Boolean algebra, conditional statements, loops.

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Lecture 04

Boolean algebra

Variable of valuable type can have only two values

- true (Matlab use `1` to indicate it, actually everything but zero)
- false (Matlab uses `0`)

There are three logical operators which are used in boolean algebra

- $\neg$ - logic **not**, Matlab `~`

$$\neg\text{true} = \text{false}$$
$$\neg\text{false} = \text{true}$$

- $\wedge$ - logic **and**, Matlab `&`

$$A \wedge B = \begin{cases} \text{true, if A=true and B=true,} \\ \text{false, otherwise} \end{cases}$$

- $\vee$ - logic **or**, Matlab `|`

$$A \vee B = \begin{cases} \text{false, if A=false and B=false,} \\ \text{true, otherwise} \end{cases}$$

Boolean operators precedence in Matlab

If $A = \textit{false}$, $B = \textit{true}$, $C = \textit{true}$

$$A | \sim B \& C$$

$\sim$ has highest precedence, then $\&$, and then $|$

$$A | ((\sim B) \& C)$$

Thus

$$A | \sim B \& C = \textit{false}$$

“Cat is an animal and cat is not an animal”
is false statement

$$\sim Z \& Z = \textit{false}$$

Boolean logic examples

There is an island, which is populated by two kind of people: liars and truthlovers.

- Liars always lie and never speak a word of truth.
- Truthlovers always speak only truth.

Suppose, you are landed on this island and met a person. What will be the answer to your question “Who are you?”

- The answer always will be “Truthlover”.

Now you see a person who answers to your question. “I am a liar.”
Is it possible?

- This makes a paradox and should not ever happen on this island.

Matlab boolean logic examples

- `123.3 & 12=` 1
- `~ 1232e-6 =` 0

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231     0
34.3430    12.0000
```

```
~B
```

```
ans =
0      1
0      0
```

```
B|~B
```

"To be or not to be"

```
ans =
1      1
1      1
```

Matlab boolean logic examples

```
>> B=[1.22312, 0; 34.343, 12]
B =
1.2231     0
34.3430    12.0000

>> A=[56, 655; 0, 24.4]
A =
56.0000   655.0000
0         24.4000
```

```
B&A
```

```
ans =
1      0
0      1
```

```
A|~B
```

```
ans =
1      1
0      1
```

Comparison operators

Math	Matlab
$=$	`==` double equal sign!
$\neq$	`~=`
$<$	`<`
$\leq$	`<=`
$>$	`>`
$\geq$	`>=`

```
x=[1,2,3,4,5]
x =
   1   2   3   4   5
```

```
x >= 3
```

```
ans =
   0   0   1   1   1
```

```
% chose such 'x' where x>=3
x(x >= 3)
```

```
ans =
      3   4   5
```

Comparison with matrices

```
>> A=[1,2;3,4]
A =
1       2
3       4
```

```
>> B=[33,11;53,42]
B =
22       11
53       42
```

```
A>=2
```

```
ans =
0       1
1       1
```

```
A(A>=2)
```

```
ans =
3
2
4
```

```
B(A>=2)
```

```
ans =
53
11
42
```

if-else-end statement

`if` *expression*
this part is executed only if *expression* is true
`else`
this part is executed only if *expression* is false
`end`

`if` *hungry*
buy some food
`else`
keep working
`end`

```
if (x>=0)
  y=sqrt(x);
else
  error('cannot do');
end
```

Common mistake in the 'if' statement

```
if (x=y)
  D=4;
  Z=45;
  C=12;
else
  D=2;
end
```

the value of 'D' is always 4, except the case when y=0

someone used assignment operator (=) instead of comparison (==)

Short form of 'if-end' statement

`if` *expression*
this part is executed only if *expression* is true
`end`

`if` *won a million*
go party
`end`

```
if (deviation<=0)
  exit;
end
```

The 'while' statement

```
while expression
this part is executed
while expression is
true
end
```

```
while hungry
keep eating
end
```

```
i=1;
while (i<=10)
  c=a+b;
  z=c*4+5;
  i=i+2;
end
```

`while` loop is extremely useful but they are not guaranteed to finish. For a bit more complicated conditional statement and loop it is impossible to predict if the loop will finish.

Yet another common mistake is

```
i=1;
while (i<=10)
  c=a+b;
end
```

not updating the term leading to fulfillment of the `while` condition

The 'for' statement

`for` variable = *expression*
do something
`end`
In this case variable is assigned concequently with columns of the *expression*, and then statements inside of the loop are executed

```
sum=0;
x=[1,3,5,6]
for v=x
sum=sum+v;
end
```

```
>> sum
sum =
  15
```

`for` loops are guaranteed to complete after predictable number of iterations (the amount of columns in *expression*).

Example

$$S = \sum_{i=1}^{100} i = 1 + 2 + 3 + 4 + \cdots + 99 + 100$$

```
S=0; i=1;
while(i<=100)
  S=S+i;
  i=i+1;
end
```

```
S=0;
for i=1:100
  S=S+i;
end
```

Example

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k < 10^{-5}$, where $a_k = k^{-k}$.

```
S=0; k=1;
while( (k<=100) & (k^-k >= 1e-5) )
  S=S+k^-k;
  k=k+1;
end
```

```
>> S
S =
  1.2913
```

```
S=0; k=1;
while( k<=100 )
  a_k=k^-k;
  if (a_k < 1e-5)
    break;
  end
  S=S+a_k;
  k=k+1;
end
```

```
>> S
S =
  1.2913
```

Same example with 'for' loop

$$S = \sum_{k=1} a_k$$

Until k<=100 and $a_k < 10^{-5}$, where $a_k = k^{-k}$.

```
S=0;
for k=1:100
  a_k=k^-k;
  if (a_k < 1e-5)
    break;
  end
  S=S+a_k;
end
```

```
>> S
S =
  1.2913
```

Interest rate related example

Suppose bank gave you 50% interest rate (let's call it 'x'), and you put one dollar in.
How much would you get at the end of the year?

- one payment at the end of the year

$$M_1 = 1 * (1 + x) = 1 * (1 + .5) = 1.5$$

- interest payment every half a year

$$M_2 = 1 * (1 + x/2) * (1 + x/2) = 1 * (1 + .5/2)^2 = 1.5625$$

- interest payment every month

$$M_{12} = 1 * (1 + x/12)^{12} = 1.6321$$

Interest rate related example

Now let's find how you money growth (M_N) depends on the number of payments per year

```
N_max=100;
N=1:N_max;
M=0*(N);
x=.5;
for i=N
  M(i)=(1+x/i)^i;
end
plot(N,M,'-');
xlabel('N, number of payments per year');
ylabel('Money grows');
title('Money grows vs number of payments per year');
```

Interest rate related example