Lecture 10

Thus radiation of black body

is Energy emitted per unit area, per unit time into unit solid angle, per spectral unit of wavelength

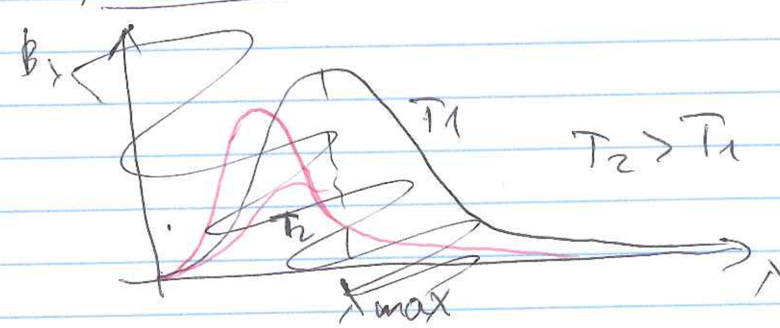
$[B_\lambda] \Rightarrow \frac{W}{m^2 \cdot s \cdot m \cdot sr}$ ← solid angle

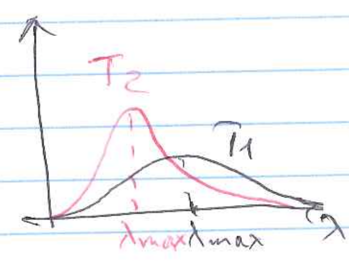
Area, time, wavelength

$$dI_\lambda = d\lambda\, d\Omega\, B_\lambda = \frac{4\pi}{\lambda^5}\hbar c^2 \frac{d\lambda\, d\Omega}{e^{\frac{\hbar 2\pi c}{\lambda}/kT} - 1}$$

recall that $\hbar = h/2\pi$

$$\boxed{B_\lambda\, d\lambda\, d\Omega = \frac{2hc^2}{\lambda^5} \frac{d\lambda\, d\Omega}{e^{\frac{hc}{\lambda kT}} - 1}}$$





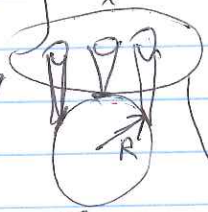
Wein's law displacement $\quad \lambda_{max} T = const \simeq (500nm)(5800K)$

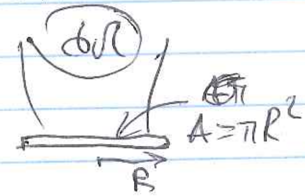
Q: Why same temperature stars have different Brightness?

A: Stefan - Boltzman law

Total energy emitted per unit time from the surface

$$L = \int u_\lambda \cdot c \, d\lambda \cdot dA = 4\pi R^2 c \int u_\lambda \, d\lambda =$$





$$= \cancel{4\pi R^2 c \cdot 16\pi} \iff$$

Black Body Emit in all directions

star

$$= \pi \int_A d I_\lambda \, dA =$$

extra π

$$= (\pi \cdot 4\pi R^2) \cdot \underline{16\pi^2} \hbar c^2 \int_0^\infty \frac{1}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda kT}} - 1}$$

$$/ \quad x = \frac{hc}{\lambda kT} \Rightarrow d\lambda = d\left(\frac{hc}{x kT}\right) = -\frac{hc}{x^2 kT} dx \quad /$$

$$\lambda = \frac{hc}{kTx}$$

$$= \pi \cdot 4\pi R^2 \cdot 16\pi^2 \hbar c^2 \int_0^\infty \left(\frac{kT}{hc}\right)^5 x^5 \frac{\frac{dx}{x^2}\left(\frac{hc}{kT}\right)}{e^x - 1} d\lambda$$

$$P = L = \pi \cdot 4\pi R^2 \, 16\pi^2 \hbar c^2 \left(\frac{k_B}{hc}\right)^4 T^4 \underbrace{\int_0^\infty x^3 \frac{1}{e^x - 1} dx}_{\frac{\pi^4}{15}}$$

↑ power

Q: $\boxed{L = \sigma 4\pi R^2 T^4}$

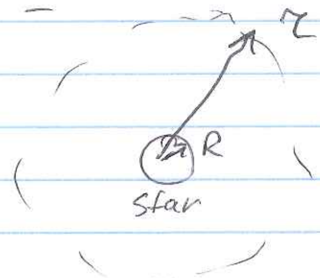
↑ star Area of surface

$$\boxed{\sigma = \frac{16}{15} \pi^7 \hbar c^2 \left(\frac{k_B}{hc}\right)^4}$$

Physiological term

Appeared Brightness proportional to received power of radiation. i.e. Flux times area of detector

$$\text{Flux} = \frac{\text{Emitted power}}{\text{Area}} =$$



$$= \frac{L}{4\pi r^2}$$

$$\boxed{F = \frac{4\pi R^2 \sigma T^4}{4\pi r^2} = \left(\frac{R}{r}\right)^2 \sigma T^4}$$

Larger star looks brighter

notice that this is not the same as a physics definition of brightness

more distant star looks dimmer,

hoter star is alco brighter

$$\Rightarrow F_\lambda = \frac{B_\lambda 4\pi^2 R^2 d\lambda}{\pi r^2}$$

Relative magnitudes

Eye persive flux in somewhat logariphmic scale (similar to ear).

So astronomers agreed to say

that if Flux increased ~~by~~ by 100 times than it went down by 5 magnitude

$$\Rightarrow \alpha \log_{10} \frac{100 F_0}{F_0} = -m = -5$$

$$\alpha \cdot 2 = -5 \Rightarrow \alpha = -2.5$$

So to calculate ~~flux~~ relative magnitude $(=m)$ we just compare fluxes

$$\boxed{m = -2.5 \log_{10} \frac{F_1}{F_2}}$$

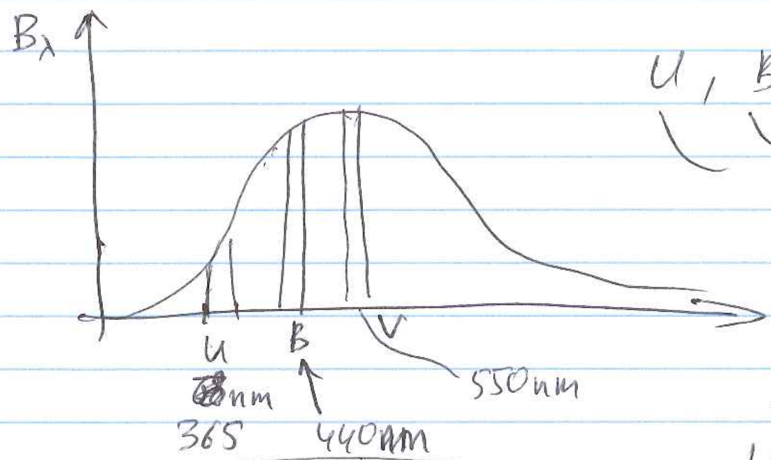
if $\boxed{m = 1} \Rightarrow \boxed{F_1 = F_2 \cdot 10^{\frac{1}{2.5}}} = \boxed{= 2.51 \cdot F_2}$

Absolute magnitude is when we compare flux as if a star placed at 10pc

$M_{\odot} = +4.74$ $\qquad M = m - 5 \log_{10} \left(\frac{d}{10pc}\right)$

$M_{Sirius} = \approx 1.42$

Color index

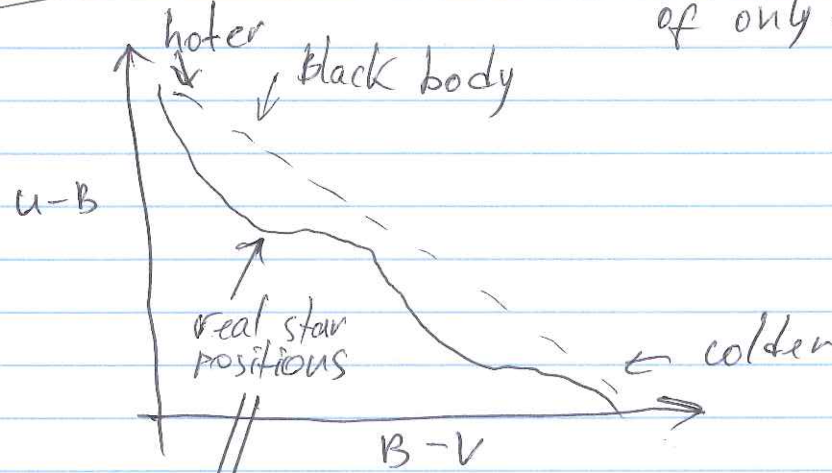


U, B, V magnitudes

ie $\sim \log_{10} B(\lambda)$ $\leftarrow \lambda_U, \lambda_B, \lambda_V$

easy to see that $U-B$ and $B-V$ for black body is fixed and is function of only temperature

$\boxed{\text{Physiology: ~~Why~~ no Green stars? but red and Blue}}$



thus stars are not ideal Black Bodies. strickly speaking star temperature is not a constant across its body.