Homework 08

General comments:

- Do not forget to run some test cases.

Problem 1 (5 points)

Solve numerically (using built-in `ode45` solver) the following physical problem if the oscillatory motion

$$x''(t) = -x(t)^p$$

with following initial conditions

$$x(0) = 0$$
$$v(0) = x'(0) = v_0$$

Here the x is position of the particle, t is time , v is velocity, v_0 is initial velocity, and p is a parameter which takes odd values.

When $p = 1$ the problem resembles the equation of motions for the well known harmonic oscillator with $k/m = 1$.

Solve this problem (i.e. plot $x(t)$ and $v(t)$) for two values of the parameter $p = 1$ and 5, and the initial velocity $v_0 = 1$. Make sure to choose final time large enough so you see at least 10 periods.

Problem 2 (5 points)

Show that the period of the oscillation is independent of v_0 for the harmonic oscillator and depends on v_0 for the case of $p = 5$. Do it for at least five different values of v_0 to convince yourself.

Problem 3 (5 points)

Have a look at the predator and prey model (the `ode_predator_prey_model.m` file provided with lecture 20 notes).

Find non trivial solution (i.e. $x_0 \neq 0$ and $y_0 \neq 0$) for which population of wolves and rabbits is independent of time (i.e. $dx/dt = dy/dt = 0$). You should get a system of two linear equations which is super simple, however I ask you to solve it using matlab numerical solver methods which we discussed during the lecture 21, i.e. form matrix A and column B, and find x such that $A * x = B$. Note: use constants a, b, c, and d provided in matlab file.

So we see that it possible to have stable populations (or economy with out ups and downs) but you need to be smart about initial conditions.

What is expected shape of the plot of the wolves population vs rabbits with calculated above initial conditions? Plot it.

Problem 4 (5 points)

It is possible to draw a parabola through any 3 point in a plane. Using matlab linear equations solver find coefficients a, b and c for parabola $y = ax^2+bx+c$ which passes through points $p_1 = (-10, 10), p_2 = (-2, 12)$, and $p_3 = (12, 10)$.