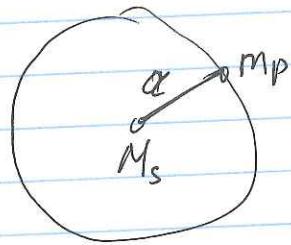


Lecture 8

~~Exoplanet search~~



$$P^2 = \frac{4\pi^2}{G(M_s + M_p)} a^3$$

$$v_p = \frac{2\pi a}{P}$$

$$v_s = v_p \frac{M_p}{M_s} = \frac{2\pi a}{P} \frac{M_p}{M_s}$$

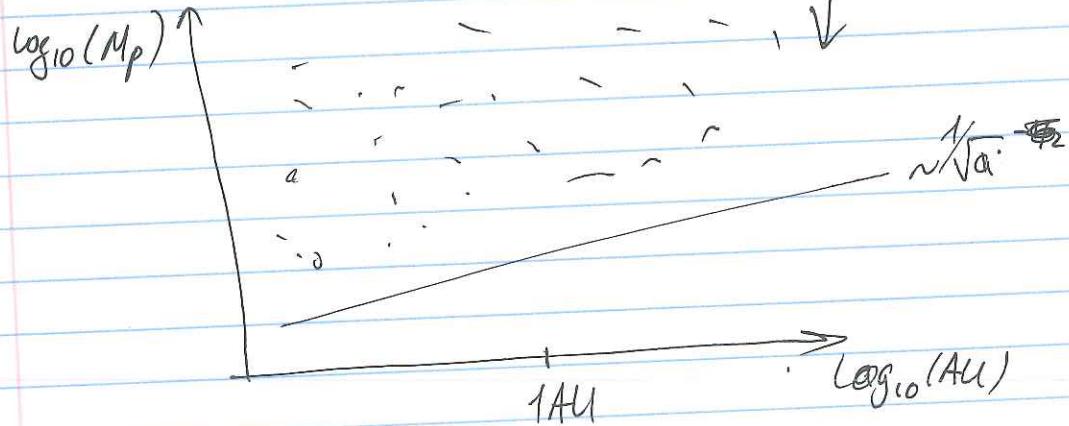
$$= 2\pi \frac{a}{\sqrt{\frac{4\pi^2}{G(M_s + M_p)} a^3}} \frac{M_p}{M_s} =$$

$$= 2\pi \cdot \frac{1}{\sqrt{a} \sqrt{\frac{4\pi^2}{G}}} \frac{M_p}{(M_s + M_p) \frac{M_s}{\sqrt{M_s + M_p}}}$$

$$\Rightarrow M_p \ll M_s$$

Current
sensitivity
60 cm/s

$$= \sqrt{\frac{2\pi}{\frac{4\pi^2}{G}}} \frac{M_p}{\sqrt{a} \cancel{M_s}}$$



Planet search.

Assume circular orbit

$$v_p = \frac{2\pi a_{\text{planet}}}{P} \quad v_{\text{star}} = \frac{2\pi a_{\text{star}}}{P}$$

$$v_p = \cancel{2\pi} \quad \frac{a_{\text{planet}}}{a_{\text{Earth}}} = \frac{v_{\text{planet}}}{v_{\text{star}}}$$

Earth motion

$$v_E = \frac{2\pi / 1 \text{ AU}}{1 \text{ year}} = \frac{2\pi \cdot 1.5 \cdot 10^{11}}{\pi \cdot 10^4} = 3 \cdot 10^8 \approx 30 \text{ km/s}$$

$$v_{\text{sun}} = v_E \cdot \frac{M_E}{M_S} = 3 \cdot 10^8 \cdot \frac{6 \cdot 10^{24}}{2 \cdot 10^{30}} \approx \\ \approx 9 \cdot 10^{-2} \approx 9 \text{ cm/s}$$

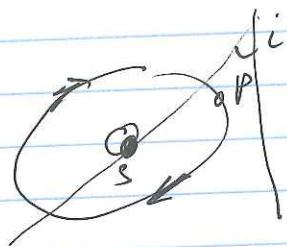
First planet report in 1996.

Overall about 860 planet reported in 2013

So far they are way heavier than Earth \approx Jupiter like, and usually way closer to star than earth

Problem with radial velocity method

we do not know inclination of the orbit



$$v_s \text{ observed} = v_s \cdot \sin i$$

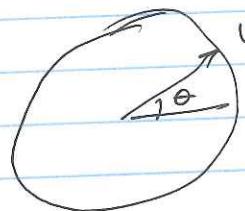
\Rightarrow radial velocity alone gives only lower bound on mass of the planet

Also we do not know ' e '

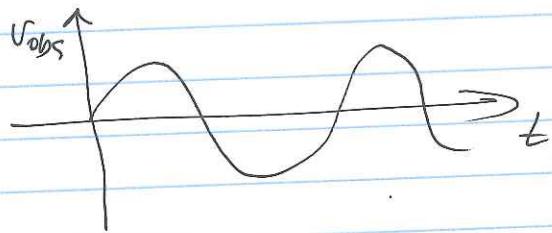
But if we can trace v_s vs t

We can deduce some things

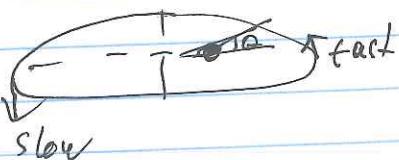
Ex. 1. circular orbit



$$v_{\text{obs}} \sim v \cdot \sin \theta = v \sin(\omega t)$$

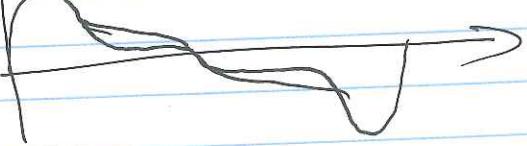


Ex. 2



v_{obs}

something like
that

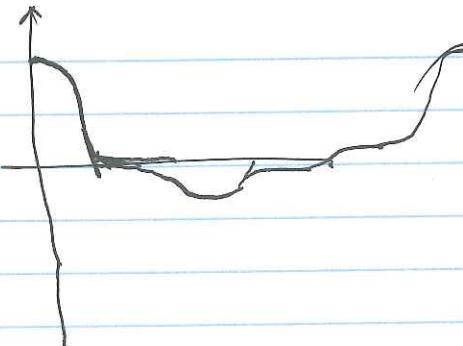


ex. 3

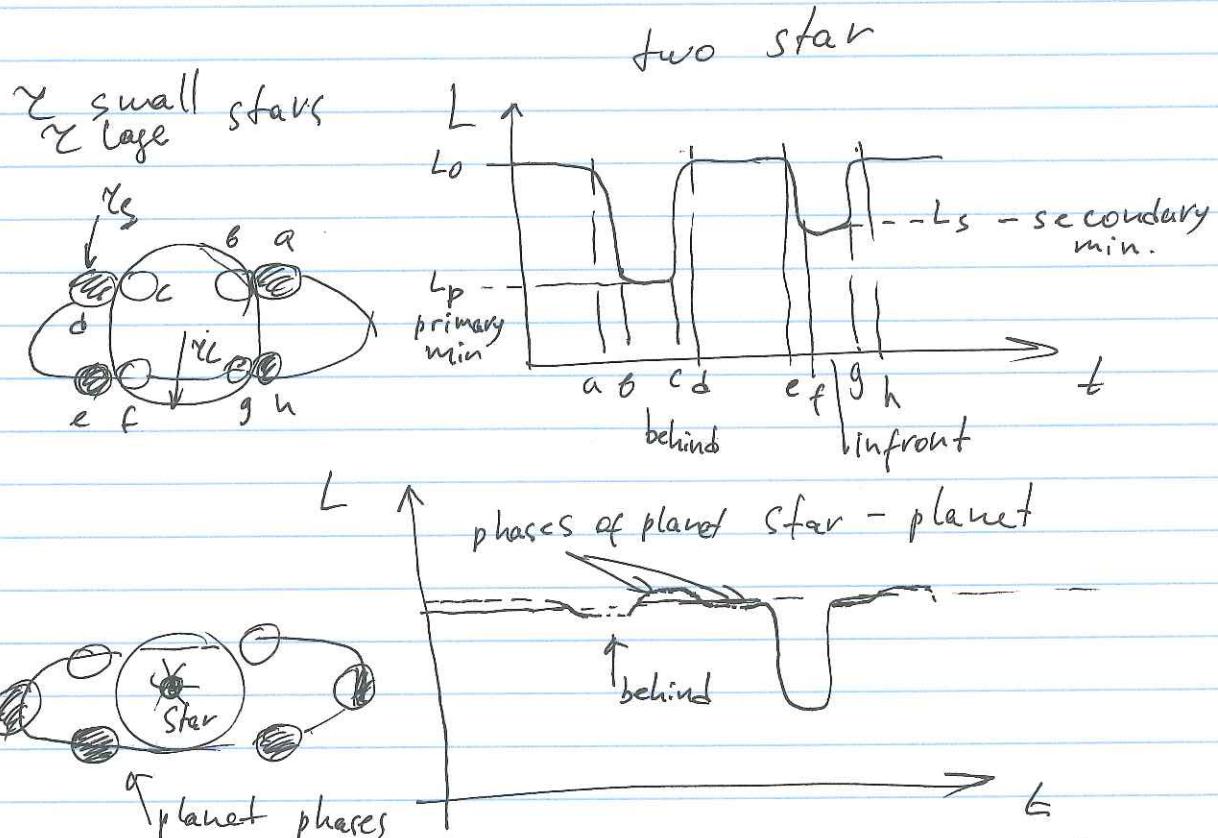
Δt



V_{obs}



Luminosity method



By the way - this how sun radius was measured
 by transit of Venus

$$r_s = \frac{v_{orb}}{2} \cdot (t_B - t_A)$$

depends on inclination

$$r_e \approx \frac{v_{orb}}{2} (t_B - t_A)$$

Relative temperatures

Stefan-Boltzmann

$$\text{Flux from star} \approx A_{\text{star}} \cdot (\sigma T_{\text{star}}^4)$$

$$L_0 \approx F_e + F_s = \pi r_e^2 T_e^4 + \pi r_s^2 T_s^4 \quad \text{shade}$$

$$L_p = \pi r_e^2 T_e^4 ; L_s = \pi r_e^2 T_e^4 \left(\pi r_s^2 T_e^4 \right) + \pi r_s^2 T_s^4$$

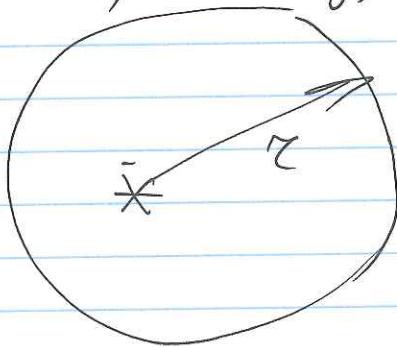
$$\frac{L_0 - L_p}{L_0 - L_s} = \frac{\pi r_s^2 T_s^4}{\pi r_s^2 T_e^4} = \left(\frac{T_s}{T_e} \right)^4$$

Lecture 8
Stops here

Radiant flux F (all 'normal' people call it intensity)

Energy per unit area per time

luminosity (Energy emitted per second)



$$F = 4\pi r^2 = L$$

$$F = \frac{L}{4\pi r^2}$$

inverse square law for light

If flux differs by factor of 100 we say that magnitude different by 5

$$\text{i.e. } \frac{F_{(m+1)}}{F(m)} = 100^{1/5} = 2.512$$

the brighter the smaller

$$\Rightarrow \frac{F_2}{F_1} = \frac{F_0 \cdot 100^{(m_2 - m_1)/5}}{F_0 \cdot 100^{-m_1/5}} = 100^{(m_2 - m_1)/5} = 10^{\frac{2}{5}(m_2 - m_1)}$$

$$\log_{10} \frac{F_2}{F_1} = \frac{2}{5} (m_1 - m_2) = \log_{10} \left(\frac{100^{(m_2 - m_1)/5}}{10^{-m_1/5}} \right) =$$

$$\Rightarrow \log_{10} \left(10^{\frac{2}{5}(m_2 - m_1)/5} \right) =$$

$$\boxed{m_1 - m_2 = -2.5 \log_{10} \frac{F_1}{F_2}} = \frac{2}{5} \log_{10} 10^{-\frac{2}{5}}$$

Absolute magnitude

Flux from star at distance of 10pc

$$F = \frac{L}{4\pi d^2} \Rightarrow F_{10\text{pc}} = \frac{F \cdot d^2}{10\text{pc}^2} = F \left(\frac{d}{10\text{pc}}\right)^2$$

$$\frac{F_{10}}{F} = 100^{(m-M)/5}$$

$$m - M = 2.5 \log \left(\frac{d}{10\text{pc}}\right)^2 = +5 \log \left(\frac{d}{10\text{pc}}\right)$$

$$M = m - 5 \log \left(\frac{d}{10\text{pc}}\right)$$

$$\Rightarrow F = \frac{L}{4\pi d^2}$$

$$\frac{F}{F_\odot} \text{ at } 10\text{pc} = \frac{L_s}{L_\odot} = 100^{(M_{\text{sun}} - M_{\text{st}})/5}$$

$$\Rightarrow \boxed{M = M_{\text{sun}} - 2.5 \log_{10} \left(\frac{L}{L_\odot}\right)}$$