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Collective transport in bilayer quantum Hall systems

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Abstract

Filling factor $\nu = 1$ incompressible states in ideal bilayer quantum Hall systems have spontaneous interlayer phase coherence and can be regarded either as easy-plane pseudospin ferromagnets or as condensates of excitons formed from electrons in one layer and holes in the other layer. In this paper we discuss efforts to achieve an understanding of the two different types of transport measurements (which we refer to as drag and tunneling experiments, respectively) that have been carried out in bilayer quantum Hall systems by the group of Jim Eisenstein at the California Institute of Technology. In a drag experiment, current is sent through one of the two-layers and the voltage drop is measured in the other layer. We will argue that the finding of these experiments that the voltage drop in the drag layer is different from that in the drive layer, is an experimental proof that these bilayers do not have quasi-long-range excitonic order. The property that at $\nu = 1$ the longitudinal drag voltage increases from near zero when spontaneous coherence is initially established, then falls back toward zero as it becomes well established, can be explained as a competition between the broken symmetry and the gap to which it gives rise. In the tunneling experiment, current is injected in one layer and removed from the other layer. The absence of quasi-long-range order likely explains the relatively small tunneling conductance per area found in the these measurements.

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1. Introduction

Among the broken symmetry states that occur in many-particle systems, those in which long range phase coherence is established, either for bosons (see for eg. Ref. [1]) or for pairs of fermions [2], have special significance because of the quantum nature of their macroscopic order and because of the sometimes startling phenomenology. In semiconductors, the possibility of long-range phase coherence due to Bose condensation of electron–hole bound states (excitons) was first raised [3] nearly 40 years ago. The physics of excitonic Bose condensation is interesting in bilayer systems in which excitons can form from electrons in one layer and holes in the neighboring layer. This case is especially exciting because the possibility of making separate electrical contact to the two layers enables novel probes of superfluid transport phenomena. In the quantum Hall regime, because of Landau level degeneracy, magnetoexcitons can emerge from

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electrons and holes that both originate from the conduction band, vastly simplifying the task of realizing the high-density electron–hole fluids in which these phenomena are expected to be most robust. Excitonic Bose condensation in this case, it turns out, is expected to occur when the total Landau level filling factor of the bilayer system is near $\nu = 1$. Although the anomalous transport properties discovered in bilayer quantum Hall systems [4] near this filling factor are thought [5] (for recent work see Ref. [6]) to follow from excitonic condensation and spontaneous phase coherence, it has not yet been possible to provide a complete interpretation of the observations.

In the present paper we concentrate on drag transport experiments, in which current flows in only one layer, but voltages are measured in both layers. The key observation in these experiments is that the longitudinal and Hall voltages measured in the current carrying layer and the electrically open layer are similar. This property suggests that the current is carried by quasiparticles that have weight in both layers even though interlayer tunneling amplitudes are negligibly small. It is naturally accounted for by a BCS-like mean-field theory of bilayer excitonic Bose condensation in which quasiparticles that are a coherent combination of states localized in separate layers are analogous to the coherent electron-plus-hole quasiparticles of superconductors; the indefinite layer index of these quasiparticles follows from the state's broken symmetry in the same way as the indefinite charge of BCS quasiparticles follows from superconducting order. In this picture, because the quasiparticles carry current that is divided equally between the layers, the drag experiment constraint that net current flow only through one layer forces an excitonic supercurrent through the bilayer. However, as we explain in greater detail below, the experimental observation that the voltages measured in the two layers are similar but not identical implies [7] that this collective superflow is not completely dissipationless. In the phenomenology that we explain below, dissipation is accounted for by the flow of vortices in the order-parameter field.

2. Phenomenology

In bilayers DC transport is characterized by a 4×4 conductivity tensor since its labels have both layer and two-dimensional Cartesian indices. For balanced layers, invariance under interchange of layer indices guarantees that even $(+)$ and odd $(-)$ channel response functions decouple. For isotropic layers these 2×2 tensors are characterized by their Hall and longitudinal conductivities so that there are four independent linear response coefficients:

$$j_{\pm,x} = \sigma_{\pm} \frac{E_{T,x} \pm E_{B,x}}{2} + \sigma^{H}_{\pm} \frac{E_{T,y} \pm E_{B,y}}{2},$$

$$j_{\pm,y} = -\sigma^{H}_{\pm} \frac{E_{T,x} \pm E_{B,x}}{2} + \sigma_{\pm} \frac{E_{T,y} \pm E_{B,y}}{2}, \qquad (1)$$

where $j_{\pm,\alpha} \equiv j_{T,\alpha} \pm j_{B,\alpha}$ and T and B label top and bottom layers, respectively. These relations can be inverted to find the four corresponding independent resistivity coefficients, $\rho_{\pm}$ and $\rho^{H}_{\pm}$. In the absence of excitonic condensation, apart from the weak inter-layer scattering processes that give rise to small drag voltages under normal circumstances and are neglected here, currents in a layer produce an electric field only in the same layer implying that $\rho_+ = \rho_-$ and $\rho^{H}_{+} = \rho^{H}_{-}$. When an excitonic Bose condensate with quasi long-range order is present, i.e. at temperatures below the Kosterlitz–Thouless temperature, the odd $(-)$ channel linear response Hall and longitudinal resistivities should vanish because [7] any difference in electric field between the layers would be shorted out by electron–hole pair condensate superflow. Experiments demonstrate that the even $(+)$ and odd $(-)$ channel resistivities in bilayers differ dramatically only for closely spaced layers and only for $\nu \approx 1$, thus strongly supporting the belief that collective excitonic transport is occurring in these systems. However, experiments also show that *the odd channel resistivities do not vanish*. It is these finite but non-zero odd channel resistivities that we concentrate on in this paper, since we believe that they are very revealing probes of the order that occurs in the system.

Weakly resistive transport in two-dimensional superfluids is normally understood in terms of Magnus-force driven vortex motion. Vortex flow leads to a steady rate of change of interlayer phase that differs at different points in the sample, and therefore via the Josephson relationship, gives rise to an odd $(-)$ channel electric field. There are two reasons we expect vortex-motion induced dissipation to be significant in quantum Hall bilayers. First of all, vortices in $\nu = 1$ quantum Hall superfluid carry electrical charge

$e^* = e/2$, and the finite-energy integer-charge elementary excitations of quantum Hall bilayers can be thought of, at least approximately, as being composed of bound vortex pairs [8,9] with opposite vorticity. When the filling factor of a quantum Hall bilayer deviates from $\nu = 1$, many charges of this nature are nucleated in the incompressible state background. Even at $\nu = 1$, long-length scale inhomogeneity in the system will nucleate charged quasiparticles. Measurement of a finite odd channel linear resistivities suggests that in real samples some of these vortices are free even in the absence of the Magnus force associated with pair condensate currents; free vortices will always lead to voltages linear in current. The second unique feature of quantum Hall superfluids which opens up an opportunity for vortex transport to play an important role is that the quantum Hall effect causes both even (+) and odd (−) channel longitudinal quasiparticle resistivities to vanish in the limit of zero temperature, even when there is no collective transport. In effect, in quantum Hall ferromagnets, we are able to look at vortex-flow dissipation in a conductor which is nearly dissipation-free even in the normal state.

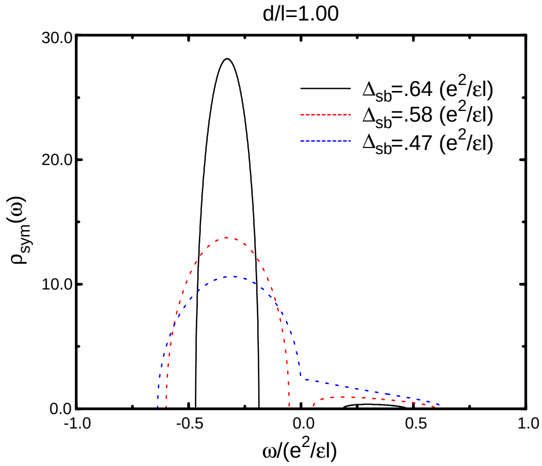

Fig. 1. Symmetric quasiparticle density-of-states, $\rho_{\rm sym}(\omega)$ for a bilayer system with $\nu = 1$ and different degrees of disorder, characterized by different values of $\Delta_{\rm sb}$. $\Delta_{\rm sb}$ is the exchange self-energy that favors symmetric quasiparticles over antisymmetric quasiparticles when the coherence phase angle is set to zero, a quantity that is proportional to the order parameter in SCBA+HF theory. For $\nu = 1$, the antisymmetric quasiparticle density-of-states satisfies $\rho_{\rm asym}(-\omega) = \rho_{\rm sym}(\omega)$. The odd (−) channel conductivity is finite only when both densities of states are finite at the Fermi energy $\omega = 0$.

3. SCBA calculations

We argue below that it is possible to separate quasiparticle and condensate contributions to transport coefficients in these systems. This argument is based partly on microscopic self-consistent Born approximation calculations for disordered quantum Hall superfluids that we now discuss. The point of view taken below in assessing the experimental results is based primarily on the type of result presented in this section, which, in turn, is based on an approximation for charge and pseudospin response in quantum Hall superfluids explained in detail in previous work [10,11]. The discussion presented here is purely qualitative. These calculations treat interactions via a generalized RPA approximation and disorder via a self-consistent Born approximation. In this treatment, excitonic superfluidity (incorrectly [6]) occurs at any layer separation d no matter how large, but the phase transition between ordered and disordered states can be (correctly) driven by increasing the degree of disorder in the system. In the ordered state, the occupied quasiparticle states spontaneously develop interlayer phase coherence. For coherence angle equal to zero and balanced bilayers, the quasiparticles experience an anomalous self-energy of collective origin that acts like a strong interlayer tunneling term (real with amplitude $\Delta_{\rm sb}/2$) which establishes the interlayer coherence, in addition to the random potentials that exist in each layer. Without disorder the Landau level density-of-states $\rho(\omega)$ of the quasiparticles would consist of two delta function pieces, the symmetric and antisymmetric branches separated in energy by $\Delta_{\rm sb}$. The ordering energy competes with the random potential by broadening the Landau levels and limiting the extent to which the quasiparticles can take advantage of a difference in potential between the layers. The densities of states in Fig. 1 are plotted for three different disorder strengths, or equivalently, three different order parameter values. In the presence of disorder all quasiparticle states have greater weight in one layer than in the other and have mixed character when projected onto symmetric and antisymmetric states. The order parameter of the bilayer can be defined as the difference per Landau level orbital of quasiparticle symmetric and antisymmetric

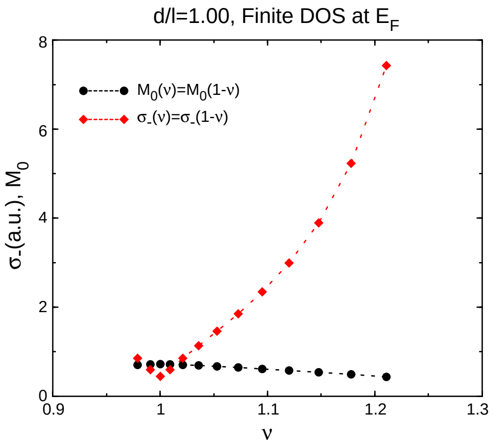

Fig. 2. Odd (−) channel conductivity (in arbitrary units) and order parameter as a function of filling factor for a fixed random potential strength. The conductivity and the order parameter are symmetric around $\nu = 1$. These results follow from SCBA calculations of linear response functions for a model with a random disorder potential in each layer and no correlations between the potentials in the different layers. A finite odd (−) channel conductivity requires quasiparticles that can carry different currents in the two layers and is reduced as order develops because the degree of layer polarization of typical quasiparticles is proportional to Γ/Δ_{sb}, where Γ is the Landau level width.

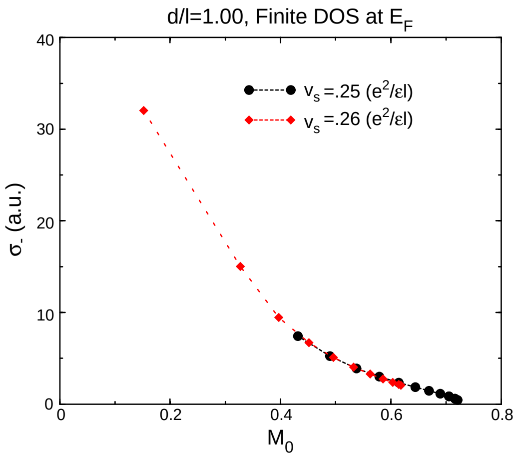

Fig. 3. Odd (−) channel conductivity (in arbitrary units) as a function of order parameter M_0. These results are calculated for various filling factors and two different disorder strengths. The approximate coincidence of these two curves demonstrates that the order parameter M_0 which depends on the same two variables, is the most important factor in determining the conductivity value.

projections summed over all occupied levels. By this measure, the order parameter at $\nu = 1$ approaches one as the disorder in the system weakens. In the SCBA, an artificial gap in the quasiparticle density of states arises when the disorder is sufficiently weak; in a more realistic calculation the quasiparticle density of states at the Fermi energy would decline continuously with the strength of the order but never vanish.

In the normal state, current is carried independently in the two layers, implying that the even (+) and odd (−) channel conductivities are identical; currents produce voltages only in the layer in which they flow. In the ordered state, the strong effective tunneling amplitude leads to quasiparticle states that tend to have their charge evenly divided between the two layers. These quasiparticle states tend to carry currents that are also nearly equally divided between the two layers, causing the odd (−) channel conductivity to be much smaller than its even (+) channel counterpart. The odd (−) channel conductivity and the order parameter are plotted as a function of filling factor in Fig. 2; we see here that the odd channel conductivity is strongly suppressed near $\nu = 1$, where the spontaneous coherence is most well developed. In Fig. 3 we plot the odd (−) channel conductivity as a function of the order parameter for two different disorder strengths and various filling factors, demonstrating that the suppression is more strongly connected to the degree of order than it is to either the filling factor or the disorder potential strength. Since the density of states at Fermi energy is strongly suppressed near $\nu = 1$, both conductivities are actually reduced in the ordered state, but the odd channel conductivity is reduced much more significantly.

Because we expect (and know from experiment) that Hall angles are very large in the quantum Hall regime, the longitudinal resistivities in each channel should be nearly proportional to the longitudinal conductivities. The difference between even (+) and odd (−) channel conductivities explained above should therefore lead to the same relative magnitudes for even (+) and odd (−) channel longitudinal resistivities, and therefore, to electric fields that have the same sign in drive and drag layers. This is opposite to what is observed experimentally. We believe that the experimental observations can be explained only by positing a condensate contribution to the currents carried through the system.

4. Condensate conductivity

Electron–hole pairs can carry opposing currents in the two layers and therefore a condensate can contribute to the odd channel conductivity. However, any vortices that are not pinned, will flow in the presence of a condensate current and produce odd channel electric fields. These electric fields will also drive extended state quasiparticles to carry current as discussed in the previous section. When vortices are unpinned, their thermal and quantum fluctuations must lead to a loss of long-range phase coherence. The observation of odd channel electric fields therefore suggests that the bilayers that have been studied have unpinned vortices.

Since the quasiparticles and the condensate phase (through its Josephson relation) see the same electric field, it follows that their conductivities σ^{Q} and σ^{C} add, i.e. they can be regarded as two separate contributions that carry odd channel current in parallel:

$$\sigma_- = \sigma_-^{\mathrm{Q}} + \sigma_-^{\mathrm{C}},$$

$$\sigma_-^{\mathrm{H}} = \sigma_-^{\mathrm{QH}} + \sigma_-^{\mathrm{CH}}. \tag{2}$$

The discussion in the previous section allowed only for σ_-^{Q}. The observed sign of the longitudinal Hall voltage implies that $\sigma_- \gg \sigma_+$, and since we have argued that σ_-^{Q} must be smaller than σ_+, this implies that $\sigma_- \approx \sigma_-^{\mathrm{C}}$. Measuring the odd channel longitudinal conductivity should therefore provide a direct measurement of condensate current in the bilayer.

As emphasized above, we do not believe that it is possible to interpret the experiments without positing such a collective current. σ_-^{Q} is small when order is well developed partly because the quasiparticles of the bilayer in an ordered system experience an in-plane pseudospin field that reduces the degree of layer polarization due to fluctuations in the local difference between random potentials in the two layers. Because the quasiparticles have little layer polarization even in the presence of disorder $\sigma_-^{\mathrm{Q}}/\sigma_+$ is $\sim (\Gamma/\Delta_{\mathrm{qp}})^2$ where Γ is the Landau level width and Δ_{qp} is the mean-field charge gap.

The quantum Hall effect occurs in the even $(+)$ channel, not in the odd $(-)$ channel. We should therefore expect that the odd $(-)$ channel Hall conductivity vanishes in the limit of zero temperature if both quasiparticles and vortices are localized in this limit. Indeed, this property is already suggested by current experiments, since the Hall voltages measured in drive and drag layers seem to approach each other at very low temperatures. From the SCBA linear response theory for the quasiparticle conductivities, it is clear that for $\nu \approx 1$ the quasiparticle Hall angles will be similar and large in both even and odd channels, i.e. that $\sigma_+^{\mathrm{H}} \gg \sigma_+$ and $\sigma_-^{\mathrm{QH}} \gg \sigma_-^{\mathrm{Q}}$. We do not, however, have a clear idea at present of the Hall angle for the condensate conductivity, which is related to the relationship between vortex flow and condensate current directions. We believe that further experiments, analyzed with the picture explained here, should allow this subtle issue, long controversial (see for eg. Ref. [12]) in the case of superconductors, to be settled for the case of quantum Hall ferromagnets.

5. Other open issues

We have seen that the drag experiments in bilayer quantum Hall systems simultaneously provide strong evidence that collective transport by an electron–hole pair condensate occurs in these systems, and that it is accompanied by dissipation suggesting that unpinned vortices are always present. The presence of free vortices may help explain the surprisingly small interlayer conductance that can be inferred from the tunneling experiments. In our view the well developed even $(+)$ channel quantum Hall effect that is seen in these systems is evidence of a large *local* order parameter. If there were long-range order, this property would [13] be difficult to reconcile with the experimentally measured layer to layer conductance per electron which is many orders of magnitude smaller than e^2/h. When current is injected on one side of one layer and extracted from the opposite side of the other layer, the overall resistance is still limited by weak hopping between the layers. It appears that collective tunneling of electrons between the layers is strongly suppressed [6]. Experiments [4] can give us results for the temperature-dependent vortex-flow resistivity, do give us results for the height and width of the low-bias peak in the tunneling conductance, and do give us results for the in-plane field scale at which the tunneling I–V characteristic changes character. The challenge for theory is to provide a common explanation for all these phenomena which, it appears, must have a common origin.

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