Final Exam

The water bottle rocket competition.

Due date for electronic submission is Tuesday December 13th of 2011 at 12pm.

This is the time by which the electronic report submission must be done.

We will meet at 2pm on Tuesday December 13th of 2011 for the final presentation and project defense.

During this time, your team will give a 10 min presentation about key aspects of your algorithm with other relevant information presented (equations, algorithms, plots, etc). Plan ahead and distribute presentation time among teammates so everyone has a chance to talk. Presentation value is 25 points and will be assigned to each speaker separately.

Members of other teams encouraged to ask questions related to the project and will be granted extra points depending on questions relevance.

Figure 1:

Algorithm evaluation:

Your answers will be compared against those of the other teams. Each team's solution will be ranked based on the highest altitude reached by their rocket. If several teams rockets get the same altitude the speed of the algorithm will decide who gets the largest bonus. The best team will get 2.5 points, next best team, 2.0 points, and so on till 0.

The competition will have two rounds: one with optimal parameters calculated based on posted on the web problem file for this final and another where the rocket problem file will be supplied during the final exam according to the specification outlined in the text below.

Your code should provide **parameters** for the bottle rocket (see requirements below). Even though to solve the problem you need your own simulation code, it will not be used in the competition. The simulation of the solution will be done with the instructor's computer and code using the input parameters supplied by your algorithm.

All test will be run during the competition on the desktop computer available in the class room. You are welcome to test your algorithm there before the competition.

- Discuss relevant equations, describe your solution, show results.
- All Matlab code/scripts must be present in the carbon copy as well.
- Make all you calculations in the S.I. units (m, kg, s).

You goal is to build the best water bottle rocket - the one which flies the highest. 125 potential points total (95 for writeup + 5 maximum possible for competition + 25 for the presentation.

You are participating in a water bottle rocket competition. You goal is to make the best possible rocket but first you need to understand the physics of it.

A simple water bottle rocket is typically made of a soda bottle (see figure 1), which we treat as a cylinder with the height H_B and the top/bottom area A_B (see figure 2). The empty bottle has mass M_B. The bottle is pumped with compressed gas to initial inner pressure P_{i_0} higher than the atmospheric pressure P_A and filled with a liquid (typically water) up to the height h_{L_0}. The bottom has an opening for a nozzle with the area A_N which allows liquid with the density ρ_L to escape from the bottle.

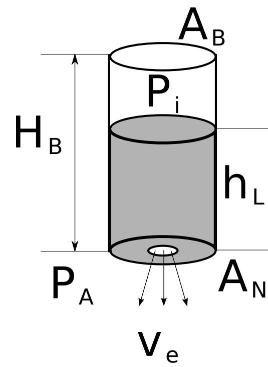

Figure 2: The water bottle rocket sketch

For simplicity we assume, that **the rocket is flying** right away once the nozzle is open. The acceleration due to gravity is g. And the mass of compressed gas can be neglected.

Please, **do not make any assumptions about above and below parameters**, since we can easily imagine that this competition runs at the different atmospheric pressure, bottle shape, water can be replaced with a different liquid, and even the planet at which competitions runs could be other than the Earth in the future. We want to have a solver as general as possible. The list of initial parameters/possibilities will be supplied in the problem file.

In order to solve this problem, we need to understand basic physics of the problem. There are three forces acting on the rocket: gravity $\vec{F_G}$, drag due to atmosphere $\vec{F_d}$, and trust of the rocket governed by escape velocity of the water from the nozzle $\vec{F_T}$. Then ODEs describing the motion of the rocket and its position $\vec{r}$

$$\vec{r}' = \vec{v}_R, \tag{1}$$

$$\vec{r}'' = \vec{v}_R' = \vec{a}_R = \frac{1}{M_R}\left(\vec{F_G} + \vec{F_d} + \vec{F_T}\right), \tag{2}$$

$$\vec{F_G} = M_R\vec{g} = -M_R g\vec{e}_y, \tag{3}$$

$$\vec{F_d} = -\frac{1}{2}\rho_A C_{d_B} A_B v_R \vec{v}_R, \tag{4}$$

$$\vec{F_T} = \rho_L v_e^2 A_N \tag{5}$$

where ρ_A is the density of the atmosphere, C_{d_B} is the bottle drag coefficient, $M_R = M_B + M_L$ is the mass of the rocket, M_L is the mass of the remaining liquid in the bottle ($M_L = \rho_L A_B h_L$), and v_e is the exhaust velocity of the liquid governed by the following formula (easily derived from the Bernoulli's equation)

$$v_e^2 = \frac{2}{\rho_L}\left(P_i - P_A\right). \tag{6}$$

Above formula correct only when **there is some liquid left** and the nozzle area is much smaller then area of the bottle. So make sure that your nozzle area $A_N \leq \alpha A_B$, where alpha is some coefficient. Also implement the check for the liquid level.

Since water level changes in the bottle, so must the inner gas pressure. Generally the flight of the bottle is short, so we assume the adiabatic inner gas expansion process i.e. no extra energy taken or delivered to the rocket. In this case there is a simple link between the inner pressure (P_i) and gas volume $V_g = A_b(H_B - h_L)$

$$P_{i_1} V_{g_1}^{\gamma} = P_{i_2} V_{g_2}^{\gamma} = \text{constant} \tag{7}$$

where γ is adiabatic index which depends on the number of degrees of freedom for the given gas. Monoatomic ideal gases have $\gamma = 5/3$, diatomic ideal gases or compositions have $\gamma = 7/5$. Air for example has γ very close to $7/5$ since main components of air are nitrogen (N_2) and oxygen (O_2) molecules. Choice of γ will be provided in the problem file.

Finally the last needed dependence is the change of the liquid level with exhaust liquid velocity

$$\frac{dh_L}{dt} = -v_e \frac{A_N}{A_b} \tag{8}$$

Problem 1 (10 points)

Write a function which for a chosen parameters of the rocket returns column vectors of times from 0 till some final time (`final_time`) and corresponding y-position of the rocket. This function may return other variables, which you might find useful in following problems. Assume that the rocket is lunched from the ground level.

Problem 2 (10 points)

Using the above function create another one which finds the time when the given rocket hits the ground.

Problem 3 (40 points)

Assuming one dimensional problem i.e. the rocket is launched directly up. Create a function **strictly following the following specification** which for a given problem file (`ParametersDataFileName`) finds the best possible parameters for the bottle rocket. Below is the function template which is also available online:

```
function [ AN, rhoL, gammaG, hLiquid0,   ...
           time, yRocket,   ...
           hLiquid, FTrust, FDrag ] ...
         = best_rocket( ParametersDataFileName )
```

Where

An the bottle rocket nozzle

gammaG the adiabatic index of the chosen inner gas

rhoL the chosen liquid density

hLiquid0 the initial liquid level in the rocket

are the best/optimal parameters found/chosen by the function and the variables describing relevant parameters of the trajectory are

time the time column vector with time from lunch time till the Earth **hit** time

yRocket the column vector of y position of the rocket at the time points governed by `time` vector

hLiquid the column vector of the liquid level in the rocket at the same time points points

FTrust the column vector of the trust force magnitude at the same time points points

FDrag the column vector of the drag force magnitude at the same time points points

Parameters data file will have the following variables set:

MB the empty bottle mass

AB the area of the bottle

HB the hight of the bottle

PA the pressure of the atmosphere

PIgas0 the initial inner gas pressure

rhoL_vec the row vector of the available liquids densities

gamma_vec the row vector of the available adiabatic indexes for the inner gases

g magnitude of the acceleration due to gravity

rhoA the density of the atmosphere

Cdr the rocket drag coefficient

alpha maximum ratio of A_N/A_B (α)

This function will be used during the in class competition so make sure that everything is according to the specifications. If you fail to meet the specifications you will be automatically excluded from the competition.

Problem 4 (20 points)

Find the best rocket parameters for a provided at the web problem data file. Save (using command `save`) only found optimal parameters into the file "solution1.mat", save only `time, yRocket, hLiquid, FTrust, FDrag` into the separate file "solution1trajectory.mat" Attach both files with the electronic submission. Plot for this optimal rocket dependencies of y-position vs time, the liquid level vs time, and the rocket trust vs time. Make sure to plot only till the time when the rocket hits the ground.

Problem 5 (15 points)

This is a problem to solve at home. The code solution code needs to be supplied in the report.

To launch the rocket you need the access codes for the lunch mechanism. This code is delivered to you as a radio voice message. However the other teams know about it and try to jam the transmission. So voice message recording will have a lot of noise. Each team will get its own code transmission voice record, available on the web site. What is the your activation code (it should be a 6 digit number)?