

Optimization problem

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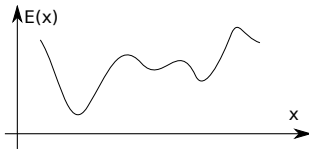
The College of William & Mary



Lecture 13

Notes

Introduction to optimization



Find \vec{x} that minimize $E(\vec{x})$ subject to $g(\vec{x}) = 0, h(\vec{x}) \leq 0$

\vec{x} design variables

$E(\vec{x})$ merit or objective or fitness or energy function

$g(\vec{x})$ and $h(\vec{x})$ constrains

There is no guaranteed way (algorithm) which can find global minimum (optimal) point.

Easy to see that maximization problem is the same as minimization once $E(\vec{x}) \rightarrow -E(\vec{x})$.

Notes

Analytical solution of 1D

If we have 1D case and $E(x)$ has analytical derivative, optimization problem can be restated as

Find $f(x) = 0$
where $f(x) = dE/dx$

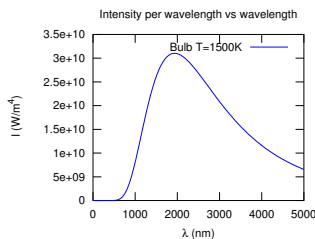
since at maximum or minimum derivative must be zero.
Since we already know how to find the solution of $f(x) = 0$ the rest is easy.

Notes

Example: max of black body radiation spectrum

According to Plank's law energy density per of black body radiation

$$I(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$



where

h is Planck constant $6.626 \times 10^{-34} \text{ J}\cdot\text{s}$,

c is speed of light $2.998 \times 10^8 \text{ m/s}$,

k is Boltzmann constant $1.380 \times 10^{-23} \text{ J K}^{-1}$,

T is body temperature,

λ is wavelength

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Solution with Matlab built in 1D minimization - fminbnd

```
function I_lambda=black_body_radiation (lambda,T)
% black body radiation spectrum
% lambda – wavelength of EM wave
% T – temperature of a black body
h=6.626e-34; % Plank constant
c=2.998e8; % speed of light
k=1.380e-23; % Boltzmann constant

I_lambda = 2*h*c^2 ./ (lambda.^5) ./ (exp(h*c./(lambda*k*T))-1);
end
```

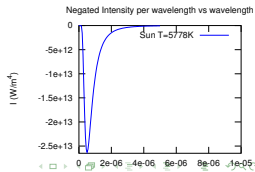
First we flip/negate function since our algorithm is suited for min search and set particular T

```
T=5778;
f = @(x) - black_body_radiation(x,T);
```

Next, we find optimal solution

```
fminbnd(f,1e-9,2e-6,optimset('TolX',1e-12))
ans = 5.0176e-07
% i.e. maximum radiation is at 502 nm
```

Then we plot it to find a bracket



Golden section search algorithm

If you have an initial bracket for solution i.e. found a, b points such that there is a point x_p satisfying $a < x_p < b$ and $E(x_p) < \min(E(a), E(b))$.

Then $h = (b - a)$

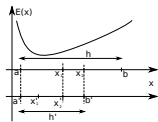
- 1 assign new probe points $x_1 = a + R * h$ and $x_2 = b - R * h$
- 2 $E_1 = E(x_1), E_2 = E(x_2), E_a = E(a), E_b = E(b)$
- 3 if $h < \epsilon_x$ then stop otherwise do steps below
- 4 note that for small enough $h: E(x_1) < E(a)$ and $E(x_2) < E(b)$
- 5 shrink/update the bracket
 - if $E_1 < E_2$ then $b = x_2, E_b = E_2$ else $a = x_1, E_a = E_1$
- 6 update $h = (b - a)$ and assign new probe points, **with the proper** R we can reuse one of the old points either x_1, E_1 or x_2, E_2
 - if $E_1 < E_2$ then $x_2 = x_1, E_2 = E_1, x_1 = a + R * h, E_1 = E(x_1)$
 - else $x_1 = x_2, E_1 = E_2, x_2 = b - R * h, E_2 = E(x_2)$
- 7 go to step 3

R given by the golden section $R = \frac{3 - \sqrt{5}}{2} \approx 0.38197$

Derivation of the R value

at first step we have

$$\begin{aligned} x_1 &= a + R * h \\ x_2 &= b - R * h \end{aligned}$$



Suppose that $E(x_1) < E(x_2)$ then $a' = a$ and $b' = x_2$ then for the next bracket we evaluate x'_1 and x'_2

$$\begin{aligned} x'_1 &= a' + R * h' = a' + R * (b' - a') \\ x'_2 &= b' - R * h' = b' - R * (b' - a') \\ &= x_2 - R * (x_2 - a) = b - R * h - R * (b - R * h - a) \end{aligned}$$

we would like to reuse one of the previous evaluations of E so we require that $x_1 = x'_2$. This leads to equation

$$R^2 - 3R + 1 = 0 \text{ with } R = \frac{3 \pm \sqrt{5}}{2}$$

We need to choose **minus** sign since fraction $R < 1$.

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