Q: let's look at problem 2.14

Halley's comet has $P = 76$ year and $e = 0.9673$.

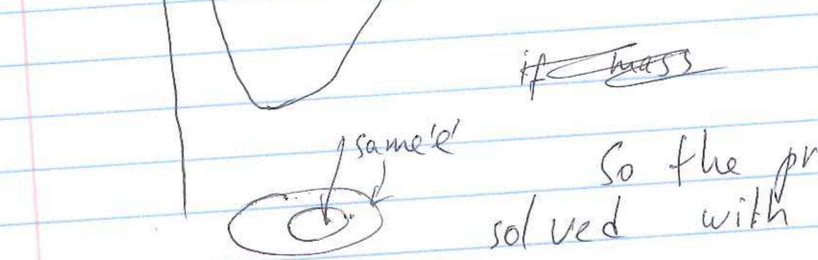
Can we find a, and ~~$M_\odot$~~ sun mass ($M_\odot$), and comet mass (m_c) from this information alone?

Lecture 6 Stops Here

$\boxed{\text{Lecture 7}}$

Well we know $P^2 = \frac{4\pi^2}{G M_{Total}} a^3$

we need M_T and a, so we need at least 1 more eq., so we can use one for 'e'.
So sounds like easy problem.



But 'e' does not bound anything ~~for~~ we can have different 'a's with the same e,

So the problem cannot be solved with only provided inform

So we need to use the fact that we know something else about solar system
i.e Earth has $P = 1$ year and $a = 1AU$ $M_\odot + M_\oplus \simeq M_\odot + \ldots$

Halley Comet cont

$P^2 = a^3$ in ~~Years~~ years and A.U.

H.C. $\Rightarrow (76)^2 = a^3$

$$a = 76^{2/3} = 17.9 \simeq 18 \text{ A.U.}$$

$$\cancel{b = \frac{a}{\sqrt{1-e^2}} = \frac{18}{\sqrt{1-0.9673^2}}} =$$

$$b = a \cdot \sqrt{1-e^2} = 4.55 \text{ A.U.}$$

Aphelion $r_a = a(1-e) = 0.586$ A.U.

perihelion $r_p = a(1+e) = 35.3$ A.U.

$$M_\odot = \frac{a^3}{P^2} \cdot \frac{4\pi^2}{G} = \frac{(18 \cdot 1.5 \cdot 10^{11})^3}{\underset{76 \cdot \pi \cdot 10^7}{(76 \cdot 365 \cdot 24 \cdot 60 \cdot 60)^2}} \frac{4\pi^2}{6.67 \cdot 10^{-11}}$$

$$= 2.028 \cdot 10^{30} \text{ kg} \quad (\text{True } M_\odot = 1.989 \cdot 10^{30} \text{ kg})$$

What about 3 or more bodyes problem

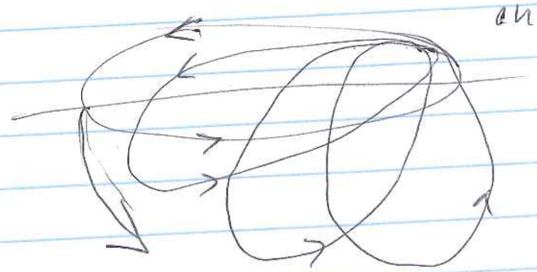
- Has no solutions analyticly

Have to use computers or some approximation i.e. massive stars and light planets

More over once we have 3rd body orbits are no more closed.

They precess

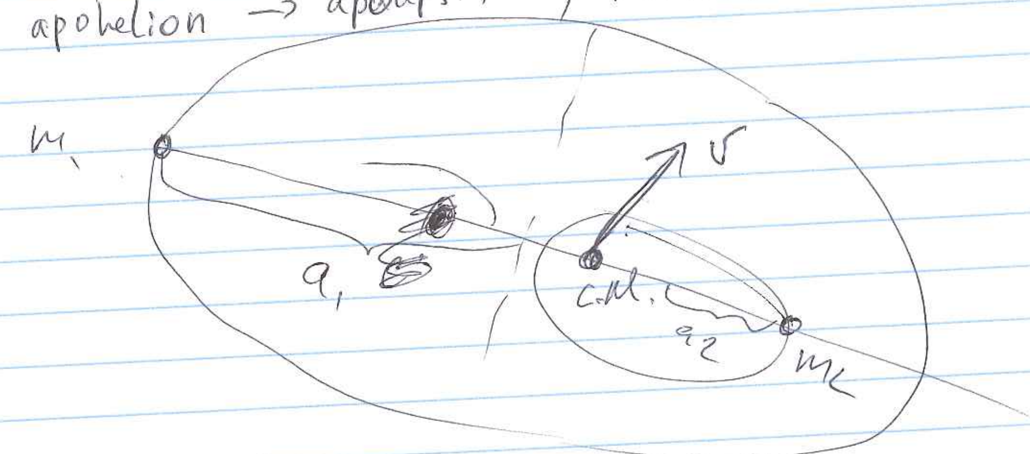
and so on



All of this is great bal what about star masses. So far we can deduce only $\underline{\underline{M_\odot}}$ from solar system data.

For this we need a binary star system with P reasonably small.

note: apohelion → apoapsis , perihelion → apoapsis

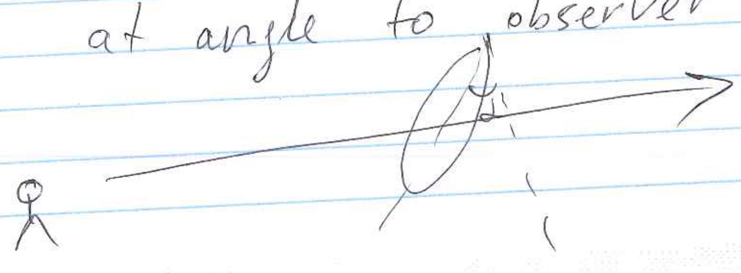


tracking stars we can deduce
a_1 and a_2

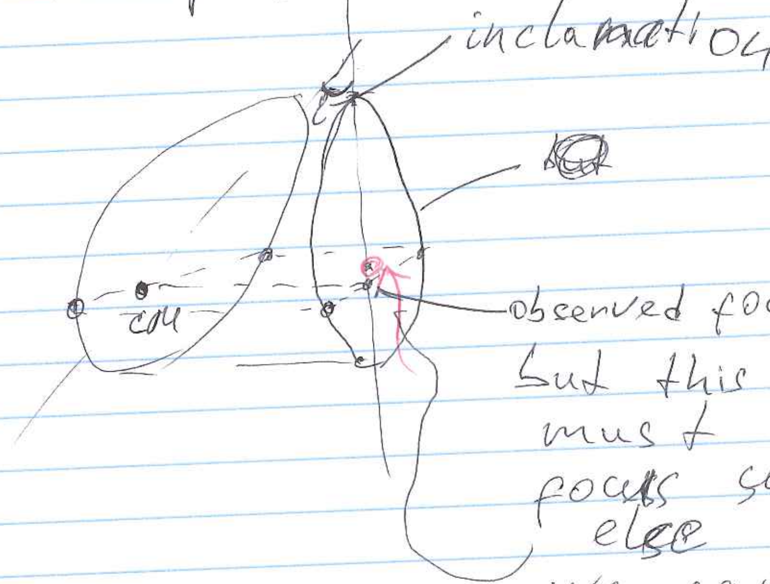
→ $a = a_1 + a_2$ ⟹ observing period or
measure deducing it from orbit
we can learn M_{total}, m_1 and m_2

so problem solved. Well there are obsticles. Ecliptic plane might be at angle to observer

so we need to compencate



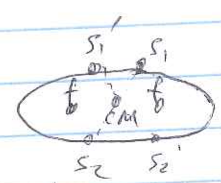
Nice part that in projected plane apparent C.M. does not coinside with projected



But this shape must have focus somewhere else so we can deduce i

Very simple example: if we have a round orbit C.M. must be in the center

but if it is inclined we will see



Such ellipce must have C.M in a focus. But projected "true" C.M is still in the midle. So we have discrepancy which helps to resolve inclanation

We cannot wait forever

For this method to work:
We need reasonably small period, $\approx$ year
$\Rightarrow$ reasonably small star separation.

$$P^2 = \frac{4\pi^2}{G(M_1+M_2)} a^3$$

Assume $M_1 = M_2 = M_\odot$
and $P = 1$ year what will be 'a'?

$P_{years}^2 = K\, a_{AU}^2$ for $M_{Total} = 1\,sun$

i.e. in this units $t \to$ years, length $\to$ A.U

$\frac{4\pi^2}{G M_\odot} = K$ so if $M_{Total} = 2M_{sun}$ then $K' = \frac{K}{2}$

So we get $P^2 = \frac{1}{2} a^3 = (1\,year)^2$

$\Rightarrow a = 2^{1/3} = \sqrt[3]{2} = 0.57\,A.$

Both stars need to be reasonably bright to see both of them.

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if we can measure a_1 and a_2 and P
$a = a_1 + a_2 \Rightarrow M_{Total}$

$a_1 = \frac{a}{m_1}$
$a_2 = a/m_2$
$\Rightarrow \begin{cases} m_1 + m_2 = M_{To} \\ \frac{a_1}{a_2} = \frac{m_2}{m_1} = \beta \end{cases}$

measured fraction

$\Rightarrow$ we can learn m_1 and m_2

Star masses. with spectroscopy.

Doppler effect. (1842)

Current limits 2012
0.6 m/s – spectrograph
Astro-comb -assisted
projected to be 0.01 $\frac{m}{s}$

Sun motion due to earth = 5 $\frac{cm}{s}$

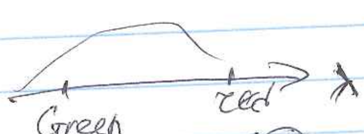
$\leftarrow \lambda, f = \frac{c}{\lambda}$ "$\vec{v}$" towards observer

$$f_{obs} = f_{rest} \cdot \left(1 \pm \frac{v}{c}\right)$$

i.e. source ref. frame

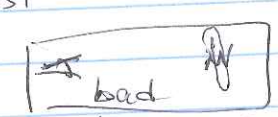
$\vec{v}$ away from observer

$\Rightarrow$ (+) $f_{obs} > f_{rest} \Rightarrow \lambda_{obs} < \lambda_{rest}$
$\Rightarrow$ red shift



(−) $\Rightarrow \lambda_{obs} > \lambda_{rest} \Rightarrow$ blue shift

important



no v component

thus if we can measure observed λ of some known spectral line and compare with the rest frame λ (our lab) we can learn star velocity

→ lecture 7 stopped here

Funny fact: Doppler thought that star color is due to Doppler effect but that would require humongous speeds and is not confirmed by more detailed studies