Homework 09

General comments:

- Do not forget to run some test cases.

Problem 1 (5 points)

A more realistic pendulum. Solve numerically (using built-in `ode45` solver) the following physical problem of a pendulum motions

$$\theta''(t) = -\frac{g}{L}\sin(\theta)$$

Here g is acceleration due to gravity (g=9.8 m/s^2), $L = 1$ is the length of the pendulum, and θ is angular deviation of the pendulum from the vertical.

$\theta(0)$ assuming that initial angular velocity is zero

$$\beta(0) = \theta'(0) = 0$$

Solve this problem (i.e. plot $\theta(t)$ and $\beta(t)$) for two values of the initial deflection $\theta(0) = \pi/10$ and $\theta(0) = \pi/3$. Make sure to choose final time large enough so you see at least 10 periods. Show that period of the pendulum depends on the initial deflection. Does is it takes longer to make one swing with larger or smaller initial deflection?

Problem 2 (5 points)

Have a look at the predator and prey model (the `ode_predator_prey_model.m` file provided with lecture 20 notes).

Find non trivial solution (i.e. $x_0 \neq 0$ and $y_0 \neq 0$) for which population of wolves and rabbits is independent of time (i.e. $dx/dt = dy/dt = 0$). You should get a system of two linear equations which is super simple, however I ask you to solve it using matlab numerical solver methods which we discussed during the lecture 21, i.e. form matrix A and column B, and find x such that $A * x = B$. Note: use constants a, b, c, and d provided in matlab file.

So we see that it possible to have stable populations (or economy with out ups and downs) but you need to be smart about initial conditions.

What is expected shape of the plot of the wolves population vs rabbits with calculated above initial conditions? Plot it.

Problem 3 (5 points)

It is possible to draw a parabola through any 3 point in a plane. Using matlab linear equations solver find coefficients a, b and c for parabola $y = ax^2 + bx + c$ which passes through points $p_1 = (-10, 10), p_2 = (-2, 12)$, and $p_3 = (12, 10)$.

Problem 4 (5 points)

Using matlab `interp1` with `spline` method, find where interpolation line crosses $y = 0$ level. The interpolation is done over the following points [(x,y) notation]: (2,10), (3,8), (4,4), (5,1), (6,-2).

Would it wise for above data point to search crossing with $x = 0$ line? Why so?