Boolean algebra, conditional statements, loops.

Eugeniy E. Mikhailov

The College of William & Mary

Lecture 03

Notes

Boolean algebra

Variable of boolean type can have only two values

- true
- false

Notes

Boolean algebra

Variable of boolean type can have only two values

- true (Matlab use `1` to indicate it, actually everything but zero)
- false

Notes

Boolean algebra

Variable of boolean type can have only two values

- true (Matlab use `1` to indicate it, actually everything but zero)
- false (Matlab uses `0`)

Notes

Boolean algebra

Variable of boolean type can have only two values
- true (Matlab use `1` to indicate it, actually everything but zero)
- false (Matlab uses `0`)

There are three logical operators which are used in boolean algebra

Notes

Boolean algebra

Variable of boolean type can have only two values
- true (Matlab use `1` to indicate it, actually everything but zero)
- false (Matlab uses `0`)

There are three logical operators which are used in boolean algebra
- $\neg$ - logic **not**, Matlab `~`

$$\neg\text{true} = \text{false}$$
$$\neg\text{false} = \text{true}$$

Notes

Boolean algebra

Variable of boolean type can have only two values
- true (Matlab use `1` to indicate it, actually everything but zero)
- false (Matlab uses `0`)

There are three logical operators which are used in boolean algebra
- $\neg$ - logic **not**, Matlab `~`

$$\neg\text{true} = \text{false}$$
$$\neg\text{false} = \text{true}$$

- $\wedge$ - logic **and**, Matlab `&`

$$A \wedge B = \begin{cases} \text{true, if A=true and B=true,} \\ \text{false, otherwise} \end{cases}$$

Notes

Boolean algebra

Variable of boolean type can have only two values
- true (Matlab use `1` to indicate it, actually everything but zero)
- false (Matlab uses `0`)

There are three logical operators which are used in boolean algebra
- $\neg$ - logic **not**, Matlab `~`

$$\neg\text{true} = \text{false}$$
$$\neg\text{false} = \text{true}$$

- $\wedge$ - logic **and**, Matlab `&`

$$A \wedge B = \begin{cases} \text{true, if A=true and B=true,} \\ \text{false, otherwise} \end{cases}$$

- $\vee$ - logic **or**, Matlab `|`

$$A \vee B = \begin{cases} \text{false, if A=false and B=false,} \\ \text{true, otherwise} \end{cases}$$

Notes

Boolean operators precedence in Matlab

If $A = false$, $B = true$, $C = true$

$$A|\sim B\&C$$

Notes

Boolean operators precedence in Matlab

If $A = false$, $B = true$, $C = true$

$$A|\sim B\&C$$

$\sim$ has highest precedence, then &, and then |

Notes

Boolean operators precedence in Matlab

If $A = false$, $B = true$, $C = true$

$$A|\sim B\&C$$

$\sim$ has highest precedence, then &, and then |

$$A|((\sim B)\&C)$$

Notes

Boolean operators precedence in Matlab

If $A = false$, $B = true$, $C = true$

$$A|\sim B\&C$$

$\sim$ has highest precedence, then &, and then |

$$A|((\sim B)\&C)$$

Thus

$$A|\sim B\&C = false$$

Notes

Boolean operators precedence in Matlab

If $A = false$, $B = true$, $C = true$

$$A|\sim B\&C$$

$\sim$ has highest precedence, then &, and then |

$$A|\ ((\sim B)\&C)$$

Thus

$$A|\sim B\&C = false$$

"Cat is an animal and cat is not an animal"

Notes

Boolean operators precedence in Matlab

If $A = false$, $B = true$, $C = true$

$$A|\sim B\&C$$

$\sim$ has highest precedence, then &, and then |

$$A|\ ((\sim B)\&C)$$

Thus

$$A|\sim B\&C = false$$

"Cat is an animal and cat is not an animal"
is false statement

Notes

Boolean operators precedence in Matlab

If $A = false$, $B = true$, $C = true$

$$A|\sim B\&C$$

$\sim$ has highest precedence, then &, and then |

$$A|\ ((\sim B)\&C)$$

Thus

$$A|\sim B\&C = false$$

"Cat is an animal and cat is not an animal"
is false statement

$$\sim Z\&Z =$$

Notes

Boolean operators precedence in Matlab

If $A = false$, $B = true$, $C = true$

$$A|\sim B\&C$$

$\sim$ has highest precedence, then &, and then |

$$A|\ ((\sim B)\&C)$$

Thus

$$A|\sim B\&C = false$$

"Cat is an animal and cat is not an animal"
is false statement

$$\sim Z\&Z = false$$

Notes

Boolean logic examples

There is an island, which is populated by two kind of people: liars and truthlovers.

- Liars always lie and never speak a word of truth.
- Truthlovers always speak only truth.

Suppose, you are landed on this island and met a person. What will be the answer to your question “Who are you?”

Notes

Boolean logic examples

There is an island, which is populated by two kind of people: liars and truthlovers.

- Liars always lie and never speak a word of truth.
- Truthlovers always speak only truth.

Suppose, you are landed on this island and met a person. What will be the answer to your question “Who are you?”

- The answer always will be “Truthlover”.

Notes

Boolean logic examples

There is an island, which is populated by two kind of people: liars and truthlovers.

- Liars always lie and never speak a word of truth.
- Truthlovers always speak only truth.

Suppose, you are landed on this island and met a person. What will be the answer to your question “Who are you?”

- The answer always will be “Truthlover”.

Now you see a person who answers to your question. “I am a liar.”
Is it possible?

Notes

Boolean logic examples

There is an island, which is populated by two kind of people: liars and truthlovers.

- Liars always lie and never speak a word of truth.
- Truthlovers always speak only truth.

Suppose, you are landed on this island and met a person. What will be the answer to your question “Who are you?”

- The answer always will be “Truthlover”.

Now you see a person who answers to your question. “I am a liar.”
Is it possible?

- This makes a paradox and should not ever happen on this island.

Notes

Matlab boolean logic examples

- `123.3 & 12=`

Notes

Matlab boolean logic examples

- `123.3 & 12=` 1
- `~ 1232e-6 =`

Notes

Matlab boolean logic examples

- `123.3 & 12=` 1
- `~ 1232e-6 =` 0

Notes

Matlab boolean logic examples

- `123.3 & 12=` 1
- `~ 1232e-6 =` 0

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000
```


Notes

Matlab boolean logic examples

- `123.3 & 12` = 1
- `~ 1232e-6` = 0

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000
```

```
~B
```


Notes

Matlab boolean logic examples

- `123.3 & 12` = 1
- `~ 1232e-6` = 0

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000
```

```
~B
```

```
ans =
0     1
0     0
```


Notes

Matlab boolean logic examples

- `123.3 & 12` = 1
- `~ 1232e-6` = 0

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000
```

```
~B
```

```
ans =
0     1
0     0
```

```
B|~B
```


Notes

Matlab boolean logic examples

- `123.3 & 12` = 1
- `~ 1232e-6` = 0

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000
```

```
~B
```

```
ans =
0     1
0     0
```

```
B|~B
```

"To be or not to be"

```
ans =
1     1
1     1
```

Matlab boolean logic examples

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000

>> A=[56, 655; 0, 24.4]
A =
56.0000  655.0000
0        24.4000
```


Notes

Matlab boolean logic examples

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000

>> A=[56, 655; 0, 24.4]
A =
56.0000  655.0000
0        24.4000
```

```
B&A
```


Notes

Matlab boolean logic examples

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000

>> A=[56, 655; 0, 24.4]
A =
56.0000  655.0000
0        24.4000
```

```
B&A
```

```
ans =
1     0
0     1
```


Notes

Matlab boolean logic examples

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000

>> A=[56, 655; 0, 24.4]
A =
56.0000  655.0000
0        24.4000
```

```
B&A
```

```
ans =
1     0
0     1
```

```
A|~B
```


Notes

Matlab boolean logic examples

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231    0
34.3430   12.0000

>> A=[56, 655; 0, 24.4]
A =
56.0000  655.0000
0        24.4000
```

```
B&A
```

```
ans =
1     0
0     1
```

```
A|~B
```

```
ans =
1     1
0     1
```


Notes

Comparison operators

Math	Matlab
$=$	== double equal sign!
$\neq$	~=
$<$	<
$\leq$	<=
$>$	>
$\geq$	>=

Notes

Comparison operators

Math	Matlab
$=$	== double equal sign!
$\neq$	~=
$<$	<
$\leq$	<=
$>$	>
$\geq$	>=

```
x=[1,2,3,4,5]
x =
   1    2    3    4    5
```


Notes

Comparison operators

Math	Matlab
$=$	== double equal sign!
$\neq$	~=
$<$	<
$\leq$	<=
$>$	>
$\geq$	>=

```
x=[1,2,3,4,5]
x =
   1    2    3    4    5
```

```
x >= 3
```


Notes

Comparison operators

Math	Matlab
$=$	== double equal sign!
$\neq$	~=
$<$	<
$\leq$	<=
$>$	>
$\geq$	>=

```
x=[1,2,3,4,5]
x =
   1    2    3    4    5
```

```
x >= 3
```

```
ans =
   0    0    1    1    1
```


Notes

Comparison operators

Math	Matlab
$=$	== double equal sign!
$\neq$	~=
$<$	<
$\leq$	<=
$>$	>
$\geq$	>=

```
x=[1,2,3,4,5]
x =
   1    2    3    4    5
```

```
x >= 3
```

```
% chose such 'x' where x>=3
x(x >= 3)
```

```
ans =
   0    0    1    1    1
```


Notes

Comparison operators

Math	Matlab
$=$	== double equal sign!
$\neq$	~=
$<$	<
$\leq$	<=
$>$	>
$\geq$	>=

```
x=[1,2,3,4,5]
x =
   1    2    3    4    5
```

```
x >= 3
```

```
% chose such 'x' where x>=3
x(x >= 3)
```

```
ans =
   0    0    1    1    1
```

```
ans =
     3    4    5
```


Notes

Comparison with matrices

```
>> A=[1,2;3,4]
A =
1      2
3      4
```

```
>> B=[33,11;53,42]
B =
22      11
53      42
```


Notes

Comparison with matrices

```
>> A=[1,2;3,4]
A =
1     2
3     4
```

```
>> B=[33,11;53,42]
B =
22     11
53     42
```

```
A>=2
```


Notes

Comparison with matrices

```
>> A=[1,2;3,4]
A =
1     2
3     4
```

```
>> B=[33,11;53,42]
B =
22     11
53     42
```

```
A>=2
```

```
ans =
0     1
1     1
```


Notes

Comparison with matrices

```
>> A=[1,2;3,4]
A =
1     2
3     4
```

```
>> B=[33,11;53,42]
B =
22     11
53     42
```

```
A>=2
```

```
ans =
0     1
1     1
```

```
A(A>=2)
```


Notes

Comparison with matrices

```
>> A=[1,2;3,4]
A =
1     2
3     4
```

```
>> B=[33,11;53,42]
B =
22     11
53     42
```

```
A>=2
```

```
ans =
0     1
1     1
```

```
A(A>=2)
```

```
ans =
3
2
4
```


Notes

Comparison with matrices

```
>> A=[1,2;3,4]
A =
1     2
3     4
```

```
>> B=[33,11;53,42]
B =
22     11
53     42
```

```
A>=2
```

```
ans =
0     1
1     1
```

```
A(A>=2)
```

```
ans =
3
2
4
```

```
B(A>=2)
```

Chose such elements of B where elements of A$\geq$2

Notes

Comparison with matrices

```
>> A=[1,2;3,4]
A =
1     2
3     4
```

```
>> B=[33,11;53,42]
B =
22     11
53     42
```

```
A>=2
```

```
ans =
0     1
1     1
```

```
A(A>=2)
```

```
ans =
3
2
4
```

```
B(A>=2)
```

Chose such elements of B where elements of A$\geq$2

```
ans =
53
11
42
```

Eugeniy Mikhailov (W&M) Practical Computing Lecture 03 8 / 19

Notes

if-else-end statement

`if` *expression*
this part is executed only if *expression* is true
`else`
this part is executed only if *expression* is false
`end`

Notes

if-else-end statement

`if` *expression*
this part is executed only if *expression* is true
`else`
this part is executed only if *expression* is false
`end`

```
if hungry
buy some food
else
keep working
end
```

Eugeniy Mikhailov (W&M) Practical Computing Lecture 03 9 / 19

Notes

if-else-end statement

`if` *expression*
this part is executed only if *expression* is true
`else`
this part is executed only if *expression* is false
`end`

`if` *hungry*
buy some food
`else`
keep working
`end`

```
if (x>=0)
  y=sqrt(x);
else
  error('cannot do');
end
```


Notes

Common mistake in the 'if' statement

```
if (x=y)
  D=4;
  Z=45;
  C=12;
else
  D=2;
end
```


Notes

Common mistake in the 'if' statement

```
if (x=y)
  D=4;
  Z=45;
  C=12;
else
  D=2;
end
```

the value of 'D' is always 4, except the case when y=0

Notes

Common mistake in the 'if' statement

```
if (x=y)
  D=4;
  Z=45;
  C=12;
else
  D=2;
end
```

the value of 'D' is always 4, except the case when y=0
someone used assignment operator (=) instead of comparison (==)

Notes

Short form of 'if-end' statement

`if` *expression*
this part is executed
only if *expression* is
true
`end`

Notes

Short form of 'if-end' statement

`if` *expression*
this part is executed
only if *expression* is
true
`end`

`if` *won a million*
go party
`end`

Notes

Short form of 'if-end' statement

`if` *expression*
this part is executed
only if *expression* is
true
`end`

`if` *won a million*
go party
`end`

```
if (deviation<=0)
  exit;
end
```


Notes

The 'while' statement

`while` *expression*
this part is executed
while *expression* is
true
`end`

Notes

The 'while' statement

`while` *expression*
this part is executed while *expression* is true
`end`

```
while hungry
keep eating
end
```


Notes

The 'while' statement

`while` *expression*
this part is executed while *expression* is true
`end`

```
while hungry
keep eating
end
```

```
i=1;
while (i<=10)
  c=a+b;
  z=c*4+5;
  i=i+2;
end
```


Notes

The 'while' statement

`while` *expression*
this part is executed while *expression* is true
`end`

```
while hungry
keep eating
end
```

```
i=1;
while (i<=10)
  c=a+b;
  z=c*4+5;
  i=i+2;
end
```

`while` loop is extremely useful but they are not guaranteed to finish. For a bit more complicated conditional statement and loop it is impossible to predict if the loop will finish.

Notes

The 'while' statement

`while` *expression*
this part is executed while *expression* is true
`end`

```
while hungry
keep eating
end
```

```
i=1;
while (i<=10)
  c=a+b;
  z=c*4+5;
  i=i+2;
end
```

`while` loop is extremely useful but they are not guaranteed to finish. For a bit more complicated conditional statement and loop it is impossible to predict if the loop will finish.

Yet another common mistake is

```
i=1;
while (i<=10)
  c=a+b;
end
```


Notes

The 'while' statement

```
while expression
this part is executed
while expression is
true
end
```

```
while hungry
keep eating
end
```

```
i=1;
while (i<=10)
  c=a+b;
  z=c*4+5;
  i=i+2;
end
```

`while` loop is extremely useful but they are not guaranteed to finish. For a bit more complicated conditional statement and loop it is impossible to predict if the loop will finish.

Yet another common mistake is

```
i=1;
while (i<=10)
  c=a+b;
end
```

not updating the term leading to fulfillment of the `while` condition

Notes

The 'for' statement

`for` variable = *expression*
do something
`end`
In this case variable is assigned concequently with columns of the *expression*, and then statements inside of the loop are executed

Notes

The 'for' statement

`for` variable = *expression*
do something
`end`
In this case variable is assigned concequently with columns of the *expression*, and then statements inside of the loop are executed

```
sum=0;
x=[1,3,5,6]
for v=x
  sum=sum+v;
end
```

```
>> sum
sum =
  15
```


Notes

The 'for' statement

`for` variable = *expression*
do something
`end`
In this case variable is assigned concequently with columns of the *expression*, and then statements inside of the loop are executed

```
sum=0;
x=[1,3,5,6]
for v=x
  sum=sum+v;
end
```

```
>> sum
sum =
  15
```

`for` loops are guaranteed to complete after predictable number of iterations (the amount of columns in *expression*).

Notes

Example

$$S = \sum_{i=1}^{100} i = 1 + 2 + 3 + 4 + \cdots + 99 + 100$$

Notes

Example

$$S = \sum_{i=1}^{100} i = 1 + 2 + 3 + 4 + \cdots + 99 + 100$$

```
S=0; i=1;
while(i<=100)
  S=S+i;
  i=i+1;
end
```

Notes

Example

$$S = \sum_{i=1}^{100} i = 1 + 2 + 3 + 4 + \cdots + 99 + 100$$

```
S=0; i=1;
while(i<=100)
  S=S+i;
  i=i+1;
end
```

```
S=0;
for i=1:100
  S=S+i;
end
```

Notes

Example

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k \geq 10^{-5}$, where $a_k = k^{-k}$.

Notes

Example

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k \geq 10^{-5}$, where $a_k = k^{-k}$.

```
S=0; k=1;
while( (k<=100) & (k^-k >= 1e-5) )
  S=S+k^-k;
  k=k+1;
end
```

Notes

Example

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k \geq 10^{-5}$, where $a_k = k^{-k}$.

```
S=0; k=1;
while( (k<=100) & (k^-k >= 1e-5) )
  S=S+k^-k;
  k=k+1;
end
```

```
>> S
S =
  1.2913
```

Notes

Example

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k \geq 10^{-5}$, where $a_k = k^{-k}$.

```
S=0; k=1;
while( (k<=100) & (k^-k >= 1e-5) )
  S=S+k^-k;
  k=k+1;
end
```

```
>> S
S =
  1.2913
```

```
S=0; k=1;
while( k<=100 )
  a_k=k^-k;
  if (a_k < 1e-5)
    break;
  end
  S=S+a_k;
  k=k+1;
end
```

Notes

Example

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k \geq 10^{-5}$, where $a_k = k^{-k}$.

```
S=0; k=1;
while( (k<=100) & (k^-k >= 1e-5) )
  S=S+k^-k;
  k=k+1;
end
```

```
>> S
S =
  1.2913
```

```
S=0; k=1;
while( k<=100 )
  a_k=k^-k;
  if (a_k < 1e-5)
    break;
  end
  S=S+a_k;
  k=k+1;
end
```

```
>> S
S =
  1.2913
```

Notes

Same example with 'for' loop and use of matrix ops

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k \geq 10^{-5}$, where $a_k = k^{-k}$.

Notes

Same example with 'for' loop and use of matrix ops

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k \geq 10^{-5}$, where $a_k = k^{-k}$.

```
S=0;
for k=1:100
  a_k=k^-k;
  if (a_k < 1e-5)
    break;
    end
  S=S+a_k;
end
```

Notes

Same example with 'for' loop and use of matrix ops

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k \geq 10^{-5}$, where $a_k = k^{-k}$.

```
S=0;
for k=1:100
  a_k=k^-k;
  if (a_k < 1e-5)
    break;
    end
  S=S+a_k;
end
```

```
>> S
S =
  1.2913
```

Often it is more elegant to use built in Matlab matrix operators

```
>> k=1:100;
>> a_k=k.^-k;
>> S=sum(a_k(a_k>=1e-5))
S =
   1.2913
```

Note

- use of the *choose elements* construct
- built in `sum` function

Notes

Interest rate related example

Suppose bank gave you 50% interest rate (let's call it 'x'), and you put one dollar in.
How much would you get at the end of the year?

- one payment at the end of the year

$$M_1 = 1 * (1 + x) = 1 * (1 + .5) = 1.5$$

Notes

Interest rate related example

Suppose bank gave you 50% interest rate (let's call it 'x'), and you put one dollar in.
How much would you get at the end of the year?

- one payment at the end of the year

$$M_1 = 1 * (1 + x) = 1 * (1 + .5) = 1.5$$

- interest payment every half a year

$$M_2 = 1 * (1 + x/2) * (1 + x/2) = 1 * (1 + .5/2)^2 = 1.5625$$

Notes

Interest rate related example

Suppose bank gave you 50% interest rate (let's call it 'x'), and you put one dollar in.
How much would you get at the end of the year?

- one payment at the end of the year

$$M_1 = 1 * (1 + x) = 1 * (1 + .5) = 1.5$$

- interest payment every half a year

$$M_2 = 1 * (1 + x/2) * (1 + x/2) = 1 * (1 + .5/2)^2 = 1.5625$$

- interest payment every month

$$M_{12} = 1 * (1 + x/12)^{12} = 1.6321$$

Notes

Interest rate related example

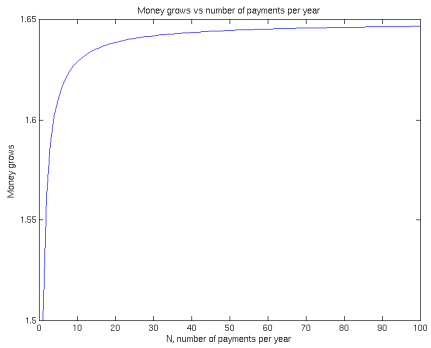
Now let's find how you money growth (M_N) depends on the number of payments per year

```
x=.5;
N_max=100;
N=1:N_max;
M=0*(N); % since N is vector M will be a vector too
for i=N
  M(i)=(1+x/i)^i;
end
plot(N,M,'-');
xlabel('N, number of payments per year');
ylabel('Money grows');
title('Money grows vs number of payments per year');
```

Of course we do not need computer to show that $M_\infty = e^x = 1.6487$ but we need it to calculate something like
$M_{1001} - M_{1000} = 2.0572 \times 10^{-7}$

Notes

Interest rate related example



Notes