Optimization problem

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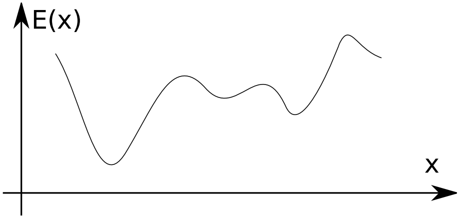
Lecture 14

Introduction to optimization



Find $\vec{x}$ that minimize $E(\vec{x})$ subject to $g(\vec{x}) = 0, h(\vec{x}) \leq 0$

$\vec{x}$ design variables

$E(\vec{x})$ merit or objective or fitness or energy function

$g(\vec{x})$ and $h(\vec{x})$ constrains

There is no guaranteed way (algorithm) which can find global minimum (optimal) point.

Easy to see that maximization problem is the same as minimization once $E(\vec{x}) \rightarrow -E(\vec{x})$.

Analytical solution of 1D

If we have 1D case and $E(x)$ has analytical derivative, optimization problem can be restated as

Find $f(x) = 0$
where $f(x) = dE/dx$

since at maximum or minimum derivative must be zero.
Since we already know how to find the solution of $f(x) = 0$ the rest is easy.

Example: wire insulation problem

Consider a metal wire with the radius '*a*', which has the heat generation '*q*', it is covered with rubber insulator of the radius 'r'. Our goal is to find such '*r*' that the wire has the smallest temperature. Problem taken from "Numerical method in engineering with matlab" by Jaan Kiusalaas.

Temperature of the wire is given by

$$T = \frac{q}{2\pi}\left(\frac{\ln(r/a)}{k} + \frac{1}{hr}\right) + T_{ambient}$$

where

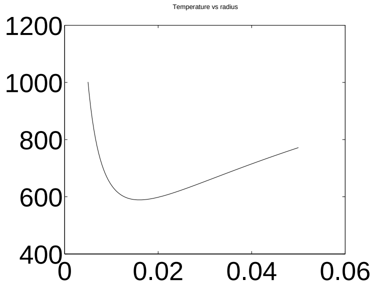
- a is the wire radius in m,
- q is the heat generation in wire in W/m,
- k is the thermal conductivity of the isolation in W/(m*K),
- h is the coefficient of convective heat-transfer in W/(m^2*K).

```
function T=wire_t(r)
  a=.02;   % wire radius in m
  q=40;    % heat generation in wire in W/m
  k=0.016; % thermal conductivity of isolation in W/(m*K)
  h=1;  % coefficient of convective heat-transfer in W/(m^2*K)
  T_ambient=280; % ambient temperature for far away points (at infinity)

  T=q/(2*pi)*(log(r/a)/k + 1/(h*r)) + T_ambient;
end
```

First, we plot the function in order to bracket.

```
r=linspace(0.001,.05,1000);
T=arrayfun(@wire_t, r);
plot(r,T)
```



Next, we find optimal solution

```
fminbnd(@wire_t, 0.001,.05)
ans =
  0.0160
```

Golden section search algorithm

If you have bracketed your solution somehow i.e. found x_1, x_2, and x_3 such that $x_1 < x_2 < x_3$ and $E(x_2) < min(E(x_1), E(x_3))$.
Then $x_1 \to a$, $x_3 \to b$, $h = (b - a)$

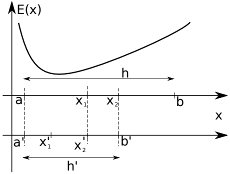
1. assign new $x_1 = a + R * h$ and $x_2 = b - R * h$
2. $E_1 = E(x_1)$, $E_2 = E(x_2)$, $E_a = E(a)$, $E_b = E(b)$
3. if $h < \varepsilon_x$ stop otherwise do steps below
4. note that for small enough h: $E(x_1) < E(a)$ and $E(x2) < E(b)$
5. find new bracket
   - if $E(x_1) < E(x_2)$ then $b = x_2$, $E_b = E_2$ else $a = x_1$, $E_a = E_1$
6. $h = (b - a)$ and reuse one of the old points x_1 or x_2 with the proper R
   - if $E(x_1) < E(x_2)$
     then $x_2 = x_1$, $E_2 = E_1$, $x_1 = a + R * h$, $E_1 = E(x_1)$
     else $x_1 = x_2$, $E_1 = E_2$, $x_2 = b - R * h$, $E_2 = E(x_2)$
7. go to step 3

R given by the golden section $R = \frac{3-\sqrt{5}}{2} \approx 0.38197$

Derivation of the R value

at first step we have

$$x_1 = a + R * h$$
$$x_2 = b - R * h$$



Suppose that $E(x_1) < E(x_2)$ then $a' = a$ and $b' = x_2$ then for the next bracket we evaluate x_1' and x_2'

$$x_1' = a' + R * h' = a' + R * (b' - a')$$
$$x_2' = b' - R * h' = b' - R * (b' - a')$$
$$= x_2 - R * (x_2 - a) = b - R * h - R * (b - R * h - a)$$

we would like to reuse on of the previous evaluations of E so we require that $x_1 = x_2'$. This leads to equation

$$R^2 - 3R + 1 = 0 \text{ with } R = \frac{3 \pm \sqrt{5}}{2}$$

We need to choose minus sign since fraction $R < 1$