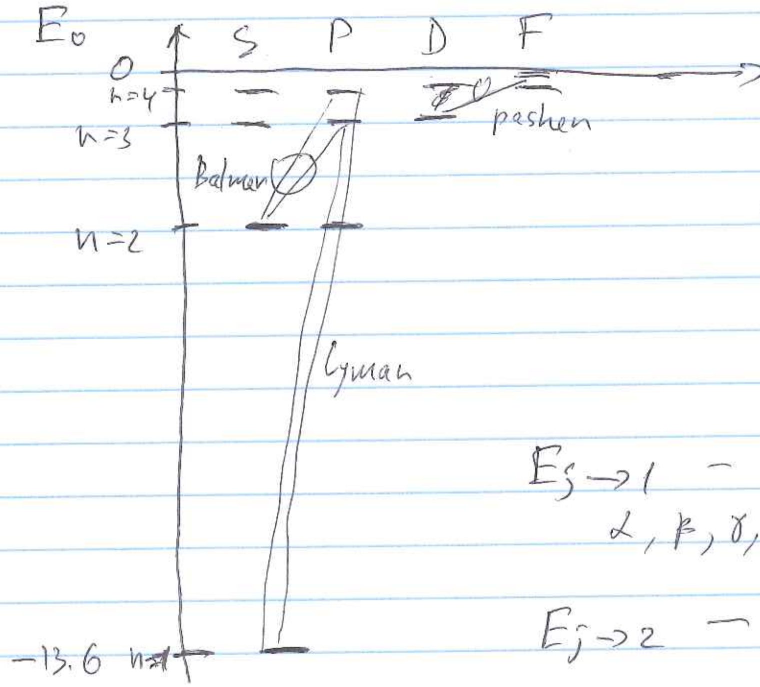
$\cancel{2m_s+1}$ $\quad$ $^{(2s+1)}L \Rightarrow$ s = spin part for hydrogen $2s+1 = 2$

$l = 0, 1, 2, 3, 4, 5 \quad n-1)$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$S \quad P \quad D \quad \underbrace{F \quad G \quad H \quad \ldots}_{\text{aephabetically}}$

sharp, principal, diffuse, fundamental



$\Delta l = \pm 1$

Δn = anything

$\Delta m = 0, \pm 1$

$E_{j \to 1}$ − Lyman

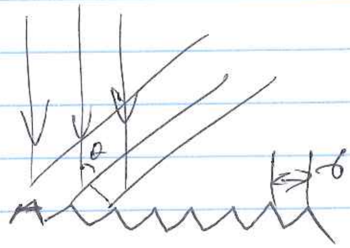
$\alpha, \beta, \gamma, \ldots$ energy dif grows

$E_{j \to 2}$ − Balmer

$E_{j \to 3} \to$ Pashen

$$\frac{1}{\lambda_{nm}} = \frac{\Delta E}{h} \cdot \frac{1}{c} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \Rightarrow R_H = \frac{13.6\,eV}{hc} = 1.096 \cdot 10^7\ m^{-1}$$

Spectral resolution



condition for max $\quad \Delta \ell = d \cdot \sin\theta = n\lambda$

recall that a slit gives a angular spread $\sim \frac{\lambda}{D}$ size of the slit

so $\underbrace{\Delta\lambda}_{\text{spectral resolution}} \Longrightarrow \Delta\theta \sim \underbrace{\frac{n\Delta\lambda}{d}}_{\text{dispersion}} \simeq \underbrace{\frac{\lambda}{D}}_{\text{angular resolution}}$

$$\Delta\lambda_{sp} = \Delta\lambda = \frac{\lambda}{n}\frac{d}{D}, \qquad \frac{d}{D} \approx \frac{1}{N}$$

$\Delta\lambda$: smaller the better

N: number of groves

$$\boxed{\Delta\lambda = \frac{\lambda}{n}\frac{1}{N}}$$ resolving power

What is number of groves N needed to detect 60 cm/s at $\lambda = 600$ nm

$$\Delta\lambda = \frac{c}{f_1} - \frac{c}{f_2} \approx \frac{c}{f_1} - \frac{c}{f_1+\Delta f} \simeq$$

$$= \underbrace{\frac{c}{f_1}}_{\lambda} \frac{\Delta f}{f}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{60 \cdot 10^{-2}\,\frac{m}{c}}{c} = \frac{6 \cdot 10^{-1}}{3 \cdot 10^{8}} =$$

$$= 2 \cdot 10^{-9} = \frac{1}{N}$$

$$N = 0.5 \cdot 10^{+9} = 5 \cdot 10^{8}$$

typically 1000 groves per mm

$$\Rightarrow D = \frac{N}{1000/mm} = 5 \cdot 10^{5}\ mm$$

$\Rightarrow$ 500 m $\Leftrightarrow$ sounds impractical!!

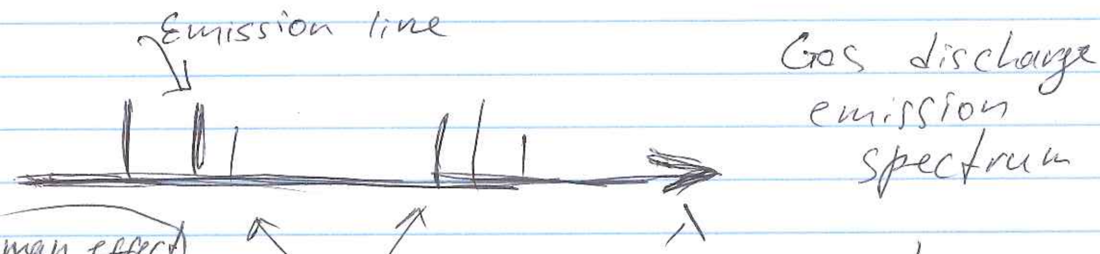
$1\,eV \Leftrightarrow kT \Leftrightarrow T = 11{,}6\,kK$

$$p(E_n) \approx (\text{degeneracy}) \cdot e^{-\frac{E_n}{kT}}$$

$$\Rightarrow \quad \frac{P(E_n)}{P(E_1)} = e^{\frac{-(E_1 - E_n)}{kT}} \approx \underline{\underline{\text{very small}}}$$

for $T < 10\,kK$

Similar structures but with different wavelength ~~a~~ exist for all atoms

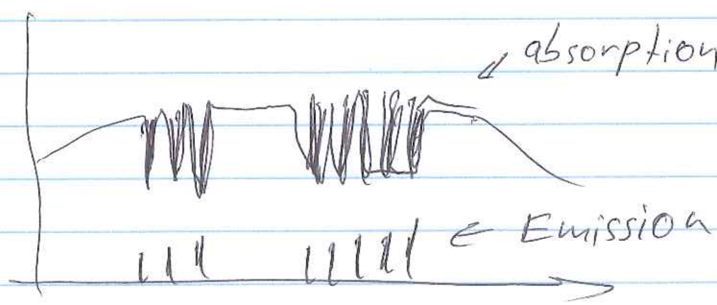


Zeeman effect

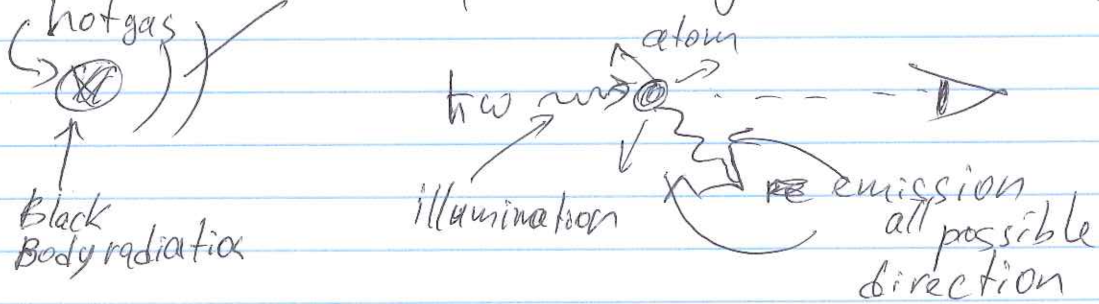
spectral fingerprint.

Splitting in magnetic field

Why in star spectrum we see absorption line i.e.



shell of cooler gas (star atmosphere)



λ_{mn} absorption is proportional to # of atoms supporting this wave length × population of the bottom level 'm' − population of upper level 'n' i.e. how many atoms are ready to grab this photon

lecture 11 stops here.

We can learn more from these spectral lines

Spectral width → related to Doppler broadening ⇒ recall

$\Delta f = f_0 \frac{v}{c}$ in thermal Maxwell distribution

$\sim v^2 e^{-\frac{mv^2}{2kT}}$

Pressure braadening → due to atoms pushing each other ⇒ pressure

& Abundance of atoms by ~~seeing~~ observing relative strength of ~~atoms~~ particular atomic lines

Periodic shifts of the line ⇒ ⇒ star speed / velocity vs time.

Magnetic field ⇔ Zeeman splitting