System of linear algebraic equations

Eugeniy E. Mikhailov

The College of William & Mary

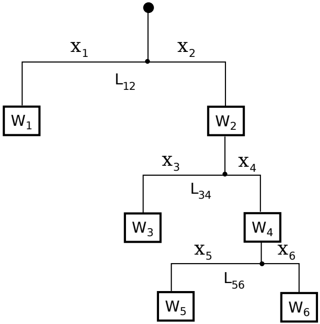
Lecture 22

Mobile problem

Suppose someone provided us with 6 weights and 3 rods. We need to calculate the positions of suspension points.

If system in equilibrium torque must be zero at any pivot point

$$
\begin{aligned}
w_1 x_1 - (w_2 + w_3 + w_4 + w_5 + w_6) x_2 &= 0 \\
w_3 x_3 - (w_4 + w_5 + w_6) x_4 &= 0 \\
w_5 x_5 - w_6 x_6 &= 0
\end{aligned}
$$



We need 3 more equation. Let's fix length of the rods

$$
\begin{aligned}
x_1 + x_2 &= L_{12} \\
x_3 + x_4 &= L_{34} \\
x_5 + x_6 &= L_{56}
\end{aligned}
$$

Mobile problem continued

Let's define $w_{26} = w_2 + w_3 + w_4 + w_5 + w_6$ and $w_{46} = w_4 + w_5 + w_6$

$$
\begin{aligned}
w_1 x_1 - w_{26} x_2 &= 0 \\
w_3 x_3 - w_{46} x_4 &= 0 \\
w_5 x_5 - w_6 x_6 &= 0 \\
x_1 + x_2 &= L_{12} \\
x_3 + x_4 &= L_{34} \\
x_5 + x_6 &= L_{56}
\end{aligned}
$$

$$\sum_j A_{ij} x_j = B_i \rightarrow \mathbf{Ax} = \mathbf{B}$$

Matlab has a lot of built in functions to solve problem of this form

$$
\begin{pmatrix}
w_1 & -w_{26} & 0 & 0 & 0 & 0 \\
0 & 0 & w_3 & -w_{46} & 0 & 0 \\
0 & 0 & 0 & 0 & w_5 & -w_6 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1
\end{pmatrix}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix}
=
\begin{pmatrix} 0 \\ 0 \\ 0 \\ L_{12} \\ L_{34} \\ L_{56} \end{pmatrix}
$$

Inverse matrix method

$$\mathbf{Ax} = \mathbf{B}$$

$$\mathbf{A}^{-1}\mathbf{Ax} = \mathbf{A}^{-1}\mathbf{B}, \quad \texttt{det}(\mathbf{A}) \neq 0$$

Analytical solution

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{B}$$

Matlab first way (not the fastest)

$$\mathbf{x} = \texttt{inv}(\mathbf{A}) * \mathbf{B}$$

Matlab second way (recommended)

$$\mathbf{x} = \mathbf{A}\backslash\mathbf{B}$$

Recall the mobile problem

If $w_1 = 20$, $w_2 = 5$, $w_3 = 3$, $w_4 = 7$, $w_5 = 2$, $w_6 = 3$, $L_{12} = 2$, $L_{34} = 1$, and $L_{56} = 3$, then $w_{26} = 20$ and $w_{46} = 12$.

$$\begin{pmatrix} 20 & -20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -3 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 1 \\ 3 \end{pmatrix}$$

Matlab mobile solution

```
A=[ ...
20, -20,  0,   0,  0,   0; ...
 0,   0,  3, -12,  0,   0; ...
 0,   0,  0,   0,  2,  -3; ...
 1,   1,  0,   0,  0,   0; ...
 0,   0,  1,   1,  0,   0; ...
 0,   0,  0,   0,  1,   1; ...
 ]
B= [ 0; 0; 0; 2; 1; 3 ]
% 1st method
x=inv(A)*B
% 2nd method
x=A\B
```

```
x =
   1.0000
   1.0000
   0.8000
   0.2000
   1.8000
   1.2000
```

Check

```
>> A*x-B
   1.0e-15 *
     0
     0
     0
     0
     0.2220
     0
```

When do and when not to do inverse matrix

Solutions based on Inverse matrix calculations involve extra (unnecessary for solution) steps and thus are slower

```
>> A=rand(4000);
>> B=rand(4000,1);
>> tic;  x=inv(A)*B; toc
Elapsed time is 54.831124 seconds.
>> tic;  x=A\B; toc
Elapsed time is 19.822778 seconds.
```

However it is handy to calculate inverse matrix in advance if you solve $\mathbf{Ax} = \mathbf{B}$ for different **B** with the same **A**.

```
>> tic; Ainv=inv(A);  toc
Elapsed time is 58.304244 seconds.
>> B1=rand(4000,1); tic; x1=Ainv*B1; toc
Elapsed time is 0.048547 seconds.
>> B2=rand(4000,1); tic; x2=Ainv*B2; toc
Elapsed time is 0.048315 seconds.
```