Lecture 4
repeat

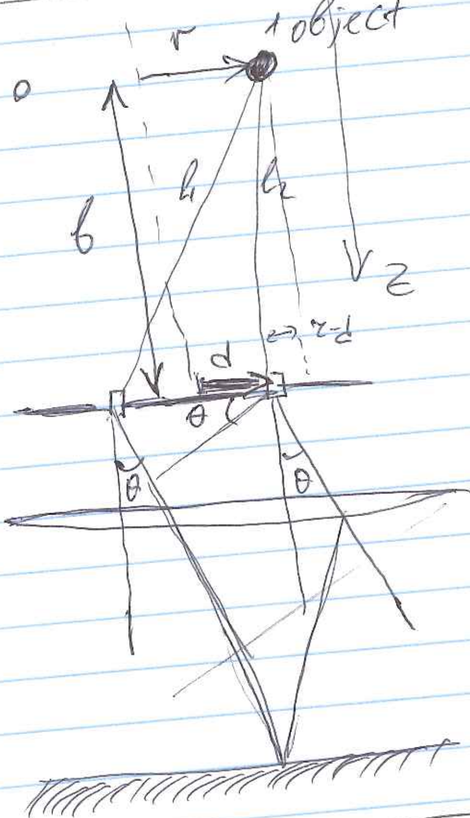
$\boxed{\text{Star separation an diameter}}$

$E_0 \cos(\omega t + \varphi(t))$ retarded signal

$t_1 \quad l \quad t$

$E_0 (\cos(\omega t_1 + \varphi(t_1)))$

$= E_0 \left(\cos\left(\omega\left(t - \frac{\Delta l}{c}\right) + \varphi\left(t - \frac{\Delta l}{c}\right)\right)\right)$



$\cancel{E = E_0 \cos(\omega t - kb + \varphi)}$

$\cancel{l_1 = \sqrt{b^2 + (r-b)}}$

$$l_1 = \sqrt{b^2 + (r-d)^2}$$

$$l_2 = \sqrt{b^2 + (r+d)^2}$$

$b \gg r \gg d$

and $\frac{r^2}{b} \ll \lambda$!!

since we compare with λ in path diffe

$$l_2 - l_1 = \sqrt{b^2 + (r+d)^2} - \sqrt{b^2 + (r-d)^2} + 2d\sin$$

$$\approx b\left(1 + \frac{1}{2}\left(\frac{r+d}{b}\right)^2\right) - b\left(1 + \frac{1}{2}\left(\frac{r-d}{b}\right)^2\right) + 2d\sin$$

$$= \frac{2rd}{b} + 2d\sin\theta = 2\left(\frac{r}{b}\right)d = 2\,\theta_{object}\, d + 2d\sin\theta$$

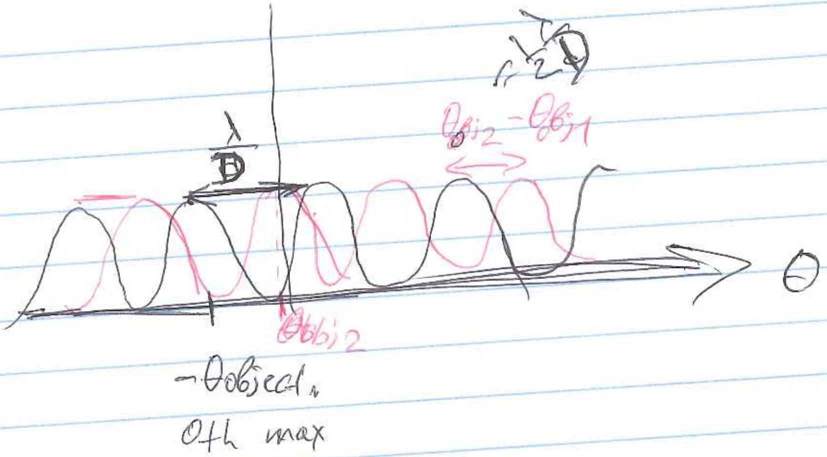
total path difference

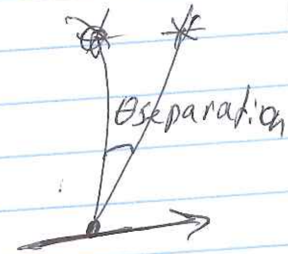
$$l_2 - l_1 + \underbrace{2d\sin\theta}_{\text{diffracted beam}} = \underbrace{(2d)}_{D \text{ base or slit separatio}} (\theta_{object} + s$$

assuming that $\varphi = \text{const}$

and that $\theta \ll 1$

$$D \cdot (\theta_{object} + \theta) = \begin{cases} \lambda m, & \text{max cond} \\ \lambda m + 1, & \text{min cond} \end{cases}$$





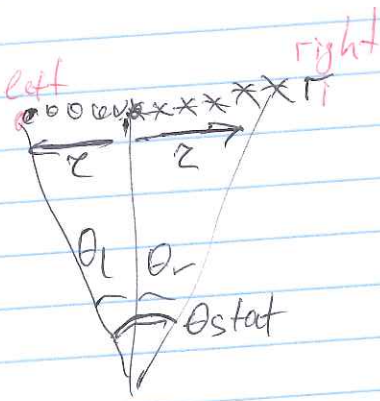
when $\frac{1}{2}\frac{\lambda}{D} = \theta_{obj_2} - \theta_{obj\,1} = \theta_{separation}$

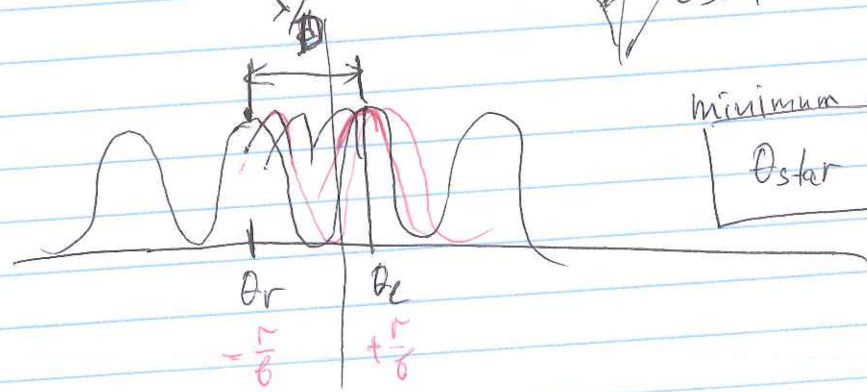
still similar to diffraction limit $\boxed{\theta_{separation} = \frac{1}{2}\frac{\lambda}{D}}$ two objects in a sky

condition of minimum visibility

But it is now differential measurement – the whole picture can move but fringes will stay constant

If we have a star





minimum visibility

$\boxed{\theta_{star} = \frac{\lambda}{D}}$

~~Why~~ fields from 2 separated objects

$$I = \left(E_1 \cos(\omega t + \varphi_1) + E_2 \cos(\omega t + \varphi_2)\right)^2$$

$$= \underbrace{E_1^2 \cos^2(\omega t + \varphi_1)}_{I_1} + \underbrace{E_2^2 \cos^2(\omega t + \varphi_2)}_{I_2}$$

$$+ \; 2 E_1 E_2 \cos(\omega t + \underset{\varphi_1(t)}{\underset{\uparrow}{\varphi_1}}) \cos(\omega t + \underset{\varphi_2(t)}{\underset{\uparrow}{\varphi_2}})$$

$$\langle I \rangle = \int_0^{T} \frac{dt \cos^2(\omega t + \varphi)}{T} = \frac{1}{2}$$

$$= \underbrace{\tfrac{1}{2} E_1^2}_{\langle I_1 \rangle} + \underbrace{\tfrac{1}{2} E_2}_{\langle I_2 \rangle} + 0 = I_1 + I_2$$

$$\langle \cos(\omega t + \varphi_1) \cos(\omega t + \varphi_2) \rangle = \left\langle \frac{\cos(\varphi_1 - \varphi_2) + \cos(2\omega t +}{2} \right.$$

if 1 and 2 not correlated $\Leftrightarrow$ independent

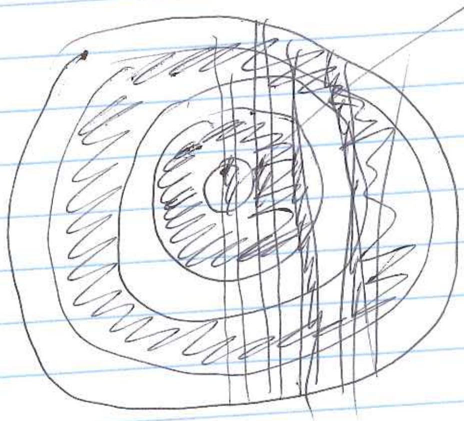
$$= \left\langle \frac{\cos(\varphi_1 - \varphi_2)}{2} \right\rangle + \left\langle \frac{\cos(2\omega t + \varphi_1 + \varphi_2)}{2} \right\rangle$$

$$\quad \downarrow 0 \qquad\qquad\qquad \downarrow 0$$

Historical notes.

Jupiter Moons diameter by Michelson
with above method 1891.
1920 - Betelgeuse diameter

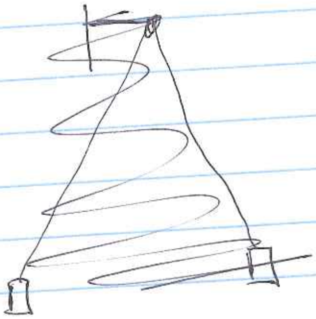
Airy disk

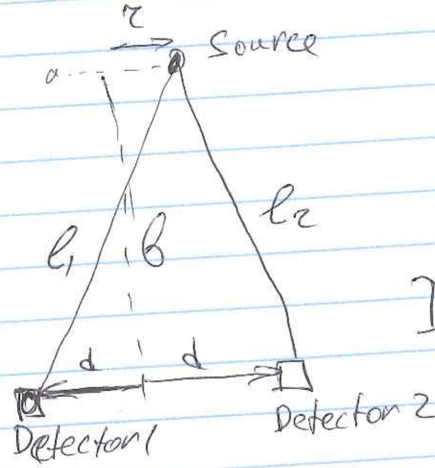


see Оптика
by ~~А~~ Ландсберг
p. 197

Hanbury Brown - Twiss experiment

star size





$D = 2d$

$$l_1 - l_2 = \frac{x \cdot d}{b} \approx D\,\theta_{source}$$

$$I_1(t) = \left[E_s\left(\cos\left(\omega\left(t - \frac{l_1}{c}\right) + \varphi_1\left(t - \frac{l_1}{c}\right)\right)\right)\right]^2$$

$$I_2(t) = \left[E_s \cos\left(\omega\left(t - \frac{l_2}{c}\right) + \varphi\left(t - \frac{l_2}{c}\right)\right)\right]^2$$

lets ask for $t - \frac{l_2}{c} = 0$ shift of the 0 time

$$t \longrightarrow t + \frac{l_2}{c}$$

$$\Rightarrow \quad I_2(t) = \left[E_s \cos(\omega t + \varphi(t))\right]^2$$

$$I_1(t) = \left[E_s \cos(\omega(t + \Delta t) + \varphi(t + \Delta t))\right]$$

$\Delta t = \frac{l_2 - l_1}{c}$

since $\varphi(t)$ is quite wild function
it is better describe

$I(t)$ as $I_{0s} + \Delta I_s(t)$ $\Rightarrow \langle I(t) \rangle = I_s$

mean

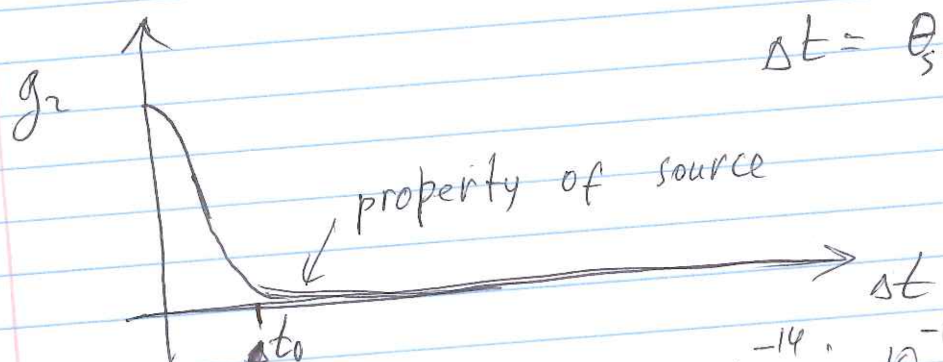
$$\langle I_{d_1} \cdot I_{d_2} \rangle = \langle (I_s + \Delta I_s(t))(I_s + \Delta I_s(t+\Delta t)) \rangle$$

$$= \langle I_s^2 \rangle + \cancelto{0}{\langle \Delta I(t) \cdot I_s \rangle} + \cancelto{0}{\langle \Delta I(t+\Delta t) I_s \rangle} +$$

$$+ \langle \Delta I(t) \cdot \Delta I(t+\Delta t) \rangle$$

$$= I_s^2 + \underbrace{\langle \Delta I(t) \cdot \Delta I(t+\Delta t) \rangle}_{g_2(\Delta t)}$$

recall that $\Delta t = \theta_s D / c$



coherence time $\approx 10^{-14} \div 10^{-15}$ s

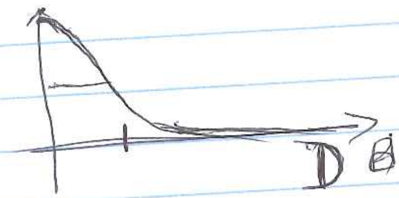
What if we have N sources

$$(\underbrace{I_{s_1} + I_{s_2} + I_{s_3}}_{\text{independent intensity adds up}}) \cdot (I_{s_1}(t_s) + I_{s_2}(t_{s_2}) + \ldots)$$

Notice $\langle \Delta I_{s_i} \Delta I_{s_j} \rangle = 0$ ← independence cond.

$$= \langle I_{d_1} \cdot I_{d_2} \rangle = I_{s_1}^2 + I_{s_2}^2 + \ldots I_{s_N}^2 + g_2(\Delta t_{s_1}) + g_2(\Delta \ldots + \ldots + g_2(\Delta t_{s_2}$$

$$\Rightarrow \quad \sum_i g_2(\Delta t_i)$$



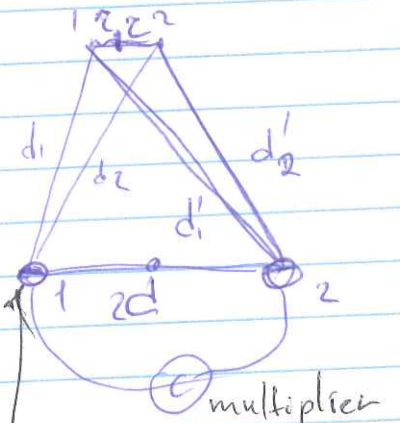
$$\frac{\theta_{star} D}{c} = t_0$$

Lecture 4 stops here

~ diameter resolution

Intensity interferometer

Hanbury & Brown and Twiss 1956



pros: no need for good telescope as long as light is on detector

search light WWII

$\phi = 156\,\mathrm{cm}$

$f = 65\,\mathrm{cm}$

$d_2' = d_1$

$d_1' = d_2$

$S_1 = I_1(t) + I_2\left(t - \frac{\Delta l}{c}\right)$

$S_2 = I_1\left(t - \frac{\Delta l}{c}\right) + I_2(t)$

$\Delta l \approx \frac{2d\tau}{6}$ see Michelson exper.

$$C = \cancel{I_1 \cdot I_2} \langle S_1 \cdot S_2 \rangle =$$

$$= \left\langle \left(I_1(t) + I_2\left(t - \frac{\Delta l}{c}\right)\right)\left(I_1\left(t - \frac{\Delta l}{c}\right) + I_2(t)\right)\right\rangle$$

$$= \langle I_1(t) I_1\left(t - \frac{\Delta l}{c}\right) + I_2(t) I_2\left(t - \frac{\Delta l}{c}\right) +$$

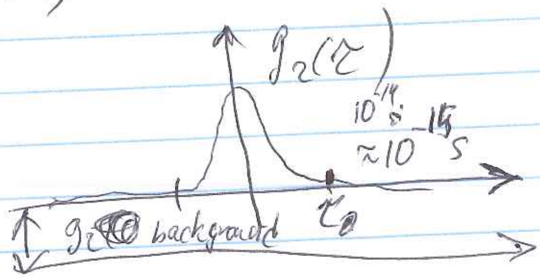
$$+ \underbrace{I_1(t) I_2\left(t - \frac{\Delta l}{c}\right)}_{\text{uncorrelated}} + \underbrace{I_1\left(t - \frac{\Delta l}{c}\right) I_2(t)}_{\text{uncorrelated}} \rangle$$

$\frac{1}{T}\int I_1(t) I_1(t-\tau)\,dt$

$$= \underbrace{\langle I_1(t) I_1\left(t - \frac{\Delta l}{c}\right)\rangle}_{g_2(\tau) = g_2\left(\frac{\Delta l}{c}\right)} + \underbrace{\langle I_2(t) I_2\left(t - \frac{\Delta l}{c}\right)\rangle}_{g_2(\tau)}$$

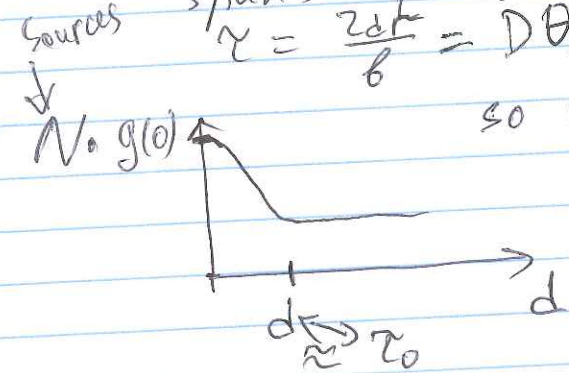
assuming that intensity are statistically the same

$$C \approx 2 g_2(\tau)$$



Now star is not just two points
so we have a sum of
all possible $g_2(\tau)$ where τ
spans from 0 (central point) to
$\tau = \frac{2dr}{6} = D\theta_{star}$

so we have a smear of pattern



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