Final Exam

Due date Monday December 6th of 2010 at 2pm.
This is the time by which the electronic report submission must be done.

We will meet at 2pm for the final presentation and project defense.

During this time, your team will give a 20 min presentation about key aspects of your algorithm with other relevant information presented (equations, algorithms, plots, etc).

Plan ahead and distribute presentation time among teammates so everyone has a chance to talk.

Presentation value is 25 points and will be assigned to each speaker separately.

Also, we will compare your answers against those of the other teams. Each team's solution will be ranked based on the number of intercepted alien rockets and the shortest time of interception (if any teams have the same number of interceptions). There will be several missiles to intercept; the combined interception time from all runs will be used. The best team or teams will get 5 points, next best team, 4 points, and so on.

Your code should provide **starting parameters** for the interceptor (see below). Even though to solve the problem you need your own simulation code, it will not be used in the competition. The simulation of the solution will be done with the instructor's computer and code using the input parameters you find.

Discuss relevant equations, describe your solution, show results. All Matlab code/scripts must be present in the carbon copy as well.

Make all you calculations in the S.I. units (m, kg, s).

Make plots of the trajectories for your report in **the polar system of coordinate** (substract the radius of the Earth from the radius of an object). Feel free to add plots in Cartesian system of coordinate if you think they have a presentation value.

We continue our quest against alien invasion (100 points total aside from the presentation/competition points)

Aliens are trying to take over the Earth. Since they were not successful in their biological attack, they decide to use nuclear warhead missiles to wipe out the human population.

The early warning radar system is able to detect the alien's missiles and provide to you the missile coordinates and the velocity (no more than 40 km/s) at the time $t = 0$. The alien missile has known drag coefficient, mass, and cross sectional area.

For simplicity, **all relevant bodies move strictly in the 'xy' plane**, latitude coincides with the polar coordinate angle ϕ measured from the axis 'x' in the counter clockwise direction. The center of the Earth coincides with the origin of system of the coordinate. Also, we will disregard the spin of the Earth.

You are the commander of the missile interception battle station (MIBS) located at the Surface of the Earth (we assume that Earth is the perfect sphere with the radius $R_e = 6,378,137$ m) at the known latitude (see Figure 1).

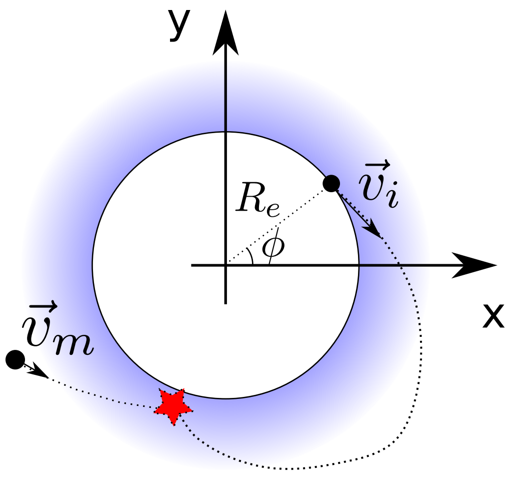

Figure 1: The problem set up (not to scale)

The interceptor is a simple ballistic projectile which is able to move with initial speed up to 14000 m/s in any direction (**make sure that it does not travel under ground which is impossible**). Your interceptor has it own logic unit and will explode if its distance to the alien missile is smaller then 100 m and in this case will destroy the missile. The interceptor has known drag coefficient, cross section area, and mass.

The interceptor and the missile motion is governed by the gravitational pull of the Earth and the air drag. (You can neglect mutual attraction of rockets and also assume that the Earth experiences negligible force from them).

So the force acting on an object is

$$\begin{aligned}
\vec{F}_o &= m_o\vec{a}_o = \vec{F}_g + \vec{F}_d \\
\vec{F}_g &= -G\frac{M_e m_o}{r_o^3}\vec{r}_o \\
\vec{F}_d &= -\frac{1}{2}\rho(h)C_{d_o}A_o v_o\vec{v}_o \\
\rho(h) &= \rho_0 2^{-h/h_{1/2}}
\end{aligned}$$

Where $\vec{F}_g$ is the gravitational pull force, $\vec{F}_d$ air drag force, m_o is the mass of the object, $\vec{r}_o$ is the position of the object with respect to the center of the Earth, $M_e = 5.9736 \times 10^{24}$ kg is the mass of the Earth, $G = 6.67428 \times 10^{-11}\ Nm^2/kg^2$ is the gravitational constant, $\vec{v}_o$ is the velocity of the object, $\vec{a}_o$ the acceleration of the object, C_{d_o} is the object drag coefficient, A_o is the object cross section area, $\rho(h)$ is the crude approximation of the air density dependence on the altitude (h) above the Earth's surface, $\rho_0 = 1.2$ kg/m^3 is the density of air at the Earth's surface, and $h_{1/2} = 6800$ m is the altitude at which density of air drops by factor of 2.

The aliens fire several missiles. You will be supplied with several data files (the web page will have a sample) which will have all relevant parameters for the battle station, interceptor, and the alien missiles. **Do not hard code these into your code; you should use the**

supplied data file parameters. The file with data parameters will contain the following matlab variables:

BattleStationLatitude the battle station latitude ϕ in **radians**

Ai the interceptor cross section area

Cdi the interceptor drag coefficient

Mi the interceptor mass

Xm the detected missile x coordinate

Ym the detected missile y coordinate

Vxm the detected missile x projection of the velocity

Vym the detected missile y projection of the velocity

Am the missile cross section area

Cdm the missile drag coefficient

Mm the missile mass

Your **single** `interceptor` function should solve all the following problems. This function might have nested function or refer to other function calls. It should conform to the following format (be sure to use proper spelling, capitolization, etc):

```
function [ShouldWeLaunchInterceptor, TimeOfLaunch, ...
        vxInterceptor, vyInterceptor, ...
        time, xInterceptor, yInterceptor, ...
        xMissile, yMissile] = interceptor( ParametersDataFileName )
```

`ShouldWeLaunchInterceptor` is the boolean true or false which depends on solution of problem 1 below.

`TimeOfLaunch` time when interceptor is launched if above is true, `NaN` if above is false.

`vxInterceptor` intitial 'x' component of the interceptor velocity at `TimeOfLaunch`

`vyInterceptor` intitial 'y' component of the interceptor velocity at `TimeOfLaunch`

`time` column vector of time from 0 until interception time, Earth impact time, or 2 hours. (whichever is the smallest)

`xInterceptor, yInterceptor, xMissile, yMissile` column vectors for positions of the interceptor and the missile during the above time. The interceptor positions should be assigned `NaN` if we do not launch the interceptor.

The interceptor function should conform to above specification and address the following problems.

Problem 1 (10 points)

Calculate the alien missile trajectory. Determine if the alien missile will hit the Earth's surface in the first 2 hours after detection. If it won't, there is no need to launch interceptor for this missile. There are plenty of other missiles to deal with that pose more immediate threats.

Problem 2 (10 points)

Calculate the interceptor trajectory for a given `TimeOfLaunch`, initial velocity `vxInterceptor`, and `vyInterceptor`.

Problem 3 (60 points)

Calculate the interceptor launch parameters: `TimeOfLaunch`, `vxInterceptor`, and `vyInterceptor`. Your goal is to intercept the missile as early as possible, however take into account calculation time. Your code takes time to calculate the trajectories, so you cannot launch your interceptor right at $t = 0$. If your time of launch is 4 seconds and the calculation took 5 seconds (measured with `tic, toc`) this is not a valid solution. You must take into account that the alien missile movies during your calculation time and compensate for it. Battle station computer might be faster or much slower than yours (take this into account). The code should be self calibrating for the real life application. Many problems can be solved with infinitively fast computers but they do not exist. So leave your self a safety margin for the launch time. Instructor will check the validity of the launch time during the competition, and assign negative points for invalid solutions.

Problem 4 (20 points)

This is a problem to solve at home. The code need to be supplied in the report.

To launch the interceptor you need the access codes. This code is delivered to you as a radio voice message (think about the president releasing the activation code right before the attack). Aliens know about it and try to jam the transmission. So voice message recording will have a lot of noise. Each team will get its own code transmission voice record, available on the web site. What is the your activation code (it should be a 6 digit number)?