

Useful circuits with Op-Amps

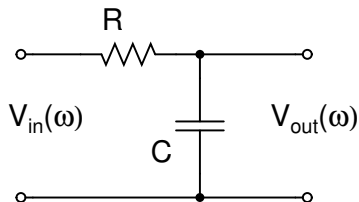
Eugeniy E. Mikhailov

The College of William & Mary



Lecture 10

Integrator



$$V_{out}(t) = V_C(t) = \frac{Q(t)}{C} = \int \frac{I(t)}{C} dt = \int \frac{V_{in}(t) - V_C(t)}{RC} dt$$

for $V_C \approx 0$

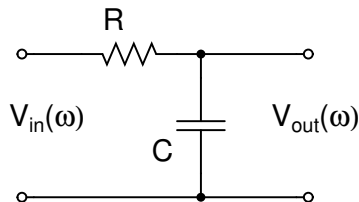
$$V_{out}(t) \approx \frac{1}{RC} \int V_{in}(t) dt$$

Integral representation in Fourier space

$$\begin{aligned} F(t) &= \int_{-\infty}^t f(t') dt' = \int_{-\infty}^t dt' \int_{-\infty}^{+\infty} f(\omega) e^{i\omega t'} d\omega \\ &= \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^t f(\omega) e^{i\omega t'} dt' \\ &= \int_{-\infty}^{+\infty} d\omega \left[\frac{f(\omega)}{i\omega} e^{i\omega t'} \Big|_{-\infty}^t \right] \\ &= \int_{-\infty}^{+\infty} d\omega \frac{f(\omega)}{i\omega} e^{i\omega t} = \int_{-\infty}^{+\infty} d\omega F(\omega) e^{i\omega t} \end{aligned}$$

$$\begin{aligned} F(t) &= \int_{-\infty}^t f(t') dt' \\ F(\omega) &= \frac{f(\omega)}{i\omega} \end{aligned}$$

Integrator



$$V_{out}(\omega) = G(\omega) V_{in}(\omega) = \frac{Z_c}{R + Z_c} V_{in}(\omega) = \frac{1}{1 + i\omega RC} V_{in}(\omega)$$

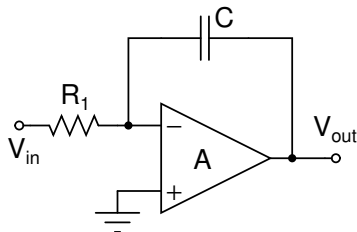
for $\omega \gg \omega_{3dB}$

$$V_{out}(\omega) \approx \frac{1}{RC} \frac{V_{in}(\omega)}{i\omega}$$

True Integrator / low-pass filter

We need to keep

$$I = \frac{V_{in}}{R}$$

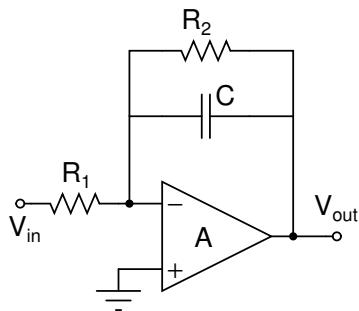


$$G(\omega) = -\frac{Z_c}{R_1} = -\frac{1}{i\omega R_1 C}$$

Only one problem remains: if any DC voltage is applied at input, output will reach a rail at power supply voltage.

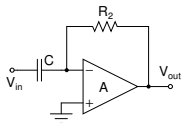
This can be thought as a lack of feedback since at DC capacitor blocks everything.

Low-pass filter / Integrator improved



$$G(\omega) = -\frac{Z_c \parallel R_2}{R_1} = -\frac{R_2}{R_1} \frac{1}{1 + i\omega R_2 C}$$

Differentiator / high-pass filter



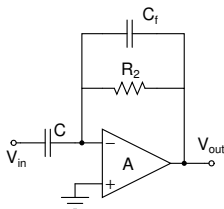
$$V_{in} = \frac{Q}{C} = \frac{1}{C} \int I dt \rightarrow I = C \frac{dV_{in}}{dt}$$
$$V_{out} = -IR_2$$

$$V_{out} = -R_2 C \frac{dV_{in}}{dt}$$

Fourier space

$$V_{out}(\omega) = -\frac{Z_{R_2}}{Z_C} = -i\omega R_2 C V_{in}(\omega) = \omega R_2 C V_{in}(\omega) e^{-i\frac{\pi}{2}}$$

Differentiator compensated



$$V_{out}(\omega) = -\frac{Z_{R_2} \parallel Z_f}{Z_c} V_{in}(\omega) = -\frac{i\omega R_2 C}{1 + i\omega R_2 C_f} V_{in}(\omega)$$

$$\omega \ll \frac{1}{R_2 C_f}$$

$$V_{out}(\omega) = -i\omega R_2 C V_{in}(\omega)$$

$$\omega \gg \frac{1}{R_2 C_f}$$

$$V_{out}(\omega) = -\frac{C}{C_f} V_{in}(\omega)$$

Thermistor linearization

$$R_{th}(T) = R_0 e^{-\gamma(T-T_0)}$$

Where $\gamma = 0.04$, $T_0 = 20\text{ }^\circ\text{C}$, $R_0 = 10\text{ k}\Omega$. Below circuit linearizes the output voltage vs temperature ($R = R_0$ as an example).

