Boolean algebra, conditional statements, loops.

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Lecture 04

Boolean algebra

Variable of valuable type can have only two values

- true
- false

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- true (Matlab use 1 to indicate it, actually everything but zero)
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- $\neg$ - logic **not**, Matlab `~`

$$\neg true = false$$
$$\neg false = true$$

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- $\neg$ - logic **not**, Matlab `~`

$$\neg\text{true} = \text{false}$$
$$\neg\text{false} = \text{true}$$

- $\wedge$ - logic **and**, Matlab `&`

$$A \wedge B = \begin{cases} \text{true, if A=true and B=true,} \\ \text{false, otherwise} \end{cases}$$

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- false (Matlab uses `0`)

There are three logical operators which are used in boolean algebra

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- $\vee$ - logic **or**, Matlab `|`

$$A \vee B = \begin{cases} false, & \text{if A=false and B=false,} \\ true, & \text{otherwise} \end{cases}$$

If $A = $ *false*, $B = $ *true*, $C = $ *true*

$$A|{\sim}B\&C$$

Boolean operators precedence in Matlab

If $A = false$, $B = true$, $C = true$

$$A | {\sim} B \& C$$

$\sim$ has highest precedence, then $\&$, and then $|$

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$$\sim Z \& Z =$$

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If $A = $ *false*, $B = $ *true*, $C = $ *true*

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$$A|\,(({\sim}B)\&C)$$

Thus

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$$\sim\!Z\&Z = \textit{false}$$

Boolean logic examples

There is an island, which is populated by two kind of people: liars and truthlovers.

- Liars always lie and never speak a word of truth.
- Truthlovers always speak only truth.

Suppose, you are landed on this island and met a person. What will be the answer to your question "Who are you?"

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Now you see a person who answers to your question. "I am a liar."
Is it possible?

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Now you see a person who answers to your question. "I am a liar." Is it possible?

- This makes a paradox and should not ever happen on this island.

- `123.3 & 12=`

Matlab boolean logic examples

- `123.3 & 12=` **1**
- `~ 1232e-6 =`

Matlab boolean logic examples

- `123.3 & 12=` 1
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Matlab boolean logic examples

- 123.3 & 12= 1
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>> B=[1.22312, 0; 34.343, 12]
B =
1.2231     0
34.3430   12.0000
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`~B`

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`B|~B`

"To be or not to be"

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ans =
1       1
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Matlab boolean logic examples

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>> B=[1.22312, 0; 34.343, 12]
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>> A=[56, 655; 0, 24.4]
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Comparison operators

Math	Matlab
$=$	$==$ double equal sign!
$\neq$	$\sim=$
$<$	$<$
$\leq$	$<=$
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```
x=[1,2,3,4,5]
x  =
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x >= 3
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% chose such 'x' where x>=3
x(x >= 3)
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```
ans =
     3     4     5
```

Comparison with matrices

```
>> A=[1,2;3,4]
A =
1       2
3       4
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```
>> B=[33,11;53,42]
B =
22      11
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```
ans =
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if-else-end statement

if *expression*
this part is executed
only if *expression* is
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else
this part is executed
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if *hungry*
buy some food
else
keep working
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if *hungry*
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```
if (x>=0)
  y=sqrt(x);
else
  error('cannot do');
end
```

Common mistake in the 'if' statement

```
if (x=y)
  D=4;
  Z=45;
  C=12;
else
  D=2;
end
```

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the value of 'D' is always 4, except the case when y=0

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the value of 'D' is always 4, except the case when y=0
someone used assignment operator (=) instead of comparison (==)

Short form of 'if-end' statement

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```
if (deviation<=0)
  exit;
end
```

The 'while' statement

`while` *expression*
this part is executed
while *expression* is
true
`end`

```
while expression
```
this part is executed
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```

```
while hungry
```
keep eating
```
end
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The 'while' statement

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this part is executed
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```
i=1;
while (i<=10)
  c=a+b;
  z=c*4+5;
  i=i+2;
end
```

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`while` loop is extremely useful but they are not guaranteed to finish. For a bit more complicated conditional statement and loop it is impossible to predict if the loop will finish.

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Yet another common mistake is

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Yet another common mistake is

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not updating the term leading to fulfillment of the while   condition

The 'for' statement

`for` variable = *expression*
do something
`end`
In this case variable is assigned
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sum=0;
x=[1,3,5,6]
for v=x
sum=sum+v;
end
```

```
>> sum
sum =
   15
```

for   variable = *expression*
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In this case variable is assigned concequently with columns of the *expression*, and then statements inside of the loop are executed

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>> sum
sum =
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```

for   loops are guaranteed to complete after predictable number of iterations (the amount of columns in *expression*).

Example

$$S = \sum_{i=1}^{100} i = 1 + 2 + 3 + 4 + \cdots + 99 + 100$$

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S=0; i=1;
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```

Example

$$S = \sum_{i=1}^{100} i = 1 + 2 + 3 + 4 + \cdots + 99 + 100$$

```
S=0; i=1;
while(i<=100)
  S=S+i;
  i=i+1;
end
```

```
S=0;
for i=1:100
  S=S+i;
end
```

Example

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k < 10^{-5}$, where $a_k = k^{-k}$.

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S=0; k=1;
while( (k<=100) & (k^-k >= 1e-5) )
  S=S+k^-k;
  k=k+1;
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```
S=0; k=1;
while( k<=100 )
  a_k=k^-k;
  if (a_k < 1e-5)
    break;
  end
  S=S+a_k;
  k=k+1;
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Same example with 'for' loop

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Interest rate related example

Suppose bank gave you 50% interest rate (let's call it 'x'), and you put one dollar in.
How much would you get at the end of the year?

- one payment at the end of the year

$$M_1 = 1 * (1 + x) = 1 * (1 + .5) = 1.5$$

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- interest payment every half a year

$$M_2 = 1 * (1 + x/2) * (1 + x/2) = 1 * (1 + .5/2)^2 = 1.5625$$

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- interest payment every half a year

$$M_2 = 1 * (1 + x/2) * (1 + x/2) = 1 * (1 + .5/2)^2 = 1.5625$$

- interest payment every month

$$M_{12} = 1 * (1 + x/12)^{12} = 1.6321$$

Interest rate related example

Now let's find how you money growth (M_N) depends on the number of payments per year

```
N_max=100;
N=1:N_max;
M=0*(N);
x=.5;
for i=N
  M(i)=(1+x/i)^i;
end
plot(N,M,'-');
xlabel('N, number of payments per year');
ylabel('Money grows');
title('Money grows vs number of payments per year');
```

Interest rate related example