

Optimization problem

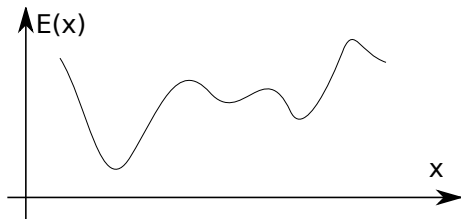
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Lecture 14

Introduction to optimization



Find \vec{x} that minimize $E(\vec{x})$ subject to $g(\vec{x}) = 0, h(\vec{x}) \leq 0$

\vec{x} design variables

$E(\vec{x})$ merit or objective or fitness or energy function

$g(\vec{x})$ and $h(\vec{x})$ constrains

There is no guaranteed way (algorithm) which can find global minimum (optimal) point.

Easy to see that maximization problem is the same as minimization once $E(\vec{x}) \rightarrow -E(\vec{x})$.

Analytical solution of 1D

If we have 1D case and $E(x)$ has analytical derivative, optimization problem can be restated as

Find $f(x) = 0$
where $f(x) = dE/dx$

since at maximum or minimum derivative must be zero.

Since we already know how to find the solution of $f(x) = 0$ the rest is easy.

Example: wire insulation problem

Consider a metal wire with the radius ' a ', which has the heat generation ' q ', it is covered with rubber insulator of the radius ' r '. Our goal is to find such ' r ' that the wire has the smallest temperature. Problem taken from "Numerical method in engineering with matlab" by Jaan Kiusalaas.

Temperature of the wire is given by

$$T = \frac{q}{2\pi} \left(\frac{\ln(r/a)}{k} + \frac{1}{hr} \right) + T_{ambient}$$

where

a is the wire radius in m,

q is the heat generation in wire in W/m,

k is the thermal conductivity of the isolation in W/(m*K),

h is the coefficient of convective heat-transfer in W/(m²*K).

Solution with Matlab built in 1D minimization - fminbnd

```
function T=wire_t(r)
    a=.02;    % wire radius in m
    q=40;    % heat generation in wire in W/m
    k=0.016; % thermal conductivity of isolation in W/(m*K)
    h=1;    % coefficient of convective heat-transfer in W/(m^2*K)
    T_ambient=280; % ambient temperature for far away points (at infinity)

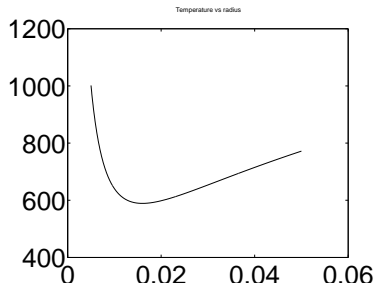
    T=q/(2*pi)*(log(r/a)/k + 1/(h*r)) + T_ambient;
end
```

First, we plot the function in order to bracket.

```
r=linspace(0.001, .05, 1000);
T=arrayfun(@wire_t, r);
plot(r, T)
```

Next, we find optimal solution

```
fminbnd(@wire_t, 0.001, .05)
ans =
    0.0160
```



Golden section search algorithm

If you have bracketed your solution somehow i.e. found x_1, x_2 , and x_3 such that $x_1 < x_2 < x_3$ and $E(x_2) < \min(E(x_1), E(x_3))$.

Then $x_1 \rightarrow a, x_3 \rightarrow b, h = (b - a)$

- 1 assign new $x_1 = a + R * h$ and $x_2 = b - R * h$
- 2 $E_1 = E(x_1), E_2 = E(x_2), E_a = E(a), E_b = E(b)$
- 3 if $h < \varepsilon_x$ stop otherwise do steps below
- 4 note that for small enough $h: E(x_1) < E(a)$ and $E(x_2) < E(b)$
- 5 find new bracket
 - if $E(x_1) < E(x_2)$ then $b = x_2, E_b = E_2$ else $a = x_1, E_a = E_1$
- 6 $h = (b - a)$ and reuse one of the old points x_1 or x_2 with the proper R
 - if $E(x_1) < E(x_2)$
then $x_2 = x_1, E_2 = E_1, x_1 = a + R * h, E_1 = E(x_1)$
else $x_1 = x_2, E_1 = E_2, x_2 = b - R * h, E_2 = E(x_2)$
- 7 go to step 3

R given by the golden section $R = \frac{3-\sqrt{5}}{2} \approx 0.38197$

Derivation of the R value

at first step we have

$$x_1 = a + R * h$$

$$x_2 = b - R * h$$

Suppose that $E(x_1) < E(x_2)$ then $a' = a$ and $b' = x_2$
then for the next bracket we evaluate x'_1 and x'_2

$$x'_1 = a' + R * h' = a' + R * (b' - a')$$

$$x'_2 = b' - R * h' = b' - R * (b' - a')$$

$$= x_2 - R * (x_2 - a) = b - R * h - R * (b - R * h - a)$$

we would like to reuse on of the previous evaluations of E so we require that $x_1 = x'_2$. This leads to equation

$$R^2 - 3R + 1 = 0 \text{ with } R = \frac{3 \pm \sqrt{5}}{2}$$

We need to choose **minus** sign since fraction $R < 1$

