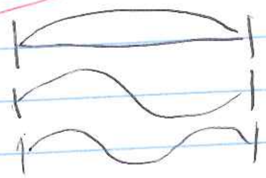
Q: why stars have different colors?

Q: how to find a star temperature

Black-Body radiation derivation

Lecture 9



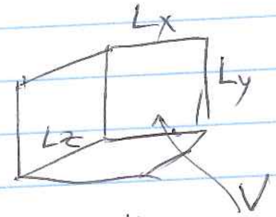
$\frac{\lambda}{2} = L$

$\lambda = L \Rightarrow L = n\frac{\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$, $n = 1, 2, 3 \ldots$

$\frac{3\lambda}{2} = L$

$$k_i = \frac{2\pi}{\lambda_i} = \frac{n}{2L_i} \qquad (i = x, y, z)$$

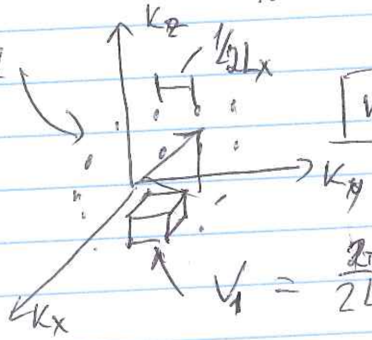
Box in equilibrium



$$\Rightarrow \vec{k}^2 = k_x^2 + k_y^2 + k_z^2 = \frac{2\pi}{\lambda}$$

$$k^2 = (2\pi)^2\left[\left(\frac{n_x}{2L_x}\right)^2 + \frac{n_y^2}{(2L_y)^2} + \frac{n_z}{(2L_z)^2}\right]$$

1 octet



$\boxed{n \geq 1}$

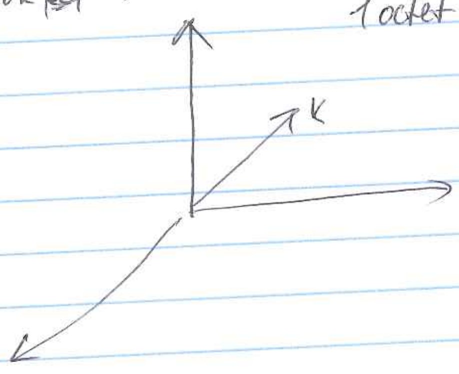
$$V_1 = \frac{2\pi}{2L_x} \cdot \frac{2\pi}{2L_y} \cdot \frac{2\pi}{2L_z} = \frac{1}{8}\frac{8\pi^3}{V}$$

number of waves with wave vector $\leq k$

$$N_k \simeq \frac{4\pi}{3}k^3 / V_1 \cdot \left(\frac{1}{8}\right) \Rightarrow N_k = \int_0^k n_k \, dk$$

1 octet

density of states



$$n_k = \frac{dN_k}{dk} = \frac{4\pi k^2}{8V_1}$$

$$n_k = \frac{4\pi k^2 V}{8\pi^3 \cdot 8} = \frac{1}{\pi^2}\frac{k^2 V}{2}$$

For light $\quad k \rightarrow E = hf = h\frac{c}{\lambda} =$

$$= \frac{2\pi}{\lambda} c \frac{h}{2\pi} = c\hbar k$$

($\frac{2\pi}{\lambda} = k$, $\frac{h}{2\pi} = \hbar$)

$\boxed{E_k = \hbar c k}$ for one photon

Quantum physics part + stat. mechanics

Photons are <u>Bosons</u> i.e. we can have them 0, 1, 2, 3 ... etc.

Average energy thus equal

$$\overline{E}_k = 0 \cdot hf \cdot p_0 + (1 \cdot hf) p_1 + (2 \cdot hf) p_2 + \dots$$

p_0: probabilities; $1 \cdot hf = E_1$; $2 \cdot hf = E_2$

Energy of one photon: E_1

$$= 1 \cdot E_1 p_1 + 2 p_2 E_1 + 3 p_3 E_1 + \dots$$

$$= E_1 \left(\sum_i i p_i \right)$$

Probabilities belong to Boltzmann distribution

$$p_i = \frac{1}{Z} e^{-E_i/kT} = e^{-nE_1/kT}$$

$$1 = \sum_0^\infty p_i = \frac{1}{Z}\left(1 + e^{-\frac{E_1}{kT}} + e^{-\frac{2E_1}{kT}} + \dots\right)$$

$$= \frac{1}{Z}\left(1 + p_1 + p_1^2 + p_1^3 + \dots\right)$$

$$= \frac{1}{Z}\frac{1}{1-p_1} = \frac{1}{Z}\frac{1}{1-e^{-E_1/kT}} = 1 \Rightarrow$$

$$\Rightarrow \boxed{Z = \frac{1}{1-e^{-E_1/kT}}}$$

$$X = \frac{1}{kT} \Rightarrow \frac{1}{Z}\frac{dZ}{dx} = \frac{1}{Z}\frac{d}{dx}\left(1 + e^{-E_1 X} + e^{-2E_1 X} + e^{-3E_1 X}\right)$$

$$= -E_1 \frac{\left(1 e^{-E_1 X} + 2 e^{-2E_1 X} + 3 e^{-3E_1 X} + \dots\right)}{Z}$$

$$=$$

$$\Rightarrow \langle E_f \rangle = -\frac{1}{Z}\frac{dZ}{dX} =$$

$$\cancel{= \frac{1}{1-e^{-E_1 X}} \cdot \frac{-1\cdot(-E_1)}{(1-e^{-E_1 X})^2}}$$

$$\cancel{= -(1-E \frac{d}{dX}\left(\frac{1}{1-e^{-E_1 X}}\right)}$$

$$= -\frac{1-e^{-E_1 X}}{1}\frac{d}{dX}\frac{1}{1-e^{-E_1 X}} =$$

$$= -\frac{(1-e^{-E_1 X})}{1}\frac{(-1)(-E_1)(-1)e^{-E_1 X}}{(1-e^{-E_1 X})^2} =$$

$$= E_1 \frac{e^{-E_1 X}}{1-e^{-E_1 X}} = E_1 \underbrace{\left(\frac{1}{e^{E_1 X}-1}\right)}_{\langle n \rangle}$$

$$\langle E_f \rangle = \underset{\substack{\uparrow \\ hf}}{E_1} \cdot \langle n \rangle \quad , \quad \langle n \rangle = \frac{1}{e^{E_1/kT}-1}$$

$$\Downarrow$$

$$\langle E_k \rangle = \hbar c k \cdot \frac{1}{e^{\frac{\hbar c k}{k_B T}}-1}$$

$$\Rightarrow \quad \frac{dE}{} = n_k \circ \langle E_k \rangle \, dk$$

$$= 2 \, \frac{4\pi k^2}{8\pi^3 V} \, \hbar c k \cdot \frac{1}{e^{\frac{\hbar c k}{k_B T}} - 1}$$

two polarizations
for each $\bar{k}$

$$\boxed{\text{Note} \quad k = \frac{\omega}{c}}$$

$$dE_k = \frac{8\pi \, k^3 \hbar c V}{8\pi^3} \, \frac{dk}{e^{\hbar c k / k_B T} - 1}$$

$$k = \frac{2\pi}{\lambda} \quad \Rightarrow \quad dk = -\frac{2\pi}{\lambda^2} \, d\lambda$$

forget it we just swap limits of integration

$$dE_\lambda = + \frac{\cancel{8\pi}}{\cancel{8\pi^3}} \left(\frac{2\pi}{\lambda} \right)^3 \hbar c V \, \frac{2\pi}{\lambda^2} \, \frac{d\lambda}{e^{\hbar \frac{2\pi}{\lambda} c / k_B T} - 1}$$

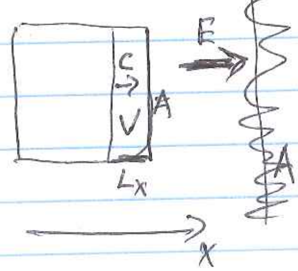
$$= \frac{16\pi^2}{\lambda^5} \, \hbar c V \, \frac{d\lambda}{e^{\hbar \frac{2\pi}{\lambda} c / k_B T} - 1}$$

So far we talked about energy inside the box.

Let see about light intensity coming from there

or energy emitted by a surface

$$I = \frac{E}{t \cdot A} \quad \text{- power}$$



u - energy density $\frac{E_v}{V}$

$$I = \frac{\overbrace{u \cdot V}^{E}}{\underbrace{(L_x/c)}_{\Delta t}} \frac{1}{A} =$$

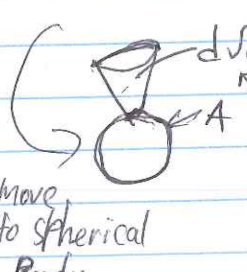
$$= u \cdot c \frac{V}{\underbrace{L_x \cdot A}_{V}} = u \cdot c$$

Thus radiation flux i.e. intensity

$$I = u \cdot c \Rightarrow \boxed{dI_\lambda = du_\lambda \cdot c = \frac{dE_\lambda}{V} \cdot c}$$

~~Lets~~ recall $dE_\lambda = \frac{16\pi^2}{\lambda^5} \hbar c V \frac{d\lambda}{e^{\hbar \frac{2\pi}{\lambda} c/k_B T} - 1}$

$$\Rightarrow \boxed{dI_\lambda = \frac{16\pi^2}{\lambda^5} \hbar c^2 \frac{d\lambda}{e^{\hbar \frac{2\pi}{\lambda} c/k_B T} - 1}}$$



Move to spherical Body

$\cancel{\oint B_\lambda d\Omega = P}$

$$\int_0^{4\pi} B_\lambda d\Omega d\lambda = dI_\lambda$$

$$\Rightarrow B_\lambda d\lambda = \frac{dI_\lambda}{4\pi}$$

Note: $$\int_0^\infty \frac{1}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda k_B T}} - 1} =$$

$$= \Big/ x = \frac{hc}{\lambda k_B T} \Rightarrow \lambda = \frac{hc}{x k_B T} \Rightarrow$$

$$\Rightarrow d\lambda = \frac{hc}{k_B T}\left(-\frac{dx}{x^2}\right) \Big/$$

$$= \int_0^\infty \left(\frac{k_B T}{hc}\right)^5 \cdot \left(\frac{hc}{k_B T}\right) \frac{x^3\,dx}{e^x - 1} =$$

$$= \left(\frac{k_B T}{hc}\right)^4 \underbrace{\int_0^\infty \frac{x^3\,dx}{e^x - 1}}_{\frac{\pi^4}{15}} =$$

$$= \frac{\pi^4}{15}\left(\frac{k_B T}{hc}\right)^4$$

Lecture 9 stops here