Homework 08

Problem 1 (10 points)

Implement the 4th order Runge-Kutta ODEs solver. It should be compatible with the call arguments and returned values of provided in class Euler's algorithm implementation. Call your solver function 'oderk4'. The function definition should be function [x,y] = oderk4(fvec, xspan, y0, N)

Check the validity of the implementation by comparing the numerical solution with analytical solution of the free fall problem shown in class. Feel free to reuse provided code.

Problem 2 (10 points)

Solve numerically (using your 'oderk4' solver) the following physical problem if the oscillatory motion

$$x''(t) = -x(t)^p$$

with following initial conditions

$$\begin{aligned} x(0) &= 0\\ v(0) &= x'(0) = v_0 \end{aligned}$$

Here the x is position of the particle, t is time, v is velocity, v_0 is initial velocity, and p is a parameter which takes odd values.

When p = 1 the problem resembles the equation of motions for the well known harmonic oscillator with k/m = 1.

Solve this problem (i.e. plot x(t) and v(t)) for two values of the parameter p = 1, 5, and the initial velocity $v_0 = 1$. Make sure to choose final time large enough so you see at least 10 periods.

Problem 3 (5 points)

Show that the period of the oscillation is independent of v(0) for the harmonic oscillator and depends on v(0) for the case of p = 5. Do it for at least five different values of v_0 to convince yourself.