Dynamics of magnetization coupled to a thermal bath of elastic modes

Enrico Rossi,[1] Olle G. Heinonen,[2] and A. H. MacDonald[1]
[1]*Department of Physics, University of Texas at Austin, Austin, Texas 78712, USA*
[2]*Seagate Technology, 7801 Computer Ave., South, Bloomington, Minnesota 55435, USA*
(Received 18 May 2005; revised manuscript received 29 August 2005; published 10 November 2005)

We study the dynamics of magnetization coupled to a thermal bath of elastic modes using a system plus reservoir approach with realistic magnetoelastic coupling. After integrating out the elastic modes we obtain a self-contained equation for the dynamics of the magnetization. We find explicit expressions for the memory friction kernel and hence, *via* the fluctuation-dissipation theorem, for the spectral density of the magnetization thermal fluctuations. For magnetic samples in which the single domain approximation is valid, we derive an equation for the dynamics of the uniform mode. Finally we apply this equation to study the dynamics of the uniform magnetization mode in insulating ferromagnetic thin films. As experimental consequences we find that the fluctuation correlation time is of the order of the ratio between the film thickness, h, and the speed of sound in the magnet and that the linewidth of the ferromagnetic resonance peak should scale as $B_1^2 h$ where B_1 is the magnetoelastic coupling constant.

DOI: 10.1103/PhysRevB.72.174412 PACS number(s): 76.20.+q, 75.70.−i, 75.80.+q, 75.40.Gb

I. INTRODUCTION

Thermally induced fluctuations of the magnetization are responsible for one fundamental limit on the signal-to-noise ratio of small magnetoresistive sensors.[1] The noise scales inversely with the volume of the sensors and peaks at frequencies[2,3] that are now close to the ever increasing data rate of magnetic storage devices. The increase of data rates combined with the continuing decrease of the dimensions of the sensors makes magnetic noise inevitable and motivates work aimed at achieving a detailed understanding of its character.

The standard approach toward modeling of magnetization fluctuations is to start from the Landau-Lifshitz-Gilbert-Brown equation[4]

$$\frac{\partial \mathbf{\Omega}}{\partial t} = \frac{\gamma}{M_s}\mathbf{\Omega} \times \left(\frac{\delta E}{\delta \mathbf{\Omega}} + \mathbf{h}\right) + \alpha \mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial t}, \tag{1}$$

where γ is the gyromagnetic ratio, $\mathbf{\Omega}=\mathbf{M}/M_s$ is the magnetization direction, $\mathbf{M}$ is the magnetization, M_S is the magnitude of the saturation magnetization, E is the free energy, and $\mathbf{h}$ is a random magnetic field. This equation assumes that the characteristic time scale of the magnetization dynamics is longer than the typical time scale of the environment that is responsible for the dissipative term proportional to α. In practice the use of this equation is partially inconsistent, resulting in some practical limitations to its application.[5,6] The source of the problem is that the dissipation is local in time. Because of the fluctuations-dissipation theorem, this implicitly requires the random field to have white noise properties, i.e., to have zero autocorrelation time. Since the contribution of the random field to the magnetization dynamics $\gamma\mathbf{\Omega}\times\mathbf{h}$ depends on $\mathbf{\Omega}$, Eq. (1) exhibits white multiplicative noise.[7] It follows that in order to integrate Eq. (1) reliably we need to track the evolution of $\mathbf{\Omega}$ on very short time scales for which the white noise approximation for $\mathbf{h}$ is likely to be unphysical.

In this paper we address the physics that determines the correlation time of the random field. We start in Sec. II by considering a formal model of a magnetic system coupled to an environment and specialize in Sec. III to an environment consisting of elastic modes. In Sec. IV we consider the case in which a single magnetic mode corresponding to coherent evolution of the magnetization in a small single-domain system is coupled to the elastic environment. In Sec. V we consider a thin film geometry in which the magnetization is coupled to elastic modes of the system and its substrate. Finally in Sec. VI we conclude with a discussion of the possible role of other sources of dissipation, in particular dissipation due to particle-hole excitations in the case of metallic ferromagnets.

II. GENERIC RESERVOIR

Calling q_n the degrees of freedom of the reservoir, we consider the following form for the total Lagrangian:

$$\mathcal{L} = \mathcal{L}_S[\mathbf{\Omega}(\mathbf{x}),\dot{\mathbf{\Omega}}(\mathbf{x})] + \mathcal{L}_R[q_n,\dot{q}_n] + \mathcal{L}_I[\mathbf{\Omega}(\mathbf{x}),q_n] - \Delta\mathcal{L}[\mathbf{\Omega}(\mathbf{x})], \tag{2}$$

where $\mathcal{L}_S[\mathbf{\Omega}(\mathbf{x}),\dot{\mathbf{\Omega}}(\mathbf{x})]$ is the Lagrangian that describes the dynamics of the magnetization when not coupled to external degrees of freedom, $\mathcal{L}_R[q_n,\dot{q}_n]$ is the Lagrangian for the reservoir and $\mathcal{L}_I[\mathbf{\Omega}(\mathbf{x}),q_n]$ is the interaction Lagrangian that couples the magnetization to the reservoir degrees of freedom. The term $\Delta\mathcal{L}[\mathbf{\Omega}(\mathbf{x})]$ is a counter term that depends on $\mathbf{\Omega}$ and the parameters of the reservoir but not on the dynamic variables of the reservoir.[8,9] This term is introduced to compensate a renormalization of the energy of the system caused by its coupling to the reservoir.[8]

The Landau-Lifshitz equations for the decoupled system magnetization follow from the magnetic Lagrangian,

$$\mathcal{L}_S = \int_{V_M} \left(\frac{M_s}{\gamma} \mathbf{A}[\mathbf{\Omega}] \cdot \dot{\mathbf{\Omega}} - E_s[\mathbf{\Omega}] \right) d\mathbf{x}, \tag{3}$$

where $\mathbf{A}$ is a vector field defined by the equation $\nabla_{\mathbf{\Omega}} \times \mathbf{A}[\mathbf{\Omega}] = \mathbf{\Omega}$ and $E_S[\mathbf{\Omega}]$ is the magnetic free energy functional and V_M the volume of the ferromagnet. We model the reservoir as a set of classical degrees of freedom,

$$\mathcal{L}_R = \frac{1}{2} \sum_n m_n \dot{q}_n^2 - E_R(q_n). \tag{4}$$

The Euler-Lagrange equations for the total Lagrangian (2) yield the following coupled dynamical equations:

$$m_n \ddot{q}_n = \frac{\partial}{\partial q_n} [\mathcal{L}_R(q_n, \dot{q}_n) + \mathcal{L}_I[\mathbf{\Omega}, q_n]] \tag{5}$$

$$\frac{\partial \mathbf{\Omega}}{\partial t} = \mathbf{\Omega} \times \frac{\gamma}{M_s} \frac{\delta}{\delta \mathbf{\Omega}} [E_S[\mathbf{\Omega}, \dot{\mathbf{\Omega}}] - \mathcal{L}_I[\mathbf{\Omega}, q_n] + \Delta \mathcal{L}[\mathbf{\Omega}]]. \tag{6}$$

When $\mathcal{L}_I$ is linear in the coordinates of the bath, we can formally integrate (5) to get $q^{(n)}(t)$ as a function only of the initial conditions and $\mathbf{\Omega}$ and then insert the result in (6) to eliminate the reservoir coordinates from the dynamical equations for $\mathbf{\Omega}$, integrating out the reservoir degrees of freedom. An example of the application of this procedure for a quantum mechanical model of the interaction between magnetization and reservoir degrees of freedom can be found in Ref. 10.

III. MAGNETIZATION COUPLED TO ELASTIC MODES: GENERAL

If we consider only long wavelength vibrations we can treat the lattice as a continuous medium and use results from elasticity theory. The potential energy functional, E_R, of the elastic medium can then be expressed in terms of the strain tensor $u_{i,j}$,

$$u_{ij} \equiv \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),$$

where $\mathbf{u}$ is the displacement vector field.

We want to study the dynamics of the magnetization when coupled to elastic deformations of the system.[11] We will be interested in applying our results to polycrystalline elastic media which can be treated as isotropic to a good approximation. (It is quite straightforward, albeit quite tedious, to extend our results to the case of nonisotropic media with specific lattice symmetries.) For isotropic elastic media it follows from general symmetry considerations that, to lowest order, we can express the magnetoelastic energy in the form,[12]

$$E_I = B_1 \sum_{i,j=1}^{3} \int_{V_M} \Omega_i \Omega_j u_{ij} d\mathbf{x}, \tag{7}$$

where B_1 is the magnetoelastic coupling constant. For the case of soft ferromagnet thin films, the main contribution to the magnetoelastic energy will be given by the magnetostatic energy dependence on strain. This contribution to E_I is normally referred to as the form effect.[13] The constant B_1 can be extracted from magnetostriction data. For an isotropic elastic medium with isotropic magnetostriction, λ, we have[12] that

$$B_1 = \frac{3}{2} \lambda \frac{E}{2 - \sigma}, \tag{8}$$

where E is the Young's modulus and σ the Poisson's ratio.

The Lagrangian for an elastic reservoir $\mathcal{L}_R$ is

$$\mathcal{L}_R = \frac{1}{2} \int_V \rho \, \dot{\mathbf{u}}^2 d\mathbf{x} - E_R, \tag{9}$$

where ρ is the mass density, V the total volume of the elastic medium (magnetic film plus substrate) and E_R is given by[14]

$$E_R = \int_V \left(\frac{E}{2(1+\sigma)} \sum_{i,j=1}^{3} u_{ij}^2 + \frac{\sigma E}{2(1+\sigma)(1-2\sigma)} \sum_{i=1}^{3} u_{ii}^2 \right) d\mathbf{x}. \tag{10}$$

The equation of motion for the displacement will then be

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = - \frac{\delta}{\delta \mathbf{u}(\mathbf{x})} (E_R[\mathbf{u}] + E_I[\mathbf{\Omega}, \mathbf{u}]). \tag{11}$$

It will prove useful to expand $\mathbf{u}$ in terms of the elastic normal modes $\mathbf{f}^{(n)}$,

$$\mathbf{u} = \sum_n q^{(n)}(t) \mathbf{f}^{(n)}(\mathbf{x}), \tag{12}$$

where the functions $\mathbf{f}^{(n)}$ satisfy the boundary conditions appropriate for $\mathbf{u}$ and satisfy

$$\frac{\delta E_R(\mathbf{f}^{(n)})}{\delta \mathbf{f}^{(n)}(\mathbf{x})} = \omega_n^2 \rho \mathbf{f}^{(n)}(\mathbf{x}), \quad n \in \mathbb{N}, \tag{13}$$

$$\frac{1}{M} \int_V \rho \mathbf{f}^{(n)}(\mathbf{x}) \cdot \mathbf{f}^{(m)}(\mathbf{x}) d\mathbf{x} = \delta_{nm}, \tag{14}$$

where M is the total mass, $M \equiv \int_V \rho d\mathbf{x}$.

In terms of the degrees of freedom, $q^{(n)}$, we have

$$\mathcal{L}_I = - E_I = - B_1 \sum_n q^{(n)} \sum_{i,j} \int_{V_M} \Omega_i \Omega_j f_{ij}^{(n)} d\mathbf{x} \tag{15}$$

with

$$f_{ij}^{(n)} \equiv \frac{1}{2} \left(\frac{\partial f_i^{(n)}}{\partial x_j} + \frac{\partial f_j^{(n)}}{\partial x_i} \right).$$

We then see that the interaction Lagrangian is linear in the coordinates $q^{(n)}$, with coupling constants

$$c^{(n)}[\mathbf{\Omega}] \equiv \sum_{i,j} \int_{V_M} \Omega_i \Omega_j f_{ij}^{(n)} \mathrm{d}\mathbf{x}. \tag{16}$$

This property will allow us to integrate out the reservoir degrees of freedom to obtain an equation for the dynamics of the magnetization in term of $\mathbf{\Omega}$ alone.

Let us first discuss the dynamics of the reservoir degrees of freedom $q^{(n)}$. Using Eqs. (11)–(14) we find the dynamical equations

$$\ddot{q}^{(n)} = -\omega_n^2 q^{(n)} - \frac{B_1}{M} c^{(n)}[\mathbf{\Omega}]. \tag{17}$$

Integrating (17) we find

$$q^{(n)}(t) = q^{(n)}|_0 \cos(\omega_n t) + \frac{\dot{q}^{(n)}|_0}{\omega_n}\sin(\omega_n t) - \frac{B_1}{M\omega_n}\int_0^t \sin[\omega_n(t-t')]c^{(n)}[\mathbf{\Omega}(t')]dt', \tag{18}$$

where $q^{(n)}|_0$ and $\dot{q}^{(n)}|_0$ are the initial values of $q^{(n)}$ and $\dot{q}^{(n)}$, respectively. The coupling of the magnetization to the reservoir will cause damping and frequency renormalization. In order to be able to separate the two effects it is useful to integrate the last term on the right-hand side of (18) by parts obtaining

$$q^{(n)}(t) = q^{(n)}|_0 \cos(\omega_n t) + \frac{\dot{q}^{(n)}|_0}{\omega_n}\sin(\omega_n t) - \frac{B_1}{M\omega_n^2}c^{(n)}[\mathbf{\Omega}(t)] + \frac{B_1}{M\omega_n^2}c^{(n)}[\mathbf{\Omega}(0)]\cos(\omega_n t) + \frac{B_1}{M\omega_n^2}\int_0^t dt'\left(\cos[\omega_n(t-t')] \times \int_{V_M} \left.\frac{\delta c^{(n)}}{\delta \mathbf{\Omega}}\right|_{\mathbf{x}',t'} \cdot \left.\frac{\partial \mathbf{\Omega}}{\partial t'}\right|_{\mathbf{x}'} d\mathbf{x}'\right). \tag{19}$$

Using the expression of the interaction Lagrangian given by (15) and the definition of the coupling constants $c^{(n)}$ we have

$$\frac{\delta \mathcal{L}_I}{\delta \mathbf{\Omega}} = -B_1 \sum_n q^{(n)} \frac{\delta c^{(n)}}{\delta \mathbf{\Omega}}. \tag{20}$$

Combining Eqs. (6), (19), and (20) for the dynamics of the magnetization we find

$$\begin{aligned}\frac{\partial \mathbf{\Omega}}{\partial t} = {} & \mathbf{\Omega} \times \frac{\gamma}{M_s}\frac{\delta E_S}{\delta \mathbf{\Omega}} + \mathbf{\Omega} \times \frac{\gamma}{M_s}\frac{\delta \Delta\mathcal{L}(\mathbf{\Omega})}{\delta \mathbf{\Omega}} + \mathbf{\Omega} \\ & \times \frac{\gamma}{M_s}\sum_n \Bigg[B_1 \left.\frac{\delta c^{(n)}}{\delta \mathbf{\Omega}}\right|_{\mathbf{x},t} \\ & \times \left(q^{(n)}|_0 \cos(\omega_n t) + \frac{\dot{q}^{(n)}|_0}{\omega_n}\sin(\omega_n t)\right) \\ & - \frac{B_1^2}{M\omega_n^2}c^{(n)}[\mathbf{\Omega}(t)] \left.\frac{\delta c^{(n)}}{\delta \mathbf{\Omega}}\right|_{\mathbf{x},t} \\ & + \frac{B_1^2}{M\omega_n^2}c^{(n)}[\mathbf{\Omega}(0)]\cos(\omega_n t) \left.\frac{\delta c^{(n)}}{\delta \mathbf{\Omega}}\right|_{\mathbf{x},t} \\ & + \frac{B_1^2}{M\omega_n^2}\int_0^t dt' \int_{V_M} d\mathbf{x}' \cos[\omega_n(t-t')] \left.\frac{\delta c^{(n)}}{\delta \mathbf{\Omega}}\right|_{\mathbf{x}',t'} \\ & \cdot \left.\frac{\partial \mathbf{\Omega}}{\partial t'}\right|_{\mathbf{x}'} \left.\frac{\delta c^{(n)}}{\delta \mathbf{\Omega}}\right|_{\mathbf{x},t}\Bigg]. \end{aligned} \tag{21}$$

The counter term $\Delta\mathcal{L}$ of the total Lagrangian is defined to cancel the frequency renormalizing term

$$\mathbf{\Omega} \times \frac{\gamma}{M_s}\sum_{i,n}\frac{B_1^2}{M\omega_n^2}c^{(n)}[\mathbf{\Omega}(t)] \left.\frac{\delta c^{(n)}}{\delta \mathbf{\Omega}}\right|_{\mathbf{x},t}. \tag{22}$$

It follows from Eq. (16) that

$$\frac{\delta c^{(n)}}{\delta \Omega_l} = \sum_i \Omega_i\left(\frac{\partial f_l^{(n)}}{\partial x_i} + \frac{\partial f_i^{(n)}}{\partial x_l}\right). \tag{23}$$

To simplify and extract the physical content from these cumbersome equations, we identify the memory friction kernel tensor γ_{jm},

$$\gamma_{jm}(t,t',\mathbf{x},\mathbf{x}') \equiv \Theta(t-t')\sum_n \frac{\gamma}{M_s}\frac{B_1^2}{M\omega_n^2}\cos[\omega_n(t-t')] \times \left.\frac{\delta c^{(n)}}{\delta \Omega_m}\right|_{\mathbf{x}',t'} \left.\frac{\delta c^{(n)}}{\delta \Omega_j}\right|_{\mathbf{x},t}, \tag{24}$$

where $\Theta(t-t')$ is the Heaviside function. We also recognize the random field $\mathbf{h}$,

$$\mathbf{h}(\mathbf{x},t) \equiv \frac{B_1}{M_s}\sum_n \left(q^{(n)}|_0 \cos(\omega_n t) + \frac{\dot{q}^{(n)}|_0}{\omega_n}\sin(\omega_n t)\right)\frac{\delta c^{(n)}}{\delta \mathbf{\Omega}}. \tag{25}$$

Assuming that the distribution of initial positions of the environment degrees of freedom follows the canonical classical equilibrium density for the unperturbed reservoir we find that

$$\langle \mathbf{h}(\mathbf{x},t)\rangle = 0, \tag{26}$$

$$\langle h_j(\mathbf{x},t)h_m(\mathbf{x}',t')\rangle = \frac{2K_BT}{\gamma M_s}\gamma_{jm}(t,t',\mathbf{x},\mathbf{x}'). \tag{27}$$

In terms of γ_{jm} and $\mathbf{h}$ the dynamical equation for $\mathbf{\Omega}$ takes the form

$$\begin{aligned}\frac{\partial \Omega_l}{\partial t} = {} & \epsilon_{ijl}\Omega_i\frac{\gamma}{M_s}\frac{\delta E_S}{\delta \Omega_j} + \gamma\epsilon_{ijl}\Omega_i h_j \\ & + \epsilon_{ijl}\Omega_i\int_0^t dt'\int_{V_M} d\mathbf{x}'\sum_m \gamma_{jm}(t,t',\mathbf{x},\mathbf{x}') \left.\frac{\partial \Omega_m}{\partial t'}\right|_{\mathbf{x}'} \\ & + \epsilon_{ijl}\Omega_i\frac{\gamma}{M_s}\sum_{i,n}\frac{B_1^2}{M\omega_n^2}c^{(n)}[\mathbf{\Omega}(0)]\cos(\omega_n t)\frac{\delta c^{(n)}}{\delta \Omega_j}. \end{aligned}$$

The final term is an artifact of the assumption that in the initial state the reservoir was decoupled from the system.[9,15] Dropping this term, the dynamical equations for magnetization coupled to a thermal bath of elastic modes is

$$\begin{aligned}\frac{\partial \Omega_l}{\partial t} = {} & \epsilon_{ijl}\Omega_i\frac{\gamma}{M_s}\frac{\delta E_S}{\delta \Omega_j} + \gamma\epsilon_{ijl}\Omega_i h_j \\ & + \epsilon_{ijl}\Omega_i\int_0^t dt'\int_{V_M} d\mathbf{x}'\sum_m \gamma_{jm}(t,t',\mathbf{x},\mathbf{x}') \left.\frac{\partial \Omega_m}{\partial t'}\right|_{\mathbf{x}'} \end{aligned} \tag{28}$$

with γ_{jm} defined by (24) and $\mathbf{h}$ a random field with statistical properties given by (26) and (27). Equation (28) is quite general. In particular notice that to obtain (28) we did not perform any expansion in $\mathbf{\Omega}$. As a consequence, as long as we keep the exact form for $E_S(\mathbf{\Omega})$, Eq. (28) includes also the effects of spin wave interactions. In principle we could also include in E_S a term to take into account the scattering of spin waves due to disorder. Equation (28) does not, however, take into account the coupling between the magnetization and particle-hole excitations. As we discuss in Sec. VI, this coupling appears to be of critical importance in many metallic ferromagnets.

Equation (28) is very different from the standard stochastic Landau-Lifshitz-Gilbert (s-LLG) equation, Eq. (1). Because the magnetoelastic energy, E_I, (7), is nonlinear in the magnetization, in (28) both the damping kernel and the random field depend on the magnetization and therefore are state dependent. This is in contrast with the s-LLG equation for which both the damping kernel, $\alpha\delta(t-t')$, and the random field are independent of $\mathbf{\Omega}$.

Another difference between Eq. (28) and the s-LLG equation is that the damping kernel, γ_{jm}, is in general a tensor. The tensor character of the damping has been suggested previously on phenomenological grounds.[6] Starting from the physical coupling (7), in our approach the tensor character of γ_{jm} appears naturally as a consequence of (a) the nonlinearity in $\mathbf{\Omega}$ of the magnetoelastic coupling (7), (b) the anisotropy of the elastic modes due to the boundary conditions and/or anisotropy of the elastic properties. For small oscillations of $\mathbf{\Omega}$ around its equilibrium (up to quadratic order), the kernel γ_{jm} can be assumed to be independent of $\mathbf{\Omega}$. Even in this linearized case, the damping kernel that appears in (28) will still have a tensor form due to the anisotropy of the elastic modes.

As mentioned above, the standard s-LLG damping kernel is simply $\alpha\delta(t-t')$, i.e., the damping is frequency independent. As a consequence, from the fluctuation-dissipation theorem, we have that the spectrum of the random field that appears in (1) is also frequency independent. This differs from Eq. (28) for which the damping kernel, and therefore the spectrum of the random field, is frequency dependent.

Given the geometry and the material properties of the system we can find the elastic modes, $\mathbf{f}^{(n)}$, and then integrate Eq. (28) using a micromagnetic approach. The integration of Eq. (28) could give insight in particular on the damping of the uniform magnetization mode for different geometries and show the range of validity of the classic picture[16] of a two stage damping process in which the motion of the coherent magnetization induces nonuniform magnetic modes on short time scales that then decay to lattice vibrations.

IV. MAGNETIZATION COUPLED TO ELASTIC MODES: UNIFORM MAGNETIZATION

We now study the dynamics of the uniform magnetic mode in the case when we can neglect its interaction with spin waves and the only coupling to external degrees of freedom is magnetoelastic. Projecting Eq. (28) on the uniform mode we find that

$$\frac{d\Omega_l}{dt} = \epsilon_{ijl}\Omega_i \frac{\gamma}{V_M M_s}\int_{V_M}\frac{\delta E_S}{\delta \Omega_j}d\mathbf{x} + \epsilon_{ijl}\Omega_i\frac{\gamma}{V_M}\int_{V_M} h_j d\mathbf{x} + \epsilon_{ijl}\Omega_i\frac{1}{V_M}\int_0^t dt'\int_{V_M}d\mathbf{x}\int_{V_M}d\mathbf{x}'\sum_m \gamma_{jm}(t,t',\mathbf{x},\mathbf{x}')\frac{d\Omega_m}{dt'}. \tag{29}$$

Let us define the space averaged error field

$$\bar{\mathbf{h}}(t) \equiv \frac{1}{V_M}\int_{V_M}\mathbf{h}(\mathbf{x},t)d\mathbf{x},$$

the damping kernel

$$\bar{\gamma}_{jm}(t,t') \equiv \frac{1}{V_M}\int_{V_M}d\mathbf{x}\int_{V_M}d\mathbf{x}'\,\gamma_{jm}(t,t',\mathbf{x},\mathbf{x}'),$$

and the coefficients

$$c_l^{(n)} \equiv \int_{V_M}\frac{\delta c^{(n)}}{\delta\Omega_l}d\mathbf{x}.$$

Using the fact that $\mathbf{\Omega}$ is uniform we obtain

$$c_l^{(n)} = \sum_i \Omega_i\int_{V_M}\left(\frac{\partial f_l^{(n)}}{\partial x_i} + \frac{\partial f_i^{(n)}}{\partial x_l}\right)d\mathbf{x}. \tag{30}$$

In terms of the coefficients $c_l^{(n)}$ we can then write

$$\bar{h}_l = \frac{B_1}{M_s V_M}\sum_n c_l^{(n)}\left(q^{(n)}|_0\cos(\omega_n t) + \frac{\dot{q}^{(n)}|_0}{\omega_n}\sin(\omega_n t)\right)$$

and

$$\bar{\gamma}_{jm} = \Theta(t-t')\frac{\gamma B_1^2}{M_s M V_M}\sum_n\frac{1}{\omega_n^2}c_j^{(n)}(t)c_m^{(n)}(t')\cos[\omega_n(t-t')]. \tag{31}$$

The uniform magnetization dynamics can then be expressed in terms of the spatially averaged random field $\bar{\mathbf{h}}$ and memory friction kernel $\bar{\gamma}_{jl}$,

$$\frac{d\Omega_l}{dt} = \epsilon_{ijl}\Omega_i\frac{1}{V_M}\frac{\gamma}{M_s}\int_{V_M}\frac{\delta E_S}{\delta\Omega_j}d\mathbf{x} + \gamma\epsilon_{ijl}\Omega_i\bar{h}_j + \epsilon_{ijl}\Omega_i\int_0^t dt'\sum_m\bar{\gamma}_{jm}(t,t')\frac{d\Omega_m}{dt'} \tag{32}$$

with

$$\langle\bar{\mathbf{h}}\rangle = 0 \tag{33}$$

and

$$\langle\bar{h}_j(t)\bar{h}_m(t')\rangle = \frac{2K_BT}{\gamma V_M M_s}\bar{\gamma}_{jm}(t,t'). \tag{34}$$

V. THIN FILM UNIFORM MAGNETIZATION DYNAMICS

We now apply Eq. (32) to study the dynamics of the uniform magnetization in a thin ferromagnetic film placed on

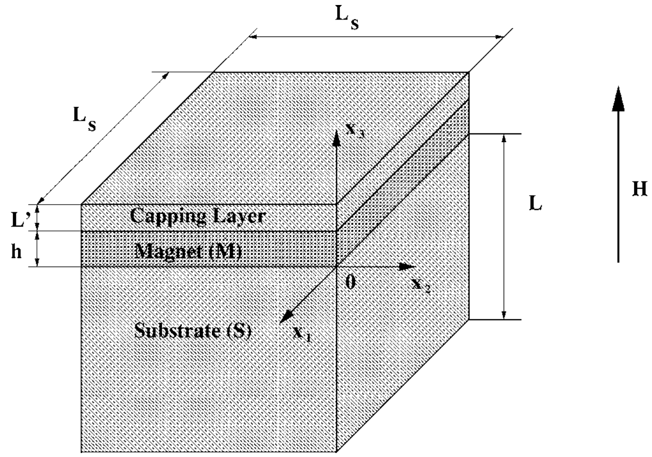


FIG. 1. Geometry considered for the case of a thin ferromagnetic film on a nonmagnetic substrate.

top of a nonmagnetic substrate and covered by a nonmagnetic capping layer, as illustrated in Fig. 1. We assume that all media are polycrystalline and treat them as isotropic. We will assume the lateral size, L_s, Fig. 1, to be much bigger than the film thickness h. Notice that if we take L_s bigger than the domain wall width our assumption that the nonuniform magnetic modes are quenched would not be valid anymore. We will consider only oscillations of the magnetization around an equilibrium position parallel to the x_3 axis so that we can calculate the damping kernel tensor γ_{jm} assuming the elastic modes to depend only on x_3. Otherwise, to find the correct damping kernel, we would have to take into account the fact that the lateral size, L_s, is finite and solve the full three-dimensional (3D) elasticity problem for the elastic modes.

A. Damping kernel and random field

To find the dynamics of the magnetization using Eq. (32) we need to evaluate the memory friction kernel γ_{jm}. The first step in this calculation is the determination of the elastic normal modes $\mathbf{f}^{(n)}$ which satisfy the following equation:

$$\omega_n^2\rho\mathbf{f}^{(n)} = -\frac{E}{2(1+\sigma)}\nabla^2\mathbf{f}^{(n)} - \frac{E}{2(1+\sigma)(1-2\sigma)}\nabla(\nabla\cdot\mathbf{f}^{(n)}). \tag{35}$$

We allow the film, the substrate, and the capping layer to have different elastic properties and solve Eq. (35) separately in the different subsystems using the appropriate elastic constants. We assume for the sake of definiteness that the substrate and capping layer material is identical. We then match solutions by imposing the continuity of displacement and stresses at the interfaces $x_3=0$, and $x_3=h$. As boundary conditions we assume the top surface of the capping layer to be free and no displacement at the bottom of the substrate.

Because in our case the elastic modes only depend on x_3, Eq. (30) simplifies to

TABLE I. Elastic properties. c_t, c_l are the transverse and longitudinal speed of sound respectively.

	Magnetic film	Substrate/capping layer
E	200 Gpa	180 Gpa
σ	0.33	0.33
ρ	5.0 g/cm^3	16.6 g/cm^3
c_t	4.0 km/s	2.0 km/s
c_l	5.0 km/s	4.1 km/s

$$c_l^{(n)} = L_s^2\sum_i \Delta f_i^{(n)}(\delta_{il}\Omega_3 + \Omega_i\delta_{3l})$$

with

$$\Delta f_i^{(n)} \equiv f_i^{(n)}(h) - f_i^{(n)}(0).$$

The spatially averaged damping coefficients have a simple expression in terms of the $\Delta f_i^{(n)}$,

$$\bar{\gamma}_{jl} = \Theta(t-t')\frac{L_s^2B_1^2}{Mh}\sum_n \frac{(\Delta f_i^{(n)})^2}{\omega_n^2}\cos[\omega_n(t-t')] \times[\delta_{ij}\Omega_3(t) + \Omega_i(t)\delta_{3j}][\delta_{il}\Omega_3(t') + \Omega_i(t')\delta_{3l}]. \tag{36}$$

Equation (36) follows from the completeness relation of the polarization vectors. Once we know the coefficients $\Delta f_i^{(n)}$, Eqs. (33), (34), and (36) completely specify the dynamical equation (32) for the magnetization.

As an example we consider the case of a polycrystalline ferromagnetic thin film, like YIG, placed on a substrate of a polycrystalline paramagnet like tantalum, Ta. As typical values we take[17] the ones listed in Table I. For the magnetostriction we assume $\lambda=2\times10^{-6}$. Using Eq. (8), we find that $B_1=4\times10^6$ ergs/cm^3. Given the elastic modes implied by these parameter values, we can calculate the coefficients $\Delta f_i^{(n)}$. Once we know the coefficients $\Delta f_i^{(n)}$ we have all the elements to completely specify Eq. (32).

We generate a stochastic field $\mathbf{h}$ with the correct statistical properties by using its Fourier representation. To obtain

$$\langle y(t)y(t')\rangle = G(t-t') \tag{37}$$

we choose[18]

$$\langle y(\omega)y(\omega')\rangle = \delta(\omega-\omega')G(\omega), \tag{38}$$

where

$$y(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} y(t)e^{-i\omega t}dt$$

and

$$G(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\tau)e^{-i\omega\tau}d\tau.$$

In our case we have from Eq. (31), that the memory friction kernel $\bar{\gamma}_{jl}$ depends separately on t and t'. As a consequence, through (34), we have that the average $\langle\bar{\mathbf{h}}(t)\bar{\mathbf{h}}(t')\rangle$ does not depend only on the time difference $\tau=t-t'$. The random field $\bar{\mathbf{h}}(t)$ therefore does not define an ergodic process and in particular we cannot use Eq. (38). For this reason it is convenient to define the auxiliary random variables,

$$x_i \equiv \sum_n \Delta f_i^{(n)} \left(q^{(n)}|_0 \cos(\omega_n t) + \frac{\dot{q}^{(n)}|_0}{\omega_n} \sin(\omega_n t) \right)$$

and the auxiliary kernels

$$g_i(t-t') \equiv \Theta(t-t') \sum_n \frac{(\Delta f_i^{(n)})^2}{\omega_n^2} \cos[\omega_n(t-t')]$$

so that we have

$$\langle x_i(t) x_j(t') \rangle = \frac{2K_B T}{M} g_i(t-t') \delta_{ij}.$$

The random variables $x_i(t)$ therefore describe an ergodic process and we can use Eq. (38) to generate them. In terms of x_i and g_i we have

$$\bar{h}_l = \frac{B_1}{M_s h} \sum_i x_i [\delta_{il} \Omega_3(t') + \Omega_i(t') \delta_{3l}],$$

$$\bar{\gamma}_{jm} = \frac{\gamma L_s^2 B_1^2}{M_s M h} \sum_i g_i [\delta_{ij} \Omega_3(t) + \Omega_i(t) \delta_{3j}] \times [\delta_{im} \Omega_3(t') + \Omega_i(t') \delta_{3m}]. \quad (39)$$

To generate the random field and calculate $\bar{\gamma}_{jl}$ we then must calculate the quantities $g_i(\tau)$ and their Fourier transforms $g_i(\omega)$. Figures 2(a), 2(b), 3(a), and 3(b) show some typical profiles for $g_i(\tau)$ and $g_i(\omega)$ using for the mechanical properties the values of Table I. We find that in general $g_i(\tau)$ does not depend on the thickness of the capping layer L'.

In the limit in which we can linearize the magnetoelastic interaction with respect to $\mathbf{\Omega}$, we have

$$\bar{\gamma}_{jm}(\tau) = \frac{\gamma B_1^2 L_s^2}{M_s M h} g_j(\tau) \delta_{jm}. \quad (40)$$

The damping kernel is diagonal with components equal, apart from an overall constant, to $g_j(\tau)$, in contrast to the s-LLG equation for which we have $\bar{\gamma}_{jl}(\tau) = \alpha \delta(\tau) \delta_{jl}$. The power spectrum of the random field component, h_j, is then proportional to $g_j(\omega)$, in contrast to the s-LLG equation for which the power spectrum of each component h_j is simply a constant. Notice that even in this limit $\bar{\gamma}_{jm}$ preserves its tensor form due to the anisotropy of the elastic modes. In our specific case we have $g_1 = g_2 \neq g_3$ due to the difference between the transverse and longitudinal speeds of sound.

From Figs. 2(a) and 2(b), we see that $g_i(\tau)$ goes to zero for times longer than $\tau_D \approx 5 \times 10^{-2}\ \tau_0 = 5h/c$, where $c \equiv c_{t,M}$ is the transverse speed of sound in the magnet. For a film 20 nm thick we then find $\tau_D \approx 10$ ps. When the relevant frequencies of $\mathbf{\Omega}$ are much lower than $1/\tau_D$, we can replace the damping kernel given by (40) with the simple kernel

$$\gamma_{jm} = \gamma_{j\mathrm{eff}} \delta(\tau) \delta_{jm}$$

with $\gamma_{j\mathrm{eff}}$ given by

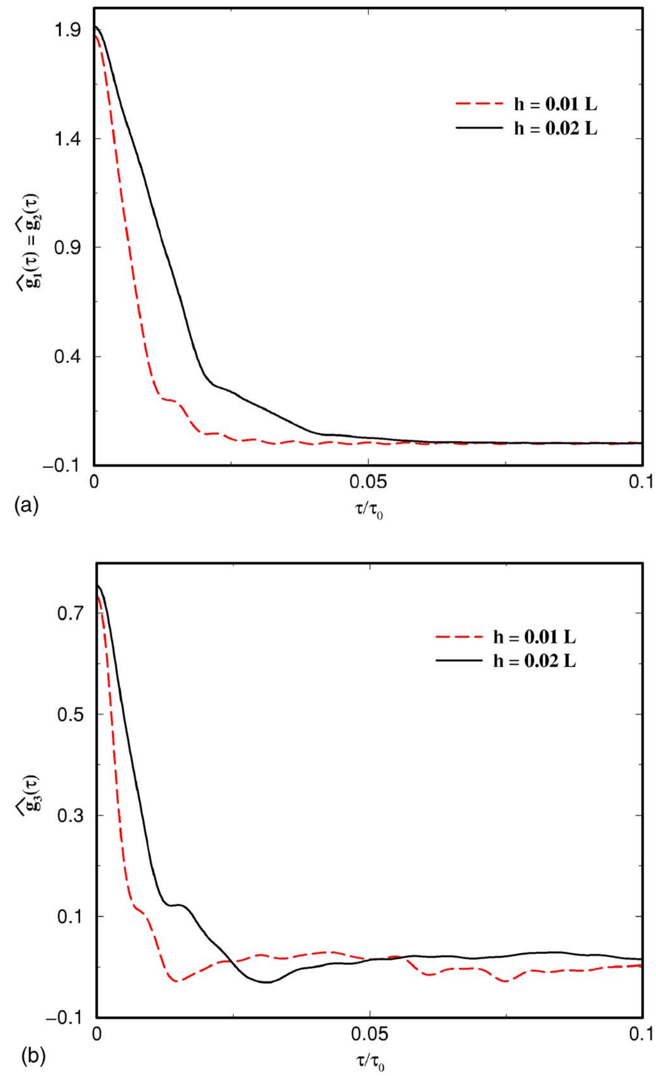


FIG. 2. (Color online) Profiles of $\hat{g}_1 \equiv g_1(\tau) c^2 / [h(L+h+L')]$ (a) and $\hat{g}_3 \equiv g_3(\tau) c^2 / [h(L+h+L')]$ (b) for the case of a thin magnetic film on a tantalum substrate; $\tau_0 \equiv L / c_{t,M}$. For the standard s-LLG equation $g_i(\tau)$ would simply be a Dirac delta centered at $\tau = 0$.

$$\gamma_{j\mathrm{eff}} = \frac{\gamma B_1^2 L_s^2}{M_s M h} \int_0^\infty g_j(\tau) d\tau. \quad (41)$$

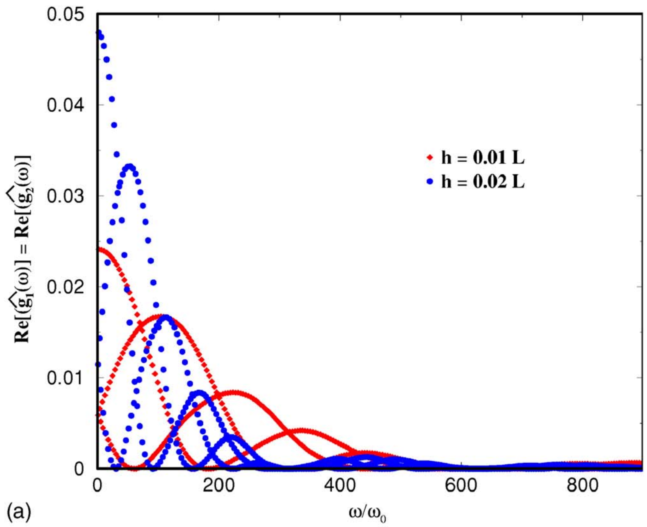
In this limit we recover a damping kernel of the same form as the one that appears in the s-LLG equation. Here $\gamma_{j\mathrm{eff}}$ is the equivalent to α in (1). From the results shown in Figs. 2(a) and 2(b) we see that we have

$$\int_0^\infty g_j(\tau) d\tau \approx \frac{h^2 (L+h+L')}{c^3}$$

and then

$$\gamma_{j\mathrm{eff}} = \frac{\gamma B_1^2 h}{M_s \rho c^3}. \quad (42)$$

We find that the damping of magnetic modes in thin films is proportional to $B_1^2 h$. Assuming the values given in Table II we find $\gamma_{1\mathrm{eff}} = \gamma_{2\mathrm{eff}} \approx 2 \times 10^{-4}$.



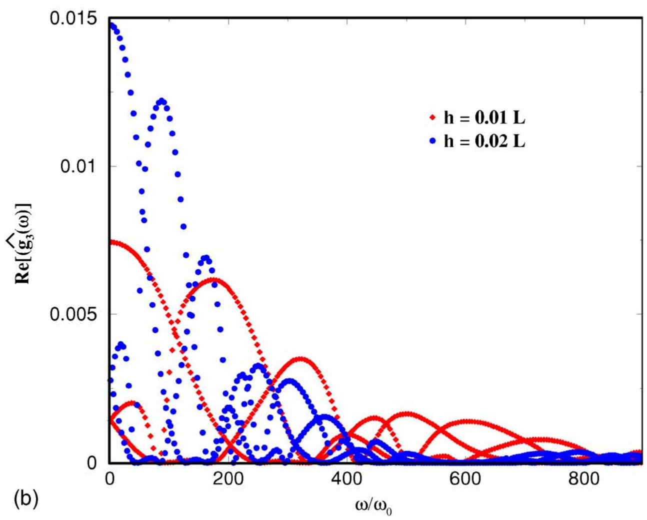


FIG. 3. (Color online) Values of $\text{Re}[\hat{g}_1(\omega)] \equiv \text{Re}[g_1(\omega)]c^2/[h(L+h+L')]$ (a) and $\text{Re}[\hat{g}_3(\omega)] \equiv \text{Re}[g_3(\omega)] \times c^2/[h(L+h+L')]$ (b) at the elastic modes frequencies $\{\omega_n\}$ for the case of a thin magnetic film on a tantalum substrate. Shown are the values for $h=0.01L$, diamonds, and $h=0.02L$, circles. For any ω_n $\text{Re}[\hat{g}_i(\omega_n)]$ is unique even though this is not completely evident from the figure because in order to show the behavior of the auxiliary kernels over a wide frequency range, the resolution is not high enough to always show the separation between the single points. For the standard s-LLG equation $g_i(\omega)$ would simply be a constant.

TABLE II. Magnetic properties and dimensions for the system studied.

Quantity	Value
γ	1.76×10^7 s^{-1} G^{-1}
B_1	4×10^6 ergs/cm^3
M_s	150 G
L	1 μm
h	20 nm

B. Integration

After generating the random field $\bar{\mathbf{h}}$ in the way described above we can proceed in integrating Eq. (32). We assume $\delta E_S/\delta\mathbf{\Omega} = -V_M M_s \mathbf{H}_{\text{eff}}$ with $\mathbf{H}_{\text{eff}} = (0,0,H_{\text{eff}})$ and H_{eff} simply a constant. Let us define the dimensionless quantities

$$\hat{t} \equiv \gamma H_{\text{eff}} t, \quad \hat{\mathbf{H}}_{\text{eff}} \equiv \frac{\mathbf{H}_{\text{eff}}}{H_{\text{eff}}}, \quad \hat{\mathbf{h}} \equiv \frac{\mathbf{h}}{H_{\text{eff}}},$$

$$\hat{\gamma}_{jm} \equiv \frac{\bar{\gamma}_{jm}}{\gamma H_{\text{eff}}}, \quad \hat{T} \equiv \frac{2K_B T}{H_{\text{eff}} M_s V_M},$$

then in dimensionless form Eq. (32) takes the form

$$\frac{d\Omega_l}{d\hat{t}} = -\epsilon_{ijl}\Omega_i \hat{H}_{\text{eff}j} + \epsilon_{ijl}\Omega_i \hat{h}_j + \epsilon_{ijl}\Omega_i \int_0^{\hat{t}} d\hat{t}' \sum_m \hat{\gamma}_{jm}(\hat{t},\hat{t}') \frac{d\Omega_m}{d\hat{t}'} \tag{43}$$

with

$$\langle \hat{h}_j(\hat{t}) \rangle = 0, \quad \langle \hat{h}_j(\hat{t}) \hat{h}_m(\hat{t}') \rangle = \hat{T} \hat{\gamma}_{jm}(\hat{t},\hat{t}'). \tag{44}$$

Similarly, for $\delta E_S/\delta\mathbf{\Omega} = -V_M M_s \mathbf{H}_{\text{eff}}$, the standard s-LLG equation, (1), for the uniform mode, takes the dimensionless form

$$\frac{d\mathbf{\Omega}}{d\hat{t}} = -\mathbf{\Omega} \times \hat{\mathbf{H}}_{\text{eff}} + \mathbf{\Omega} \times \hat{\mathbf{H}} + \alpha \mathbf{\Omega} \times \frac{d\mathbf{\Omega}}{d\hat{t}} \tag{45}$$

with

$$\langle \hat{h}_j(\hat{t}) \rangle = 0, \quad \langle \hat{h}_j(\hat{t}) \hat{h}_m(\hat{t}') \rangle = \alpha \hat{T} \delta(\hat{t} - \hat{t}'). \tag{46}$$

Using for $\bar{\gamma}_{jm}$ the expression (39) and for $g_i(\tau), g_i(\omega)$ the results shown in Figs. 2(a), 2(b), 3(a), and 3(b) and assuming $\hat{T} = 10^{-2}$ and the values given in Table II we integrate Eq. (43). We used the stochastic Heun scheme that ensures convergence to the Stratonovich solution even in the limit of zero autocorrelation time for the random field.[7] The results of the integration are shown in Figs. 4(a), 4(b), and 5(a). As initial condition we took $\mathbf{\Omega} = (0.6, 0, 0.8)$, $d\mathbf{\Omega}/d\hat{t} = 0$.

We then integrated Eq. (45) setting $\alpha = \gamma_{1\text{eff}}$ with $\gamma_{1\text{eff}}$ calculated using (41). The results of the integration are shown in Figs. 4(a), 4(b), and 5(b).

From Figs. 4(a), 4(b), 5(a), and 5(b) we see that on average Eqs. (43) and (45) give very similar results. This is expected because for the initial conditions chosen we are in the limit of small oscillations around the equilibrium position and therefore the dependence of $\hat{\gamma}_{jm}$ on $\mathbf{\Omega}$ is negligible. The main differences, for the case considered, between the results obtained using (43) and (45) are in the random fluctuations of $\mathbf{\Omega}$. This is a consequence of the different correlation in time of the random field $\mathbf{h}$ used in (43) and (45). For example, we notice that Eq. (43) seems to give a less noisy dynamics than (45) even though for both simulations $|\hat{h}|^2$ is of the same order of magnitude. If we zoom on a short time interval, Fig. 4(b), as a matter of fact, we see that on very short time scales the amplitude of the random fluctuations for the two simulations is very similar. However for (45) fluctuations with the same sign are much more likely than for

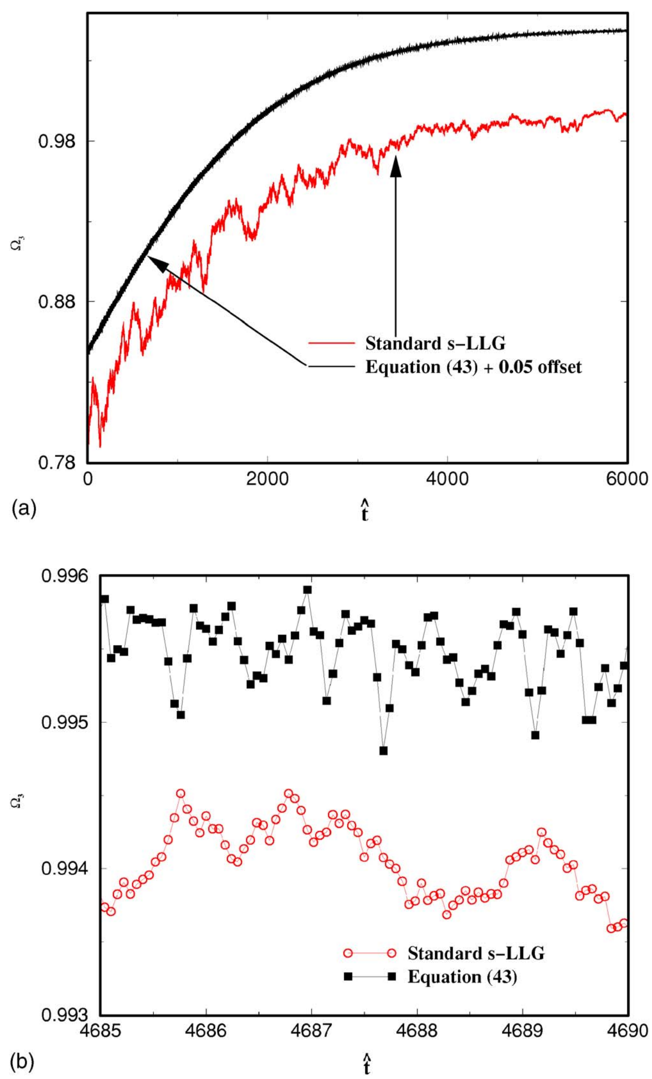


FIG. 4. (Color online) Ω_3 as a function of time obtained integrating the standard s-LLG equation, (45), and Eq. (43). In (a) the trace obtained using Eq. (43) has been offset up by +0.05 for clarity. In (b) the trace of Ω_3 is shown on a short time scale.

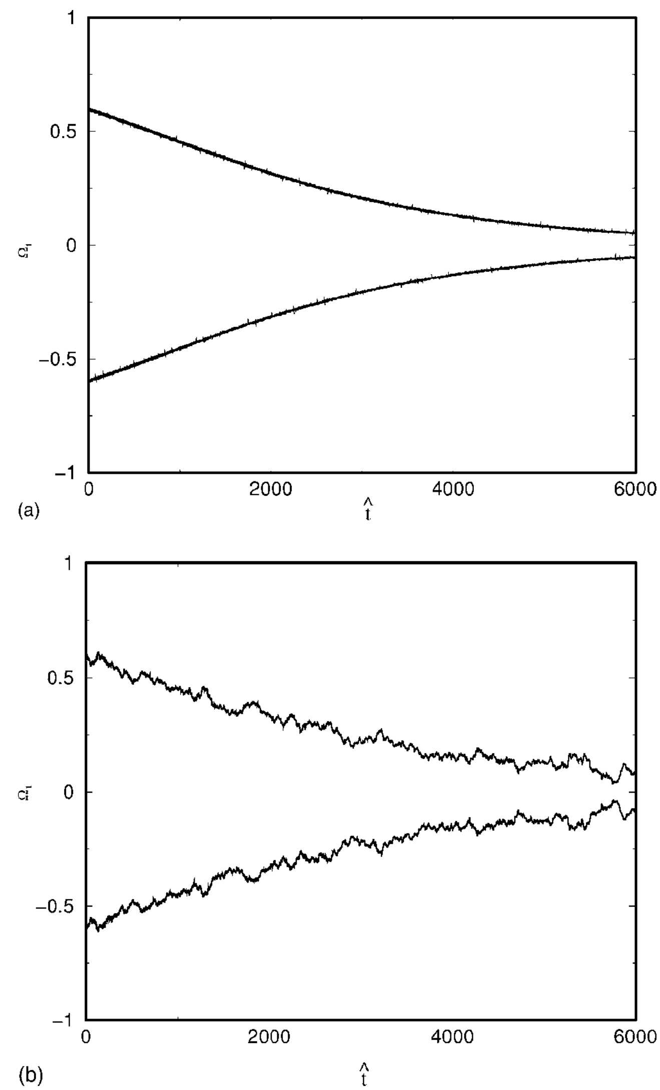


FIG. 5. Envelope curves of the trace of Ω_1 in time as obtained integrating Eq. (43) (a) and Eq. (45) (b). Ω_1 oscillates between the maximum and minimum value given by the envelope curves with frequency γH_{eff}, equal to 1 in the dimensionless units used.

(43). This is due to the different spectral density of the random field. For (45) we simply have $|\bar{h}_j(\omega)|^2 = \alpha \bar{T}$, whereas for (43) $|\bar{h}_j(\omega)|^2$ is equal to $g_j(\omega)$ (considering that for our simulation, to a good approximation, we can neglect the dependence of the random field on $\mathbf{\Omega}$). In particular for (43) $|\bar{h}_j(\omega)|^2$ has a low frequency cutoff at $\omega = \omega_0 \equiv c_{t,M}/L$, where $c_{t,M}$ is the transverse speed of sound in the magnet. This implies that for (43) we have a much lower probability than for (45) to have consecutive fluctuations of the random field with the same sign with the result that the dynamics appears less noisy.

VI. DISCUSSION AND CONCLUSIONS

In this paper we derived the equation for the dynamics of the magnetization taking into account its coupling to the lattice vibrations. The equation that we obtain, (28), is quite general. Equation (28) will have the same form also if we include spin-spin and spin-disorder interactions. To take into account these phenomena it is necessary only to add the appropriate terms to the energy functional $E_S[\mathbf{\Omega}]$.

From the general equation we derived the equation, (32), for the dynamics of the uniform magnetic mode in a thin magnetic film when nonuniform magnetic modes can be assumed frozen out. We find that in general the random field that appears in the dynamical equation for the magnetization has a correlation time, τ_D, of the order of the ratio between the film thickness, h, and the sound velocity c. When the time scale for the dynamics of the magnetization is much longer that τ_D, we recover the stochastic LLG equation. In this limit we calculated the value of the effective Gilbert damping constant, α. For typical ferromagnetic insulators, like YIG, we find $\alpha \approx 10^{-4}$, in good agreement with the values measured in experiments.[16,19] We can then conclude that for magnetic insulators magnetoelastic coupling is the main source of magnetization damping.

Our work predicts that magnetic resonance experiments on ferromagnetic insulators should be able to observe the anisotropy of the damping and as a consequence of the correlation of the thermal fluctuations. With our theory it is possible to exactly calculate the spectral density of the thermal fluctuations. The spectral densities for small samples will be different from the one observed in bulk experiments because of the discreteness of the elastic modes. It would be very interesting to test these results with new experiments on small ferromagnetic insulators samples. In particular for thin films one experimental consequence of our work is that the correlation time of the magnetic fluctuations will be of the order of h/c where h is the thickness of the ferromagnetic film and c the speed of sound in the magnet. We also found that in the limit when the magnetization evolves on time scales much bigger than h/c the damping of the magnetic modes is directly proportional to $B_1^2 h$. The linewidth of the ferromagnetic resonance peak in insulating ferromagnetic thin films should therefore scale as $B_1^2 h$, which, in principle, can be confirmed experimentally.

For ferromagnetic metals, like permalloy, we also find $\alpha \approx 10^{-4}$. This value is about two orders of magnitude smaller than the value observed experimentally.[20] The reason is that in ferromagnetic metals the electronic degrees of freedom are the main source of dissipation for the magnetization.[21,22] Starting from a model of localized d spins exchange coupled to the s-band electron, the interaction Lagrangian will be

$$\mathcal{L}_I = J_{sd} \int d\mathbf{x} \boldsymbol{\Omega}(\mathbf{x}) \cdot \mathbf{s}(\mathbf{x}),$$

where J_{sd} is the exchange coupling constant and $\mathbf{s}$ is the conduction electrons spin density

$$\mathbf{s}(\mathbf{x}) = \frac{1}{2} \sum_{a,b} \Psi_a^\dagger(\mathbf{x}) \boldsymbol{\tau}_{ab} \Psi_b(\mathbf{x}),$$

where Ψ are the s-band carrier field operators and $\boldsymbol{\tau}_{ab}$ the representation of the spin operator in terms of Pauli matrices. By integrating out the s-band degrees of freedom, in the linear response approximation Sinova *et al.*,[23] for the damping of the uniform magnetic mode find

$$\alpha = \lim_{\omega \to 0} \frac{g \mu_B J_{sd}^2}{2 M_s \hbar \omega} \int \frac{d^3 k}{(2\pi)^3} \sum_{a,b} |\langle \psi_a(\mathbf{k}) | \tau^+ | \psi_b(\mathbf{k}) \rangle|^2 \times \int \frac{d\epsilon}{2\pi} A_{a,\mathbf{k}}(\epsilon) A_{b,\mathbf{k}}(\epsilon + \hbar\omega) [f(\epsilon) - f(\epsilon + \hbar\omega)], \quad (47)$$

where $A_{a,\mathbf{k}}(\epsilon)$ and $A_{b,\mathbf{k}}(\epsilon)$ are the spectral functions for s-band quasiparticles and $f(\epsilon)$ is the Fermi-Dirac distribution. Equation (47) gives zero damping unless there is a finite-measure Fermi surface area with spin degeneracy or there is a broadening of the spectral function due to disorder.[24] Characterizing the quasiparticle broadening by a simple number $\Gamma \equiv \hbar/\tau_s$, where τ_s is the quasiparticle lifetime, we can assume

$$A_{a,\mathbf{k}}(\epsilon) = \frac{\Gamma}{(\epsilon - \epsilon_{a,\mathbf{k}})^2 + \Gamma^2/4}. \quad (48)$$

Inserting this expression for the spectral functions in (47) we find α as a function of the phenomenological scattering rate Γ. Notice that (47) includes the contribution both of intraband, and interband[25–27] quasiparticles scattering events. The intraband contribution is due to spin-flip scattering within a spin-split band and is nonzero only when intrinsic spin-orbit coupling is present. From Eq. (47), using the expression for $A_{a,\mathbf{k}}(\epsilon)$ given in (48), we see that in the limit of weak disorder, small Γ, the intraband contribution to α is proportional to $1/\Gamma$, in agreement with experimental results for clean ferromagnetic metals with strong spin-orbit coupling[28–31] and previous theoretical work.[26,27,32–35] Similarly from (47) we see that the interband contribution to α is proportional to Γ. This result agrees with the experimental results for ferromagnetic metals with strong disorder[36] and previous theoretical work.[25–27] Notice that Eq. (47) implicitly also includes the contribution due to the so-called spin-pumping effect[37–41] in which spins are transferred from the ferromagnetic film to adjacent normal metal layers as a consequence of the precession of the magnetization. In order to calculate this effect in first approximation we simply must substitute in (47) the conduction band quasiparticle states, ψ, calculated taking into account the heterogeneity of the sample. Assuming for the scattering rate, $1/\tau_s$, typical values estimated by transport experiments, from Eq. (47) we find values of α in good agreement with experiments.

In summary we have studied in detail the effect of the magnetoelastic coupling to the dynamics of the magnetization. Starting from a realistic form for the magnetoelastic coupling we have found the expression for the damping kernel, γ_{jm}. We find that in general γ_{jm} is a nondiagonal tensor nonlocal in time and space. The knowledge of the exact expression of γ_{jm} allows us to correctly take into account the autocorrelation of the noise term overcoming the zero correlation approximation of the stochastic Landau-Lifshitz-Gilbert equation. We find that for thin films for which the single domain approximation is valid, both the damping and the fluctuations correlation time are proportional to the film thickness. Our results apply to systems for which the direct coupling of the magnetization to the lattice vibrations is the main source of the magnetization relaxation. We have shown that this is the case for ferromagnetic insulators whereas for ferromagnetic metals the magnetization relaxation is mainly due to the s-d exchange coupling.

ACKNOWLEDGMENTS

It is a pleasure to thank Harry Suhl, Thomas J. Silva, Alvaro S. Núñez, and Joaquín Fernández-Rossier for helpful discussions. This work was supported by the Welch Foundation, by the National Science Foundation under Grant Nos. DMR-0115947 and DMR-0210383, and by a grant from Seagate Corporation.

APPENDIX: SIMPLE ESTIMATE OF γ_{eff}

Let us start from the definition of $\bar{\gamma}_{jm}$ [Eq. (31)],

$$\bar{\gamma}_{jm} = \Theta(t-t')A_1\sum_n \frac{1}{\omega_n^2}c_j^{(n)}(t)c_m^{(n)}(t')\cos[\omega_n(t-t')], \tag{A1}$$

where $A_1 \equiv \gamma B_1^2/M_sMV_M$. For the case of thin film we found

$$c_l^{(n)} = L_s^2\sum_i \Delta f_i^{(n)}(\delta_{il}\Omega_3 + \Omega_i\delta_{3l}),$$

where

$$\Delta f_i^{(n)} \equiv f_i^{(n)}(h) - f_i^{(n)}(0).$$

Notice that, by definition, $f_i^{(n)}$ are dimensionless and so are the quantities $\Delta f_i^{(n)}$. Assuming that at equilibrium is $\mathbf{\Omega} = (0,0,1)$ and keeping only the leading terms in $\mathbf{\Omega}$ in the expression for $c_l^{(n)}$ we have

$$c_l^{(n)} = L_s^2\Delta f_l^{(n)}.$$

Let us now expand the collective index n in its components, $\mathbf{k},s$ where s is the polarization index of the elastic modes. Then, using the completeness of the polarization vectors and the fact that the polarization directions are parallel to the axis x_1, x_2, x_3 we have

$$\bar{\gamma}_{jm} = \Theta(t-t')A_1\sum_n \frac{1}{\omega_n^2}c_j^{(n)}(t)c_m^{(n)}(t')\cos[\omega_n(t-t')]$$
$$= \Theta(t-t')A_1L_s^4\sum_{\mathbf{k},s} \frac{1}{\omega_{\mathbf{k},s}^2}\Delta f_j^{\mathbf{k},s}\Delta f_m^{\mathbf{k},s}\cos[\omega_{\mathbf{k},s}(t-t')]$$
$$= \Theta(t-t')A_1L_s^4\sum_{\mathbf{k}} \frac{1}{\omega_{\mathbf{k},j}^2}\Delta f_j^{\mathbf{k}}\Delta f_m^{\mathbf{k}}\delta_{jm}\cos[\omega_{\mathbf{k},j}(t-t')].$$

Now note that

$$M = \rho L_s^2 L(1 + \hat{h} + \hat{L}'), \quad V_M = L_s^2 h,$$

where $\hat{h} \equiv h/L$, $\hat{L}' \equiv L'/L$. Then we can write

$$\bar{\gamma}_{jm} = \Theta(t-t')\frac{\gamma B_1^2}{M_s\rho L(1+\hat{h}+\hat{L}')h} \times \sum_{\mathbf{k}} \frac{(\Delta f_j^{\mathbf{k}})^2}{\omega_{\mathbf{k},j}^2}\delta_{jm}\cos[\omega_{\mathbf{k},j}(t-t')].$$

For small enough h/L we can assume $\Delta f_j^{(k)} \approx kh$ with a cutoff for k_D such that $k_D h = 1$. We can then define the cutoff frequency $\omega_D \equiv ck_D = c/h$. With this approximation we have

$$\sum_{\mathbf{k}} \frac{(\Delta f_j^{\mathbf{k}})^2}{\omega_{\mathbf{k},j}^2}\cos[\omega_{\mathbf{k},j}(t-t')]$$
$$= \sum_{\mathbf{k}} \frac{1}{\omega_{\mathbf{k},j}^2 + \omega_D^2}\cos[\omega_{\mathbf{k},j}(t-t')]$$
$$= \frac{1}{\omega_D^2}\int_0^\infty \delta(\omega - \omega_{\mathbf{k},j})\frac{\omega_D^2}{\omega^2+\omega_D^2}\cos(\omega(t-t'))d\omega$$
$$\approx \frac{1}{\omega_D^2}\frac{1}{\omega_0}\int_0^\infty \frac{\omega_D^2}{\omega^2+\omega_D^2}\cos[\omega(t-t')]d\omega$$
$$= \frac{1}{\omega_D^2}\frac{1}{\omega_0}\omega_D e^{-\omega_D(t-t')},$$

where $\omega_0 \equiv c/L$. In this approximation we can then write

$$\bar{\gamma}_{jm}(\tau) \approx \Theta(t-t')\frac{\gamma B_1^2}{M_s\rho L(1+\hat{h}+\hat{L}')h}\frac{1}{\omega_D^2}\frac{1}{\omega_0}\omega_D e^{-\omega_D\tau}.$$

Integrating this expression between $\tau=0$ and $\tau=\infty$ we find

$$\bar{\gamma}_{\text{eff}} = \frac{\gamma B_1^2}{M_s\rho L(1+\hat{h}+\hat{L}')h}\frac{1}{\omega_D^2}\frac{1}{\omega_0} = \frac{\gamma B_1^2}{M_s\rho L(1+\hat{h}+\hat{L}')h}\frac{h^2}{c^2}\frac{L}{c}$$
$$= \frac{\gamma B_1^2}{M_s\rho(1+\hat{h}+\hat{L}')}\frac{h}{c^3}. \tag{A2}$$

To be more accurate let us define the functions

$$\hat{g}_j(\tau) \equiv \frac{1}{\hat{h}(1+\hat{h}+\hat{L}')}\frac{c^2}{L^2}\sum_{\mathbf{k}} \frac{(\Delta f_j^{\mathbf{k}})^2}{\omega_{\mathbf{k},j}^2}\cos[\omega_{\mathbf{k},j}(\tau)]$$

so that we can write

$$\bar{\gamma}_{jm} = \Theta(\tau)\frac{\gamma B_1^2}{M_s\rho L^2}\frac{L^2}{c^2}\hat{g}_j(\tau).$$

The functions $\hat{g}_j(\tau)$ are plotted in Fig. 2. Integrating $\hat{g}_j(\tau)$ between 0 and ∞ we find

$$\eta \equiv \int_0^\infty \hat{g}_j(\tau)d\tau \approx \frac{h}{c}$$

and then finally

$$\bar{\gamma}_{\text{eff}} = \frac{\gamma B_1^2}{M_s\rho c^2}\frac{h}{c},$$

analogously to what we found previously (A2).

[1] N. Smith, Appl. Phys. Lett. **78**, 1448 (2001).
[2] O. Heinonen, IEEE Trans. Magn. **38**, 10 (2002).
[3] O. Heinonen and H. S. Cho, IEEE Trans. Magn. **40**, 2227 (2004).
[4] W. F. Brown, Phys. Rev. **130**, 1677 (1963).
[5] D. A. Garanin, Phys. Rev. B **55**, 3050 (1997).
[6] N. Smith, J. Appl. Phys. **90**, 5768 (2001).
[7] J. L. García-Palacios and F. J. Lazaro, Phys. Rev. B **58**, 14937 (1998).
[8] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).
[9] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999).

[10] A. Rebei and G. J. Parker, Phys. Rev. B **67**, 104434 (2003).
[11] H. Suhl, IEEE Trans. Magn. **34**, 1834 (1998).
[12] C. Kittel, Rev. Mod. Phys. **21**, 541 (1949).
[13] E. W. Lee, Rep. Prog. Phys. **18**, 184 (1955).
[14] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon, New York, 1986).
[15] P. Hanggi, in *Stochastic Dynamics*, edited by L. Schimansky-Geier and T. Pöschel, Lect. Notes Phys. Vol. 484 (Springer, New York, 1997).
[16] M. Sparks, *Ferromagnetic Relaxation Theory* (McGraw-Hill, New York, 1964).
[17] R. M. Bozorth, *Ferromagnetism* (Van Nostrand, New York, 1951).
[18] C. W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry, and the Natural Sciences* (Springer-Verlag, New York, 1990).
[19] H. Chen, P. De Gasperis, and R. Marcelli, IEEE Trans. Magn. **29**, 3013 (1993).
[20] M. Covington, T. M. Crawford, and G. J. Parker, Phys. Rev. Lett. **89**, 237202 (2002).
[21] C. Kittel and A. H. Mitchell, Phys. Rev. **101**, 1611 (1956).
[22] E. A. Turov, in *Ferromagnetic Resonance*, edited by S. V. Vonsovskii (Pergamon, Oxford, U.K., 1966).
[23] J. Sinova, T. Jungwirth, X. Liu, Y. Sasaki, J. K. Furdyna, W. A. Atkinson, and A. H. MacDonald, Phys. Rev. B **69**, 085209 (2004).
[24] Y. Tserkovnyak, G. A. Fiete, and B. I. Halperin, Appl. Phys. Lett. **84**, 5234 (2004).
[25] B. Heinrich, D. Fraitová, and V. Kamberský, Phys. Status Solidi **23**, 501 (1967).
[26] V. Kamberský, Can. J. Phys. **48**, 2906 (1970).
[27] L. Berger, J. Phys. Chem. Solids **38**, 1321 (1977).
[28] S. M. Bhagat and L. L. Hirst, Phys. Rev. **151**, 401 (1966).
[29] S. M. Bhagat and P. Lubitz, Phys. Rev. B **10**, 179 (1974).
[30] B. Heinrich, D. J. Meredith, and J. F. Cochran, J. Appl. Phys. **50**, 7726 (1979).
[31] J. M. Rudd, J. F. Myrtle, J. F. Cochran, and B. Heinrich, J. Appl. Phys. **57**, 3693 (1985).
[32] V. Korenman and R. E. Prange, Phys. Rev. B **6**, 2769 (1972).
[33] V. Korenman, Phys. Rev. B **9**, 3147 (1974).
[34] J. Kunes and V. Kamberský, Phys. Rev. B **65**, 212411 (2002).
[35] J. Kunes and V. Kamberský, Phys. Rev. B **68**, 019901 (2003).
[36] S. Ingvarsson, L. Ritchie, X. Y. Liu, G. Xiao, J. C. Slonczewski, P. L. Trouilloud, and R. H. Koch, Phys. Rev. B **66**, 214416 (2002).
[37] L. Berger, Phys. Rev. B **54**, 9353 (1996).
[38] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, Phys. Rev. Lett. **88**, 117601 (2002).
[39] E. Simanek and B. Heinrich, Phys. Rev. B **67**, 144418 (2003).
[40] E. Simanek, Phys. Rev. B **68**, 224403 (2003).
[41] A. Rebei and M. Simionato, cond-mat/0412510 (unpublished).