Multi-D optimization problem

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Lecture 15
Find $\vec{x}$ that minimize $E(\vec{x})$ subject to $g(\vec{x}) = 0$, $h(\vec{x}) \leq 0$

$\vec{x}$ design variables
$E(\vec{x})$ merit or objective or fitness or energy function
$g(\vec{x})$ and $h(\vec{x})$ constrains

Easy to see that maximization problem is the same as minimization once $E(\vec{x}) \rightarrow -E(\vec{x})$. 
Solution with Matlab built in Multi-D minimization - fminsearch

\[ [x, fval] = \text{fminsearch}(\text{fun}, x0) \]

- **fun**: handle to the multi-variable function
- **x0**: initial 'guess' (vector)
- **x**: optimum position vector
- **fval**: value of the function at the optimum
Example

```matlab
function ret=fsample_sinc(v)
x=v(1); y=v(2);
r=sqrt(x^2+y^2);
ret= -sin(r)/r;
end
```

```matlab
x0vec=[0.5, 0.5];
[xResVec,zopt]=fminsearch(@fsample_sinc, x0vec)
xResVec = [0.2852e-4, 0.1043e-4]
zopt = -1.0000
```
It is easy to miss global minimum

Example

```matlab
function ret=fsample_sinc(v)
    x=v(1); y=v(2);
    r=sqrt(x^2+y^2);
    ret= -sin(r)/r;
end
```

Example

```matlab
x0vec=[5, 5];
[xResVec,zopt]=fminsearch(@fsample_sinc, x0vec)
xResVec = [ 5.6560 5.2621 ]
zopt = -0.1284
```
Sample problem 1

Problem 1.
Find the minimum of the function
\[ F(x, y, z) = 2x^2 + 2y^2 + z^2 + 2xy + 1 - 2y + 2xy \]
Answer: \([x,y,z]=[-1,1,1]\)
Problem 2.
Consider masses $m_1$ and $m_2$ suspended by strings with length $L_1$, $L_2$, and $L_3$.
Find the angles $\theta_1$, $\theta_2$, and $\theta_3$.

We need to minimize potential energy subject to the length constrains. See merit function in the file 'EconstrainedSuspendedWeights.m'

For the following initial conditions

\[
m_1 = 2; \quad m_2 = 2; \\
L_1 = 3; \quad L_2 = 2; \quad L_3 = 3; \\
L_{\text{tot}} = 4; \quad H_{\text{tot}} = 0;
\]

The answer should be close to $\theta_1 = -1.231; \quad \theta_2 = 0; \quad \theta_3 = 1.231$;