Midterm 03

Due date Monday November 19th of 2012 at 1pm.

Discuss relevant equations, describe your solution, show results. All Matlab code/scripts must be present in the carbon copy as well.

Make all you calculations in the S.I. units (m, kg, s).

Solar system celestial mechanics (100 points total)

We will model the motion of the planets of the solar system governed by the gravitational force. In total we will consider 8 planets (Pluto is out), the Sun, and the Earth's moon.

The model is simplified:

- assume all bodies are moving in the same xy-plane
- disregard influence of stars, asteroids and other objects

The data files with masses, initial positions, and velocities will be provided on the web. Download the file 'solar_system_data.mat', load it with

`load 'solar_system_data.mat'`

and you will obtain variables `body_names, xposition, yposition, vx, vy,` and `mass`. These are column vectors of corresponding data. To see which index corresponds to which celestial body refers to `body_names` variable. For example, index 3 corresponds to Venus, since `body_names(3)` yields 'Venus'.

You job is to solve many body system numerically (it is known that even 3 body systems are impossible to solve analytically in the general case).

Do not hardcode the number of the celestial bodies, i.e. pull/deduce this information from the data file.

Important equations.

All you need to know is Newton's second law (**Pay attention to the vector notation! If not sure consult with the class instructor**)

$$m_i \vec{r_i}\,'' = m_i \vec{a}_i = \vec{F_i} \tag{1}$$

where i is the index of the body, since force is governed by gravitational pull of **all** other bodies, we can write it down as

$$\vec{F_i} = \sum_{j \neq i} \vec{F}_{ij} \tag{2}$$

$$\vec{F}_{ij} = G \frac{m_i m_j}{r^3} \vec{r}_{ij} \tag{3}$$

$$\vec{r}_{ij} = \vec{r_j} - \vec{r_i} \tag{4}$$

Where $G = 6.67428 \times 10^{-11}\ Nm^2/kg^2$ is the gravitational constant.

Note that since we consider only the xy-plane, all vectors have no z projection (i.e. $a_z = 0$, $F_z = 0, r_z = 0, v_z = 0$).

Task 1 (50 points): Calculate the planet position for the time span of at least one orbital period of Neptune ($\approx$ 165 years). Plot y(t) vs x(t) i.e. orbit shapes for this time period.

Task 2 (10 points): Make a movie of the planet motion for the first 12 years. Mark a planet position with a circle proportional to its mass (Sun might be the exception, chose something reasonable for it), and also leave a trace of the previous planets positions with lines. Make sure that you have enough frames to show dynamics of the system, but the movie size does not exceed **2 MB**.

Task 3 (10 points): Have a close look at the path of the Earth's moon, does it circle around (something like a stretched helix) and cross itself or just wobble around the Earth? Show the representative plot leading to your conclusion. Make plots in Cartesian and polar coordinates (see a note at the very end). Which one is more convincing?

Task 4 (20 points): Have a closer look at the sun's orbit. Which planet has the most influence on the sun's orbit? Try to remove the planet of question from the system and compare the sun tracks. Show plots which support your conclusion. Note: you might need a quite long time span.

Note: removal of any celestial body will make total momentum different from zero, which results in combined drift of the whole system in a certain direction. Pay attention to this drift and the wobble around it.

Task 5 (10 points):

It is well known that the presence of Neptune was first calculated by Urbain Le Verrier and then Neptune was observed very close to the predicted location in 1846. Le Verrier observed that he needed one more planet (later named Neptune) to explain the deviation of Uranus from the calculated track.

Can you show that removal of Neptune modifies the orbit of Uranus?

Make plots in Cartesian and polar coordinates (see a note at the end). Which one is more convincing?

Note: For some of these problems it is more convenient to use the polar coordinate system when you convert x and y to $\rho = \sqrt{x^2 + y^2}$ and $\phi = \arctan(y/x)$. Use plots of ρ vs ϕ. Especially for differences in one track with respect to another.