Digital filters

Eugeniy E. Mikhailov

The College of William & Mary

Lecture 27

Notes

DFT filters (repeat)

Once you get a signal you can filter unwanted components out of it. The recipe is the following

- sample the signal
- calculate FT (fft)
- have a look at the spectrum and decide which components are unwanted
- apply filter which attenuate unwanted frequency component (remember that if you attenuate the component of the frequency f by g_f you need to attenuate the component at $-f$ by g_f^*.
- calculate inverse FT (ifft) of the filtered spectrum
- repeat if needed

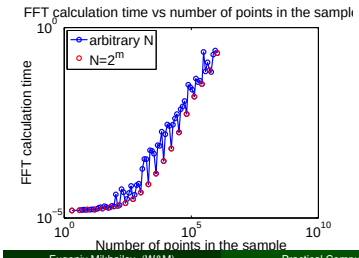
$$y_{filtered}(t) = \mathcal{F}^{-1}\left[\mathcal{F}(y(t))G(f)\right] = \mathcal{F}^{-1}\left[Y(f)G(f)\right]$$

Notes

Speed of FFT

- The main work horse of the DFT filters is FFT algorithm
- it is handy to know its performance behavior

$$y_k = \frac{1}{N}\sum_{n=0}^{N-1} c_n \exp(i\frac{2\pi(k-1)n}{N}) \quad \text{inverse Fourier transform}$$



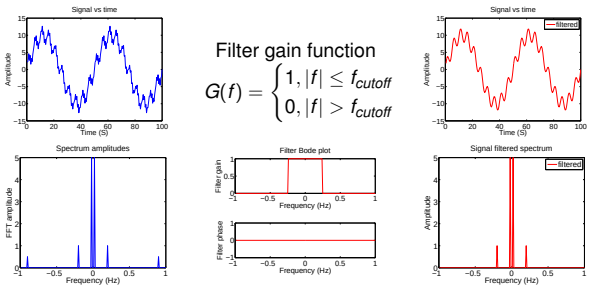
- General DFT scales $\sim N^2$ (N coefficient each involving sum of N)
- FFT scales $\sim N \log_2 N$ which is great speed up

The fastest calculation time for

$$N = 2^m$$

Notes

Brick wall low-pass filter



Filter gain function

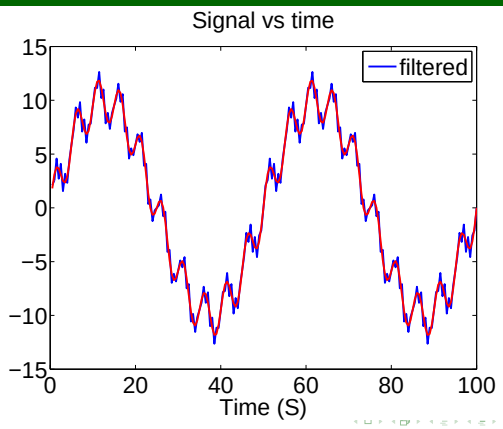
$$G(f) = \begin{cases} 1, |f| \le f_{cutoff} \\ 0, |f| > f_{cutoff} \end{cases}$$

$$y_{filtered}(t) = \mathcal{F}^{-1}\left[\mathcal{F}(y(t))G(f)\right] = \mathcal{F}^{-1}\left[Y(f)G(f)\right]$$

```
freq=fourier_frequencies(SampleRate, N);
G=ones(N,1); G( abs(freq) > Fcutoff, 1)= 0;
y_filtered  =  ifft(  fft( y ) * G  )
```

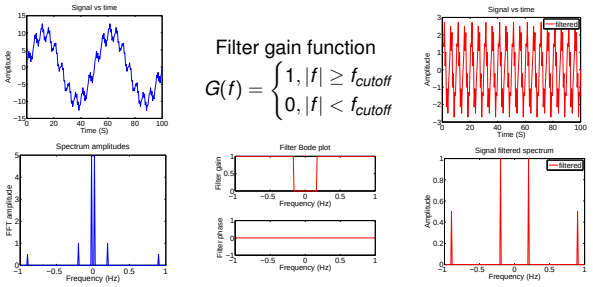
Notes

Brick wall low-pass filter (continued)



Notes

Brick wall high-pass filter



Filter gain function

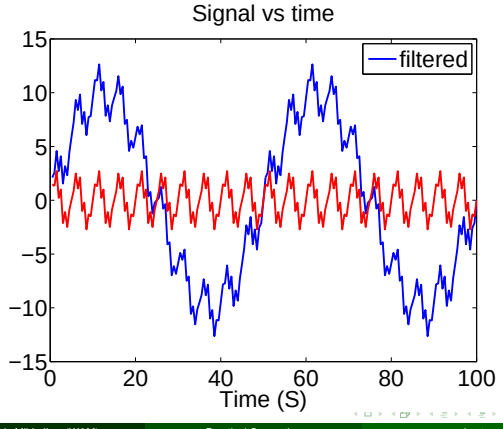
$$G(f) = \begin{cases} 1, |f| \geq f_{cutoff} \\ 0, |f| < f_{cutoff} \end{cases}$$

$$y_{filtered}(t) = \mathcal{F}^{-1}\left[\mathcal{F}(y(t))G(f)\right] = \mathcal{F}^{-1}\left[Y(f)G(f)\right]$$

```
freq=fourier_frequencies(SampleRate, N);
G=ones(N,1); G( abs(freq) < Fcutoff, 1)= 0;
y_filtered  =  ifft(  fft( y ) * G  )
```

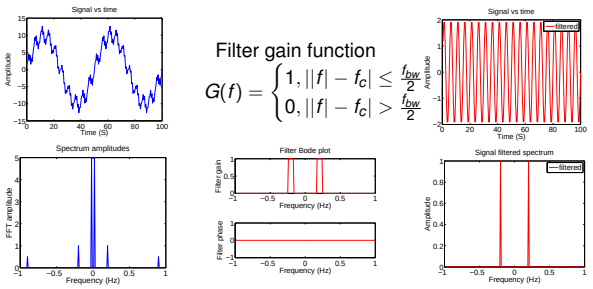

Notes

Brick wall high-pass filter (continued)



Notes

Brick wall band-pass filter



Filter gain function

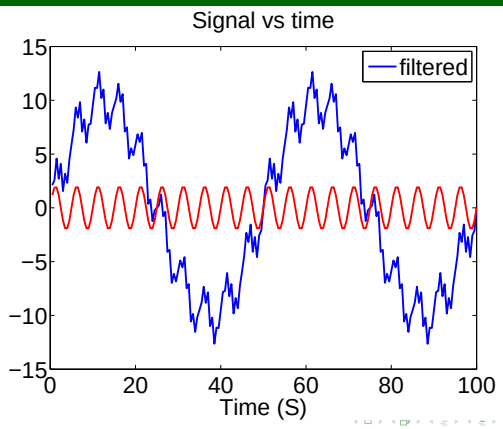
$$G(f) = \begin{cases} 1, ||f| - f_c| \leq \frac{f_{bw}}{2} \\ 0, ||f| - f_c| > \frac{f_{bw}}{2} \end{cases}$$

$$y_{filtered}(t) = \mathcal{F}^{-1}\left[\mathcal{F}(y(t))G(f)\right] = \mathcal{F}^{-1}\left[Y(f)G(f)\right]$$

```
freq=fourier_frequencies(SampleRate, N);
G=ones(N,1); G( abs(abs(freq)-Fcenter) > BW/2, 1)=0;
y_filtered  =  ifft(  fft( y ) * G  )
```

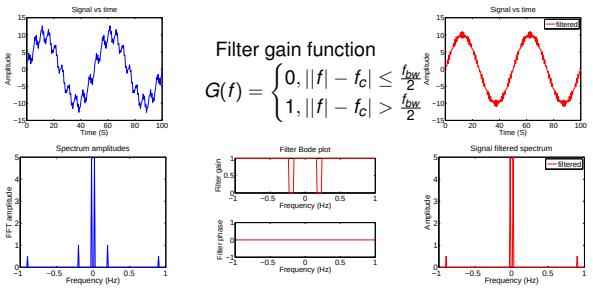

Notes

Brick wall band-pass filter (continued)



Notes

Brick wall band-stop filter

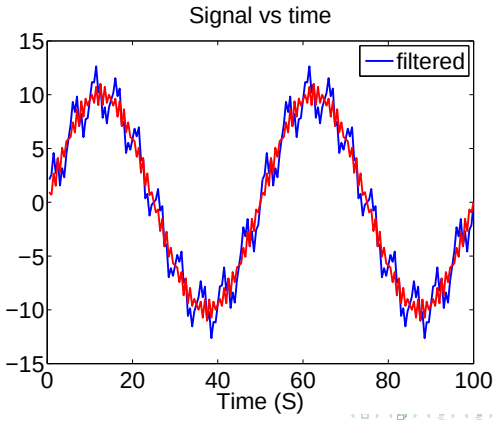


$$y_{filtered}(t) = \mathcal{F}^{-1}\left[\mathcal{F}(y(t))G(f)\right] = \mathcal{F}^{-1}\left[Y(f)G(f)\right]$$

```
freq=fourier_frequencies(SampleRate, N);
G=zeros(N,1); G( abs(abs(freq)-Fcenter) > BW/2, 1)=1;
y_filtered  =  ifft(  fft( y ) * G  )
```


Notes

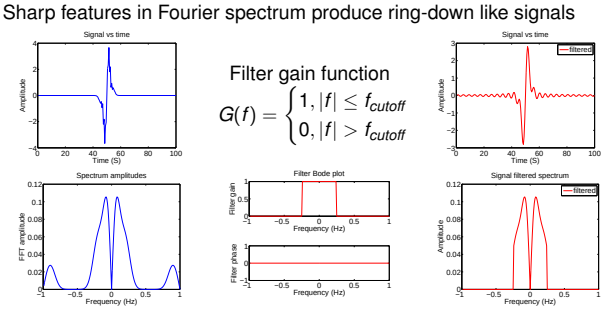
Brick wall band-cut filter (continued)



Notes

Brick wall filters artifacts

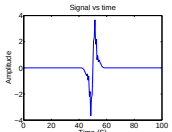
Sharp features in Fourier spectrum produce ring-down like signals



Notes

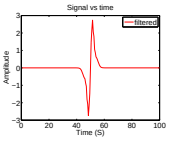
Low pass smoothed

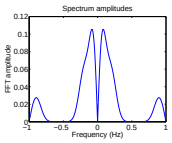
Sharp features in Fourier spectrum produce ring-down like signals

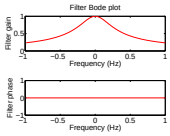


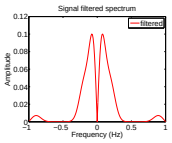
Filter gain function

$$G(f) = |\frac{1}{1 + if/f_{cutoff}}|$$



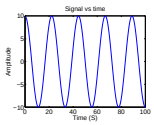






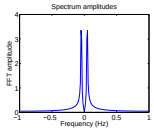
Windowing artefact

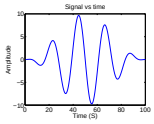
Similarly sharp features in time lead to broadening of the spectrum



Implicitly assumed Rectangular window

$$w_n = 1$$

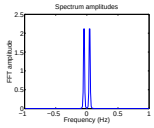




$$y_{windowed_n} = y_n w_n$$

Hann window

$$w_n = \frac{1}{2}(1 - \cos(2\pi \frac{n-1}{N-1}))$$



- Note: spectral resolution $\sim 1/T_{window}$.

Search for other windowing functions: Hamming, Tukey, Cosine, Lanczos, Triangulars, Gaussians, Bartlett-Hann, Blackmans, Kaisers.

Notes

Notes

Notes

Notes