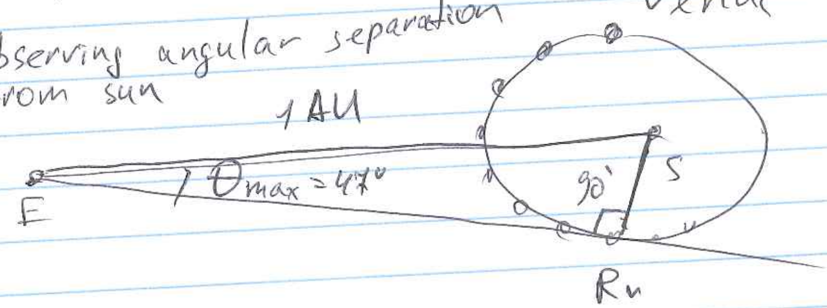
Lecture 2

~~How to~~ Plan: How ~~to~~ Greeks now relative distances to planets

* Parallax (as measure distances)
* Physiological limit for eyes angle resolution
* Diffraction limited optics, Airy disk, Rayleigh criterion

Distance in A.U. to the inner planets

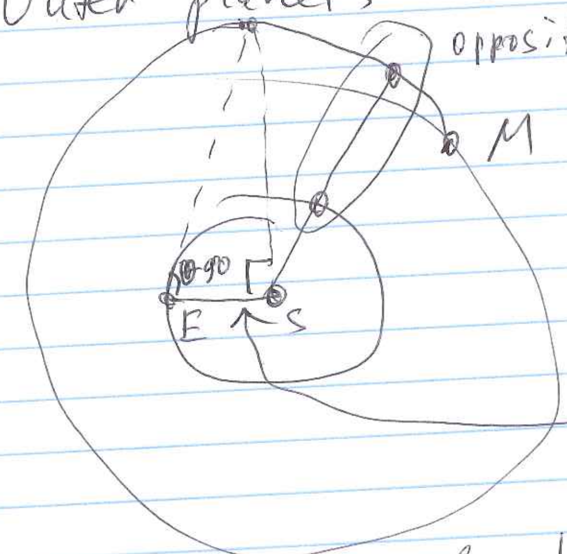
Observing angular separation from sun



$$\Rightarrow R_{venus} = 1\,AU \cdot \sin 47^\circ \approx 0.73\,AU$$

Same for Mercury

Outer planets are tricky

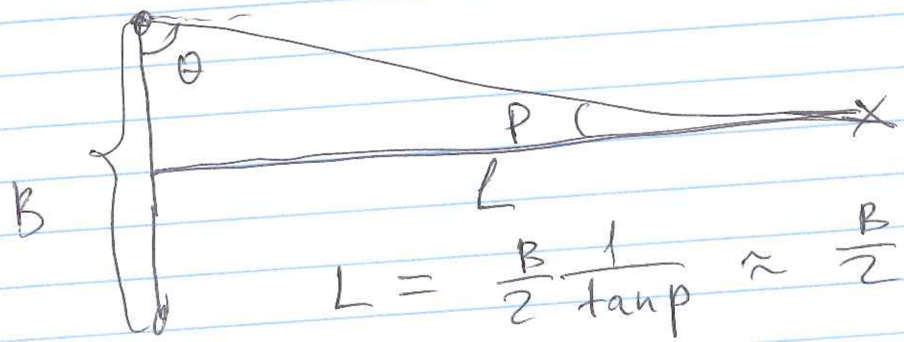


How to make 90°

Use timing from synodial periods to find ~~S~~ siderial perion and wait some time after opposition $\Rightarrow \frac{S}{4}$ ← siderial period

$$\frac{1}{S} = \frac{1}{P_E} - \frac{1}{P_M}$$

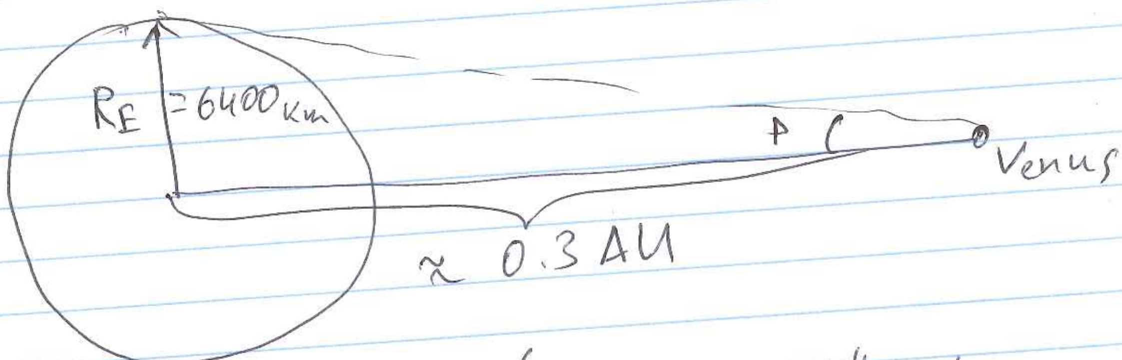
Parallax: all we need is compass and a base



$$L = \frac{B}{2}\frac{1}{\tan p} \approx \frac{B}{2}\frac{1}{p}$$

Problem: Why it is hard to measure 1 AU.

Let's try to measure of Venus.

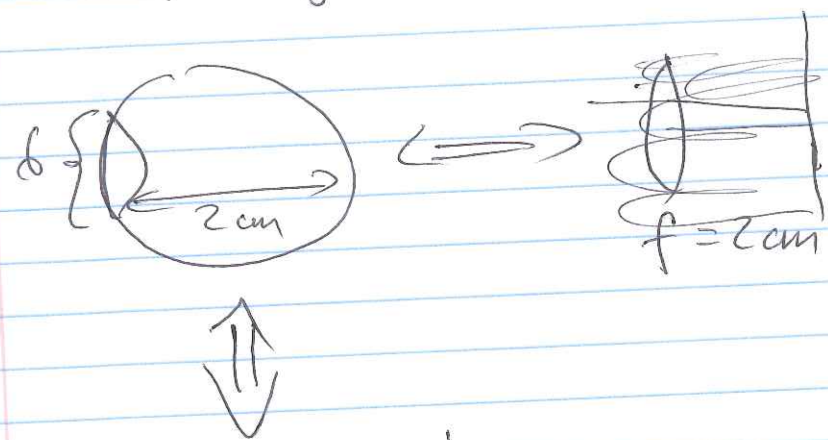


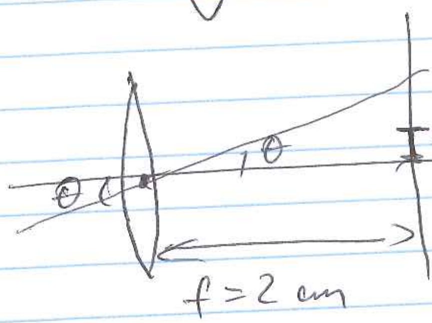
$$p = \frac{6.4\cdot 10^6}{0.3\cdot 1.5\cdot 10^{11}} \approx 1.4\cdot 10^{-4}\ \text{rad}$$

$$\simeq 1.4\cdot 10^{-4}\ \frac{180^\circ}{\pi\,\text{rad}}\cdot\frac{60'}{1^\circ} \approx 0.48'$$

Well ~~Kepler~~ Tycho can do 4' ~~but for relative~~
so no way with naked eye
and there were no way to do
Earth sized base at his time (prior 1601).
~~since~~ Magelan (made ~~the~~ his trip in 1519–152[illegible] though

Physiological limits for an eye





← 'pixel' size size of the rods and cones

$\approx 2\,\mu m$ ↖ color and sharper vision part

$$\Rightarrow \theta_{min} = \frac{2\,\mu m}{2\,cm} \approx 10^{-4}\,\text{rad} \approx 0.34'$$

Books says ("Оптика" ~~Лансберг~~ Ландсберг)

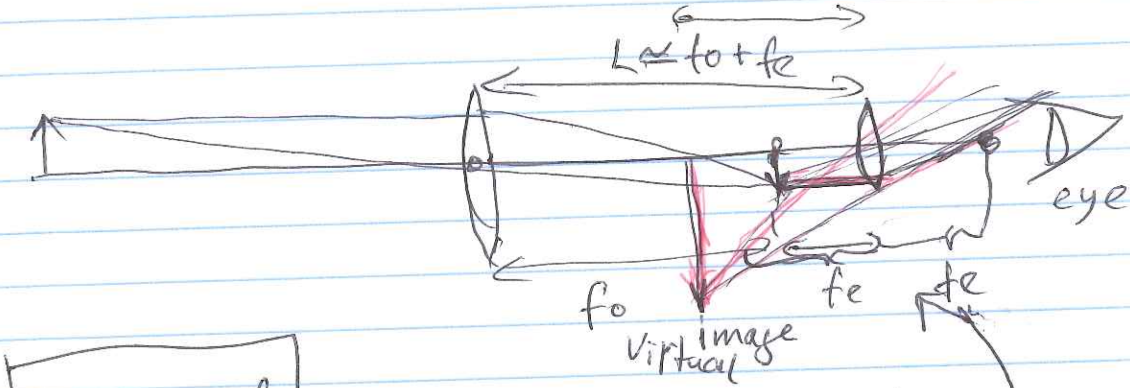
Physio limit $1''$ but for bright objects

so $4'$ of Tycho is quite impressive

NB: Amasingly Physiological limit close to diffraction limited case

$$\theta_{min} = 1.22\,\frac{\lambda}{d} = 1.22\,\frac{500\,nm}{2\,mm} \approx 3\cdot 10^{-4}$$

Telescope, is the way to lift physiological limit



$$\boxed{M = \frac{f_o}{f_e}}$$ $\Leftrightarrow$ angular magnification

$\Rightarrow \theta_{min} = \dfrac{\theta_{physiological}}{M}$

$f_o > f_e$

eye lens used as magnifying lens for image formed by Objective

lens $= \dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$$

There is a limit for 'M'
since f_e cannot be to small

$f \sim R_{of\,lens} \Rightarrow f \gg R$

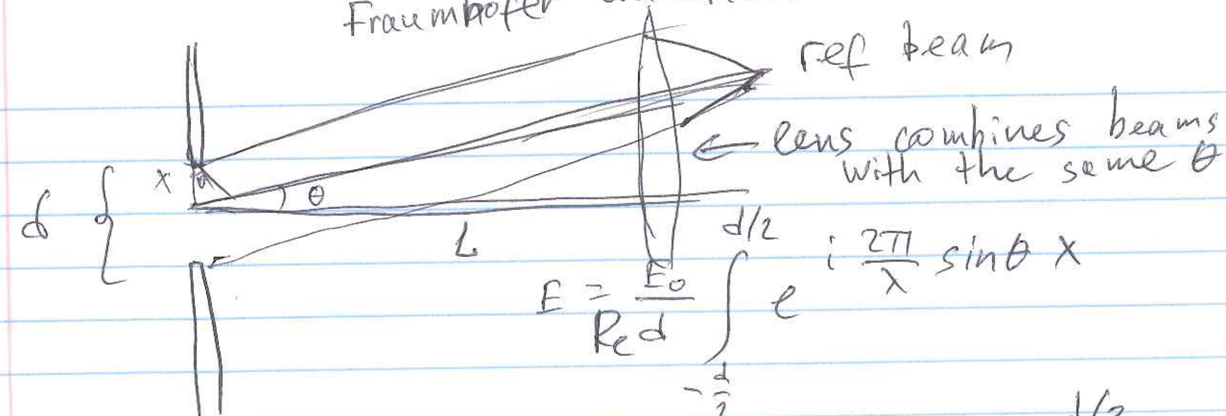
lens would look like a "glass bead"
with very limited aperture!

But there is another limit

Diffraction

diffraction on slit for simplicity

Fraunhofer condition $d \ll L$



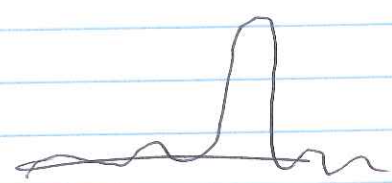
$$E = \mathrm{Re}\,\frac{E_0}{d}\int_{-\frac{d}{2}}^{d/2} e^{i\frac{2\pi}{\lambda}\sin\theta\, x}$$

$$= \mathrm{Re}\,\frac{E_0}{d}\,\frac{e^{i\frac{2\pi}{\lambda}\sin\theta\, x}}{i\frac{2\pi}{\lambda}\sin\theta}\Bigg|_{-d/2}^{d/2}$$

$$= \mathrm{Re}\,\frac{E_0}{d}\,\frac{e^{i\frac{2\pi}{\lambda}\sin\theta\, d/2} - e^{-i\frac{2\pi}{\lambda}\sin\theta\frac{d}{2}}}{i\,\frac{2\pi}{\lambda}\sin\theta}$$

$$= \frac{E_0}{d}\,\frac{\cancel{i}\,2\sin\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta\ \cancel{\text{to}}}{\cancel{i}\,\frac{2\pi}{\lambda}\sin\theta}$$

$$= \frac{2E_0}{\cancel{d}}\,\frac{\sin\left(\frac{\pi}{\lambda}d\sin\theta\right)}{\frac{\pi}{\lambda}d\sin\theta}$$



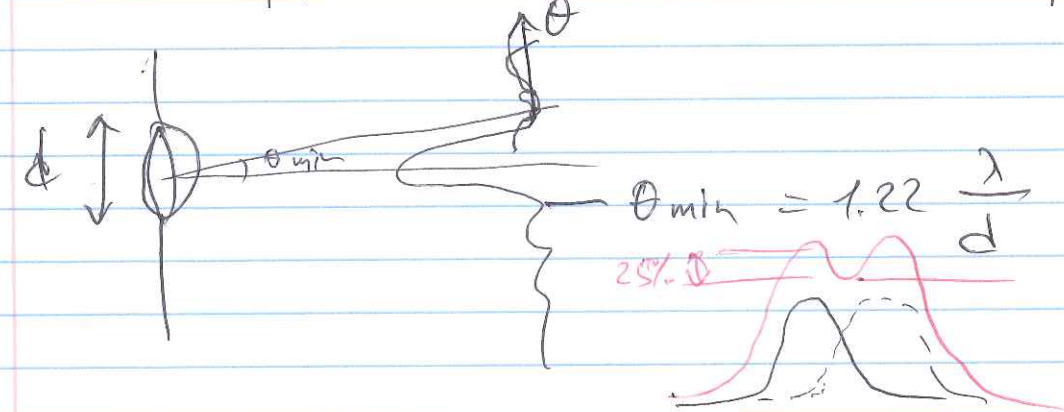
Minima condition

$$\Rightarrow \frac{\pi}{\lambda}d\sin\theta = m\pi$$

$$d\sin\theta = \lambda$$

$$\boxed{\sin\theta = \lambda/d}$$

Diffraction at the round aperture



$$\theta_{min} = 1.22 \frac{\lambda}{d}$$

Rayleigh criterion $\theta_{resolution} = \theta_{min}$

$$E(\theta) = E_0 \frac{2 J_1(kR \sin\theta)}{kR \sin\theta} \Rightarrow R = \frac{d}{2}, \quad k = \frac{2\pi}{\lambda}$$

$$= E_0 \frac{2 J_1\left(\frac{\pi}{\lambda} D \sin\theta\right)}{\frac{\pi}{\lambda} D \sin\theta}$$

Bessel function