Sorting continued

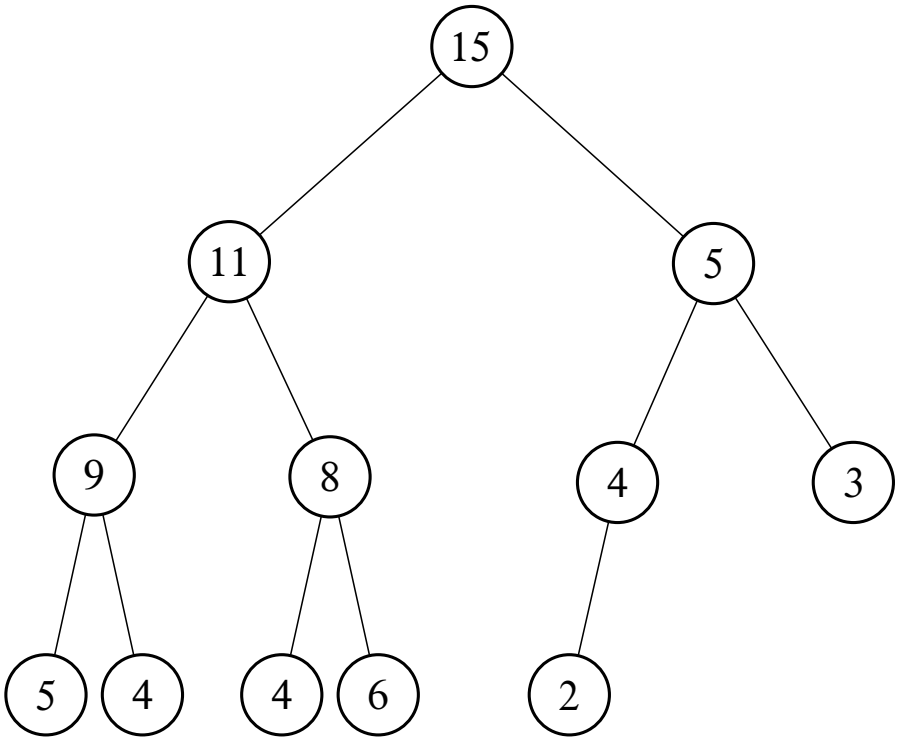
Eugeniy E. Mikhailov

The College of William & Mary

Lecture 08

Heap

Heap is a structure where parent element is larger or equal to its children.



The top most element of the heap is called root.

Heap sorting method

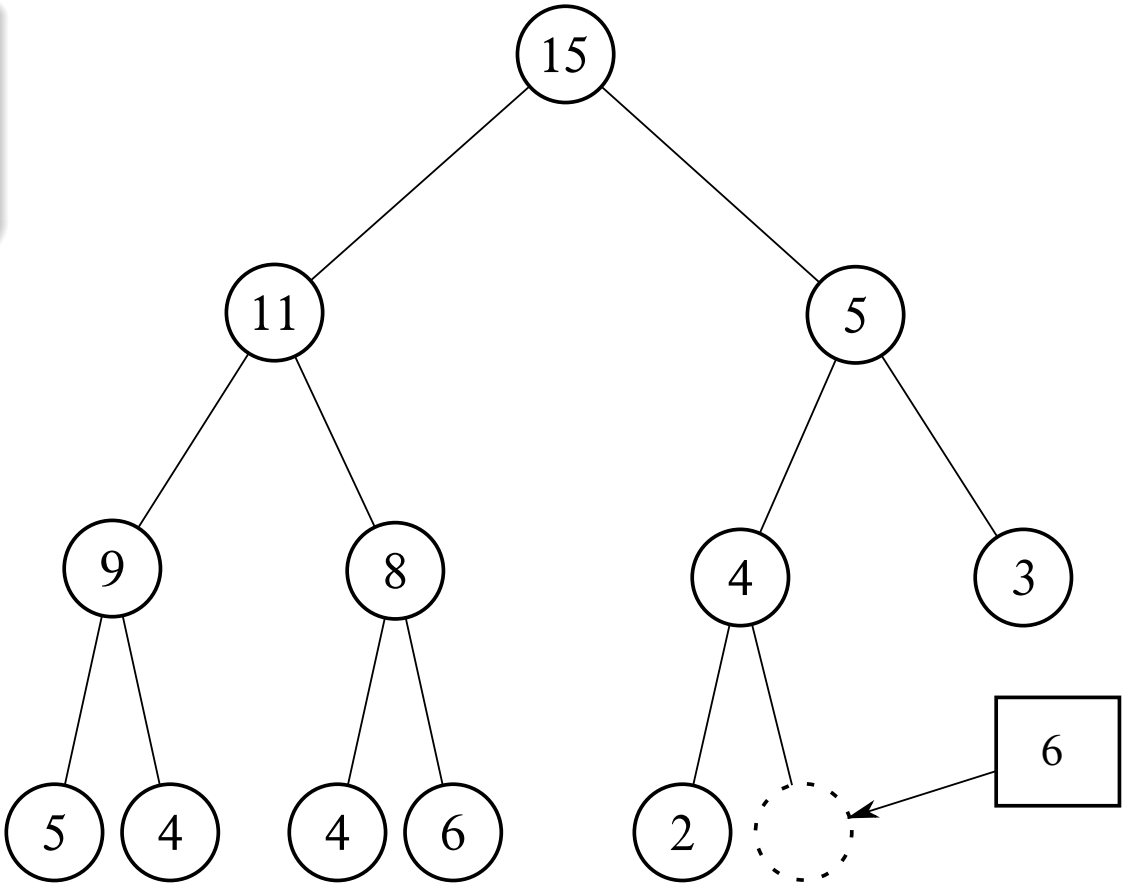
1. Fill the heap from the input vector elements
   1. take the element and place it at the bottom of the heap
   2. sift-up (bubble up) this element
   3. do the same with the next element
2. remove the root element since it is the largest
3. rearrange the heap i.e. sift-down
   1. take the last bottom element
   2. place it at the root
   3. check if parent is larger then children
      1. find the largest child element
      2. if the largest child is larger then parent swap them and repeat the check
4. repeat step 2 until no elements left in the heap

Heap sorting complexity $\mathcal{O}(N \log_2 N)$

Filling (sift-up) the heap

Step 1

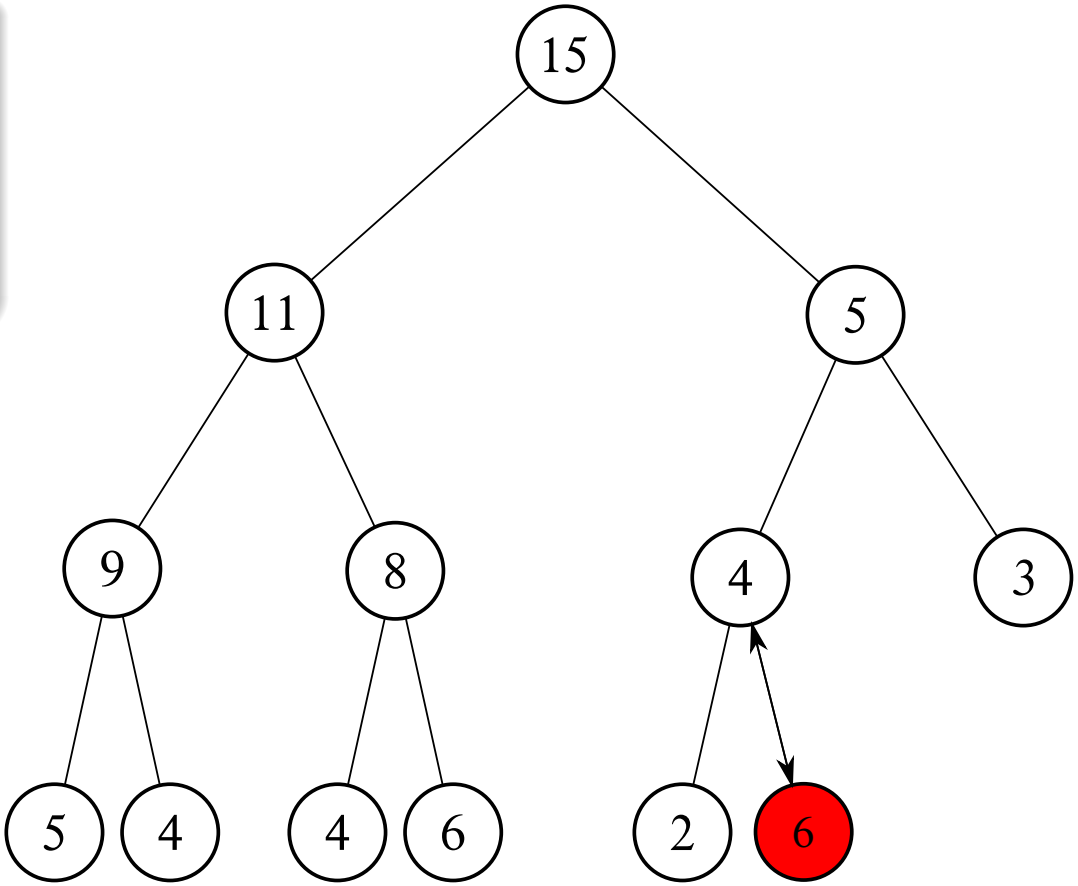
Place new element at the bottom of the heap



Filling (sift-up) the heap

Step 2

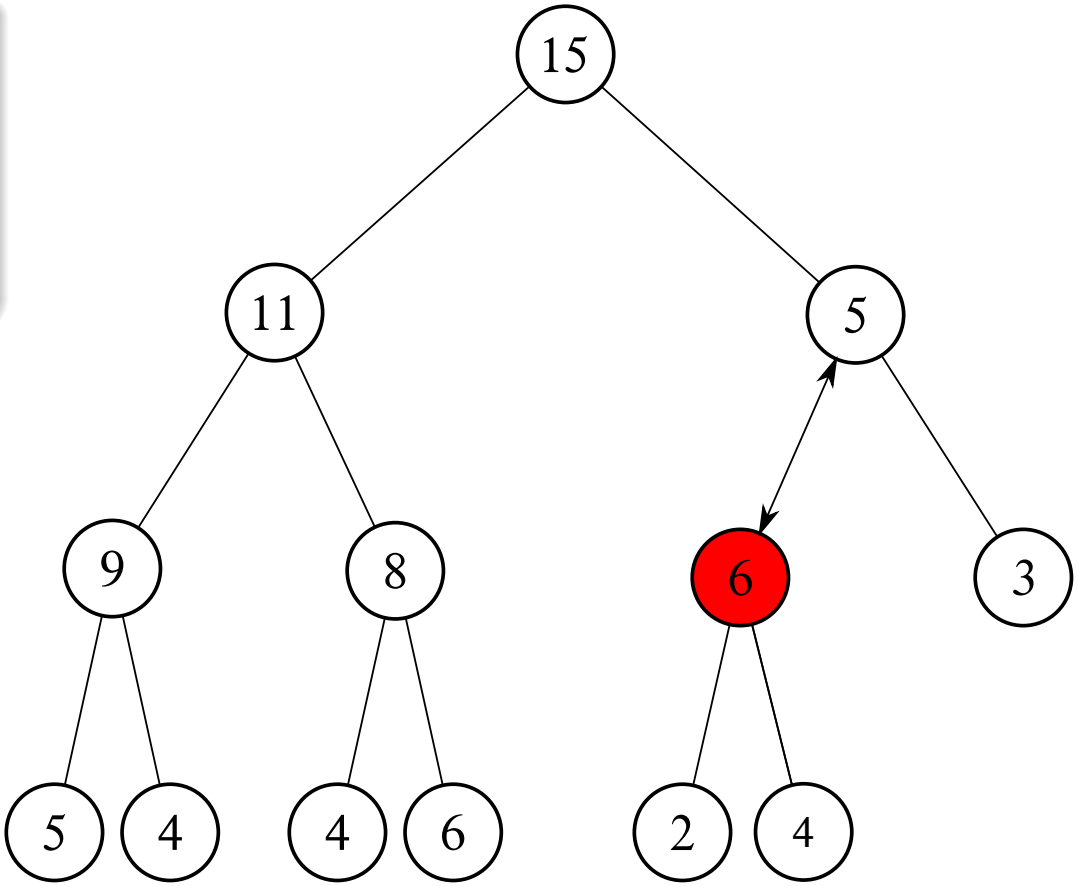
Check if parent is larger then child. If so swap them and repeat step 2.



Filling (sift-up) the heap

Step 2

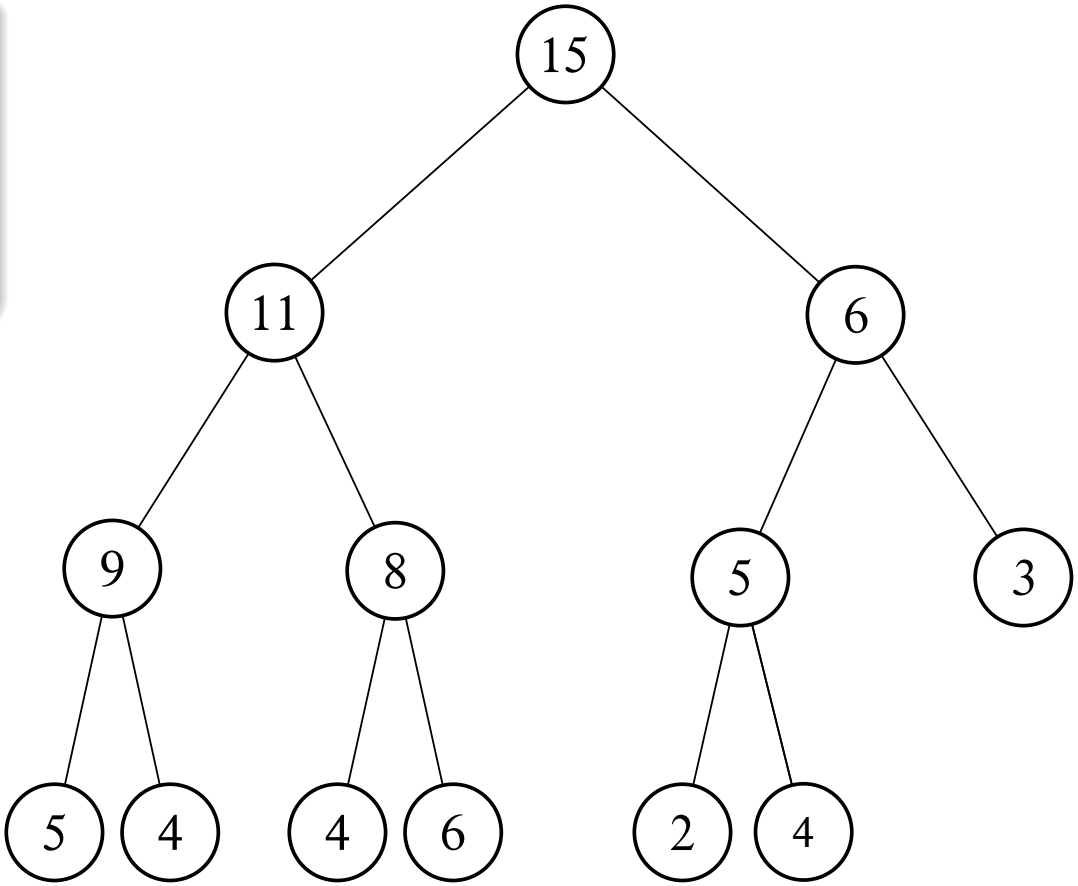
Check if parent is larger then child. If so swap them and repeat step 2.



Filling (sift-up) the heap

Step 2

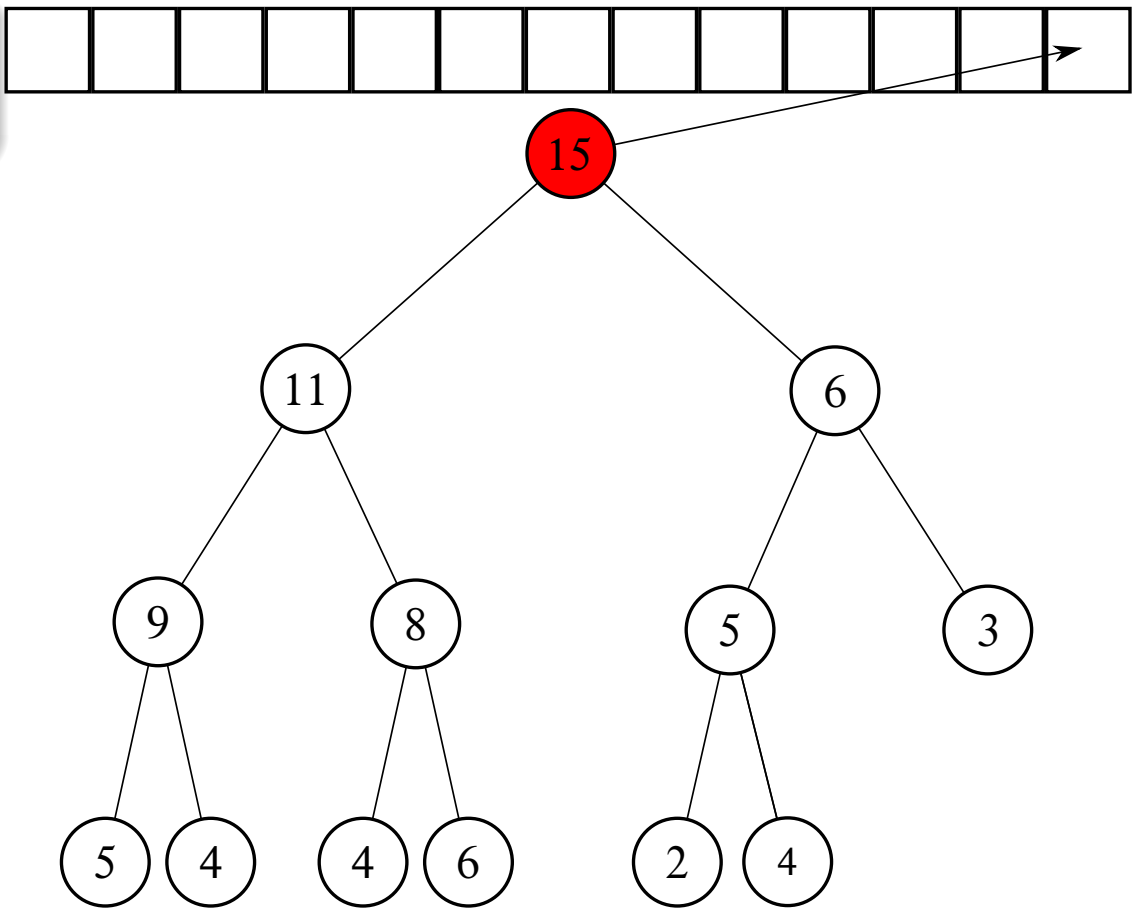
Check if parent is larger then child. If so swap them and repeat step 2.



Removing from the heap (sift-down) the heap

Step 1

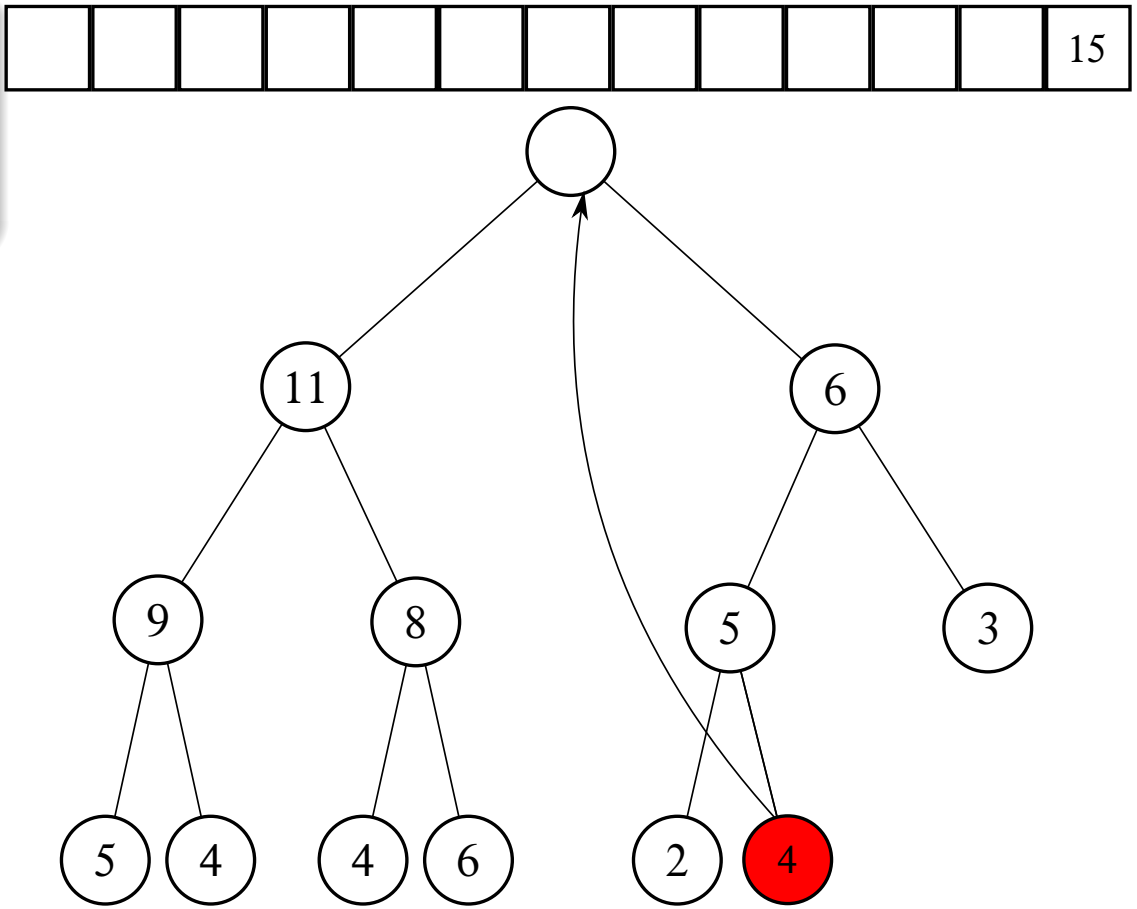
Remove the root element



Removing from the heap (sift-down) the heap

Step 2

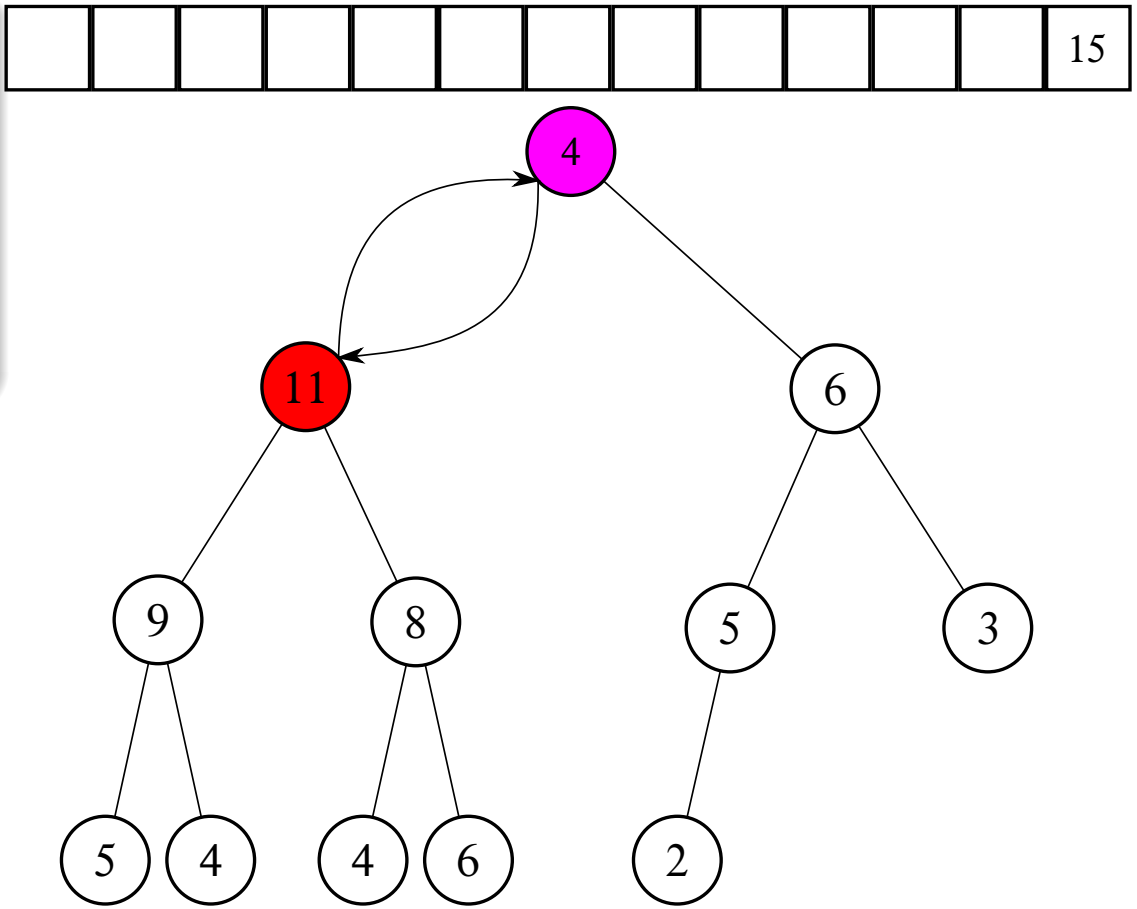
Place the last element of the heap to the root



Removing from the heap (sift-down) the heap

Step 3

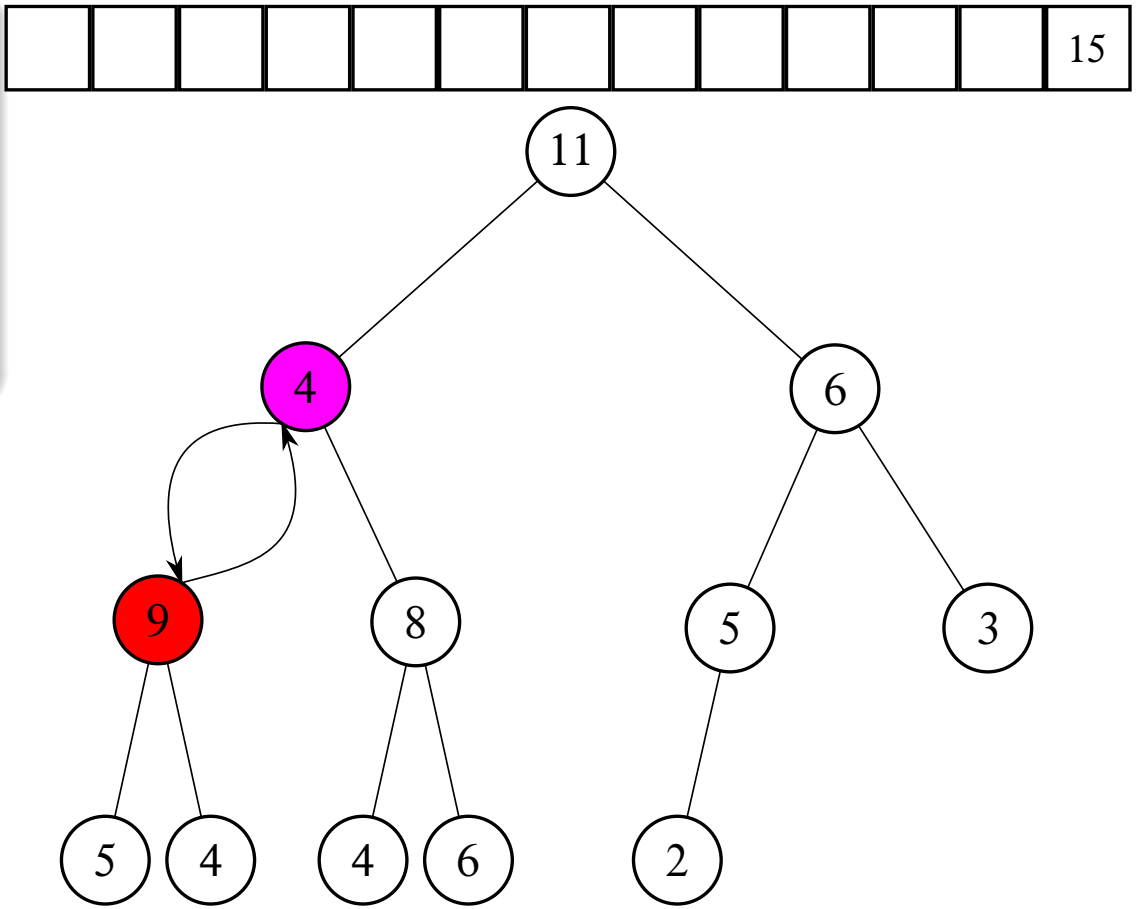
Check if parent is smaller than the largest child. If so swap and repeat step 3 else go to step 1



Removing from the heap (sift-down) the heap

Step 3

Check if parent is smaller than the largest child. If so swap and repeat step 3 else go to step 1

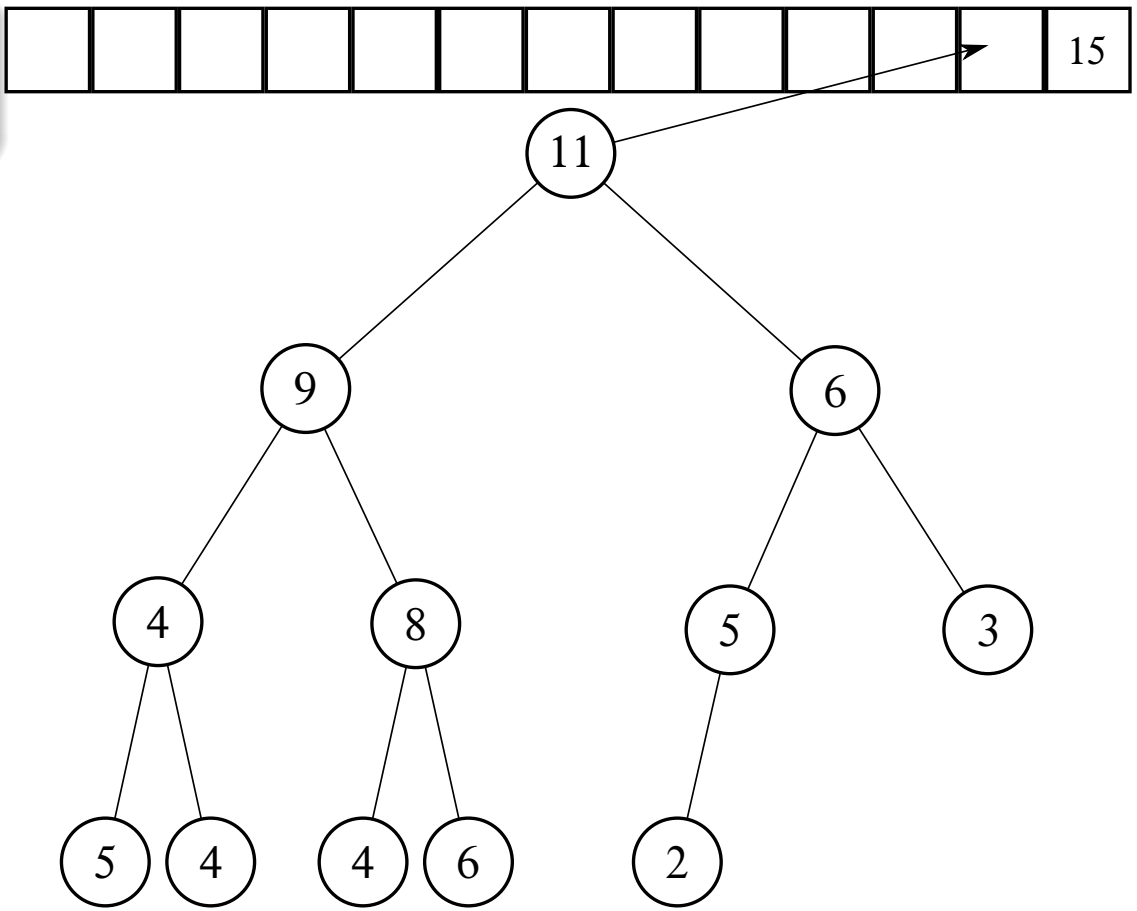


Removing from the heap (sift-down) the heap

Sequence repeats

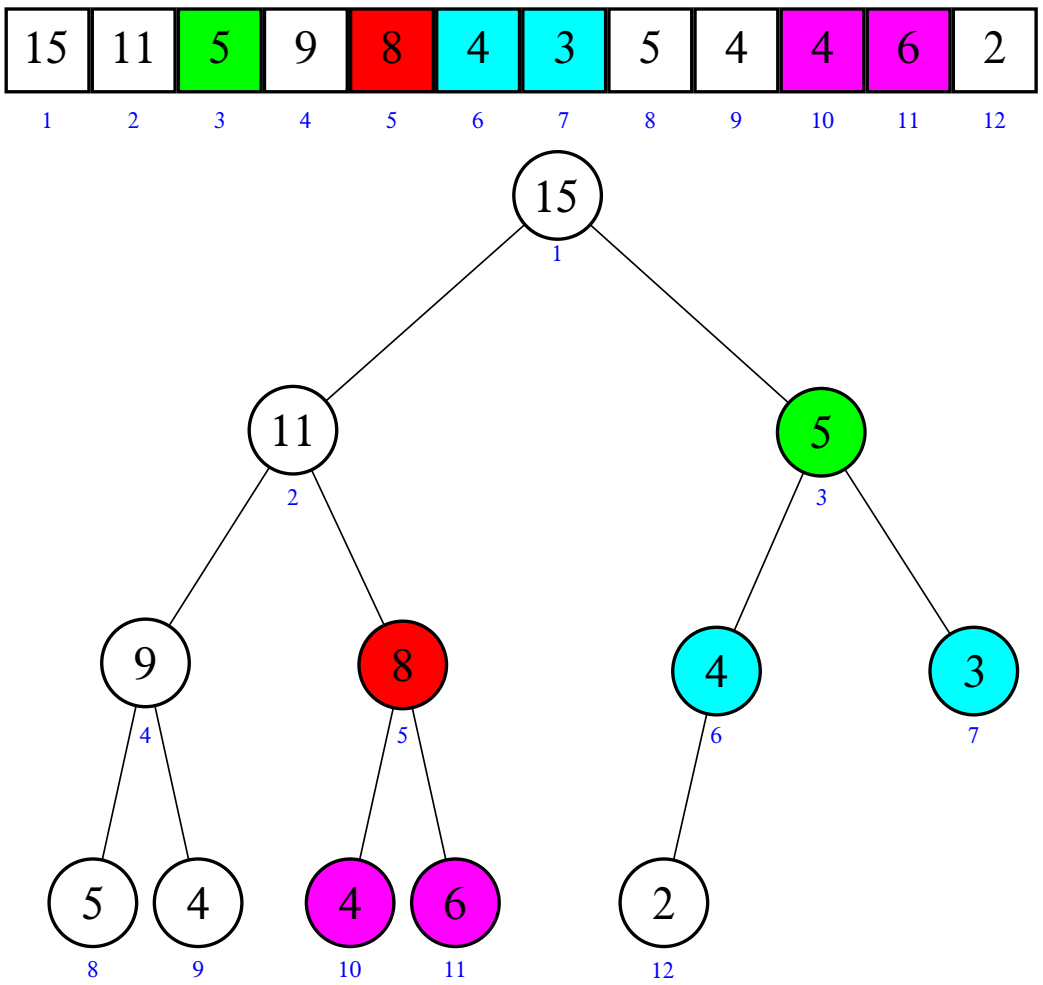
Step 1

Remove the root element



Vector heap representation

- Heap nodes are numbered consequently these numbers represent the node position in the vector.
- notice that parent and children have very simple relationship
  - if parent node index is i
    - child 1 index is $2i$
    - child 2 index is $2i + 1$
  - if we know child index (i) then
    - parent index is `floor`$(i/2)$



Matlab built in 'issorted'

Easy check if an array is sorted can be done with `issorted` which returns `true` or `false`.

```
>> x=[1,2,3];
>> issorted(x)
ans =
1
```

`issorted` checks only for ascending order, for example

```
>> x=[3,2,1];
>> issorted(x)
ans =
0
```

Recall that '0' is equivalent of `false` in Matlab