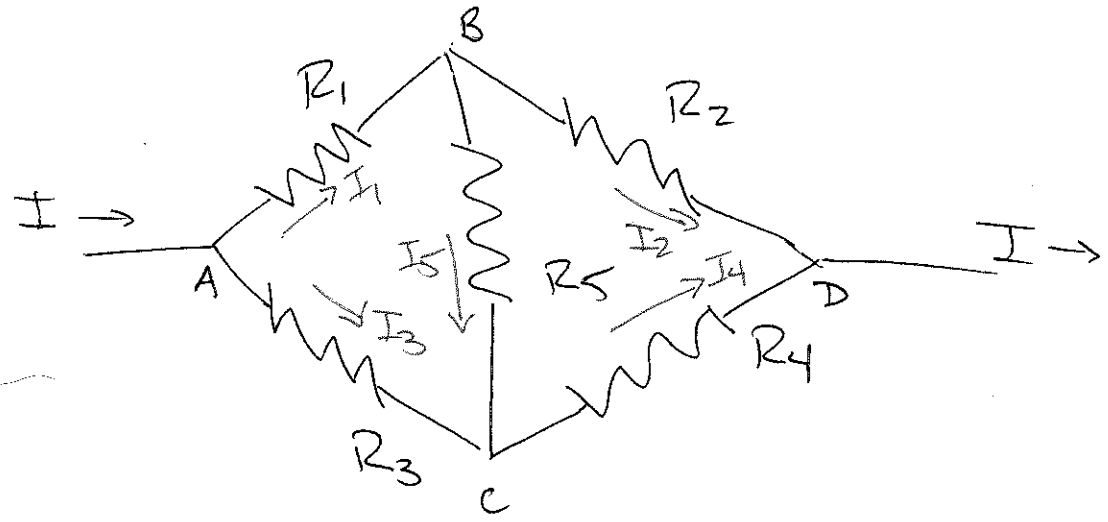
2-1



Junctions

A: $I = I_1 + I_3$

B: $I_2 = I_2 + I_5$

C: $I_4 = I_3 + I_5$

D: $I = I_2 + I_4$

Not independant.

only need,

$I_1 = I_2 + I_5$

$I_4 = I_3 + I_5$

Loops

ABCA: $-I_1R_1 - I_5R_3 + I_3R_3 = 0$

BDCB: $-I_2R_2 + I_4R_4 + I_5R_5 = 0$

want $R_{eq} = \frac{\Delta V}{I} = \frac{\Delta V_{AB} + \Delta V_{BD}}{I}$

$= \frac{I_1R_1 + I_2R_2}{I_1 + I_3}$ (from junction A)

5 unknown Currents, 4 Equations. 1 is arbitrary
Solve by hand or via Matrix. (under constrained problem)

Equations:

① $I_1 + I_2 + I_5 = 0$

② $I_3 - I_4 + I_5 = 0$

③ $I_1R_1 - I_3R_3 + I_5R_3 = 0$

④ $-I_2R_2 + I_4R_4 + I_5R_5 = 0$

forms Matrix,

$$\begin{pmatrix} 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 \\ R_1 & 0 & -R_3 & 0 & R_5 \\ 0 & -R_2 & 0 & R_4 & R_5 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

2-1

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restart;
with(LinearAlgebra) :
m := Matrix( [ [1,-1, 0, 0,-1], [0, 0,-1, 1,-1], [R1, 0,-R3, 0, R5], [0,-R2, 0, R4, R5] ]);
b := Matrix( [ [0], [0], [0], [0] ]) :
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$$\begin{bmatrix} 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 \\ R_1 & 0 & -R_3 & 0 & R_5 \\ 0 & -R_2 & 0 & R_4 & R_5 \end{bmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \textbf{(1)}$$

LinearSolve(*m*, *b*, *free* = *k*)

$$\begin{aligned} I_1 &= -\frac{k_{1,1}\left(R_3 R_5 + R_3 R_4 + R_5 R_4 + R_2 R_3\right)}{R_4 R_1 - R_2 R_3} \\ I_2 &= -\frac{k_{1,1}\left(R_3 R_5 + R_3 R_4 + R_5 R_4 + R_4 R_1\right)}{R_4 R_1 - R_2 R_3} \\ I_3 &= -\frac{k_{1,1}\left(R_5 R_1 + R_2 R_5 + R_2 R_1 + R_4 R_1\right)}{R_4 R_1 - R_2 R_3} \\ I_4 &= -\frac{\left(R_5 R_1 + R_2 R_5 + R_2 R_1 + R_2 R_3\right) k_{1,1}}{R_4 R_1 - R_2 R_3} \\ I_5 &= k_{1,1} \quad \text{(arbitrary)} \end{aligned} \quad \textbf{(2)}$$

$$R_{eq} := \text{simplify}\left(\frac{\left(-\frac{k_{1,1}\left(R_3 R_5 + R_3 R_4 + R_5 R_4 + R_2 R_3\right)}{R_4 R_1 - R_2 R_3} \cdot R_1 + -\frac{k_{1,1}\left(R_3 R_5 + R_3 R_4 + R_5 R_4 + R_4 R_1\right)}{R_4 R_1 - R_2 R_3} \cdot R_2 \right)}{-\frac{k_{1,1}\left(R_3 R_5 + R_3 R_4 + R_5 R_4 + R_2 R_3\right)}{R_4 R_1 - R_2 R_3} + -\frac{k_{1,1}\left(R_5 R_1 + R_2 R_5 + R_2 R_1 + R_4 R_1\right)}{R_4 R_1 - R_2 R_3}} \right);$$

$$\frac{R_3 R_5 R_1 + R_3 R_4 R_1 + R_5 R_4 R_1 + R_3 R_2 R_1 + R_3 R_2 R_5 + R_2 R_3 R_4 + R_2 R_5 R_4 + R_2 R_4 R_1}{R_3 R_5 + R_3 R_4 + R_5 R_4 + R_2 R_3 + R_5 R_1 + R_2 R_5 + R_2 R_1 + R_4 R_1} = R_{eq} \quad \textbf{(3)}$$

factor(*eval*(R_{eq}, $R_5 = 0$));

Balanced Bridge check

$$\frac{R_3 R_4 R_1 + R_3 R_2 R_1 + R_2 R_3 R_4 + R_2 R_4 R_1}{\left(R_2 + R_4\right)\left(R_3 + R_1\right)} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} \quad \textbf{(4)}$$

P

$$R_{eq} = \frac{I_1 R_1 + I_2 R_2}{I_1 + I_3}$$