

# Induced superconducting pairing in integer quantum Hall edge states

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Indium Arsenide (InAs) near surface quantum wells (QWs) are ideal for the fabrication of semiconductor-superconductor heterostructures given that they allow for a strong hybridization between the two-dimensional states in the quantum well and the ones in the superconductor. In this work we present results for InAs QWs in the quantum Hall regime placed in proximity of superconducting NbTiN. We observe a negative downstream resistance with a corresponding reduction of Hall (upstream) resistance. We analyze the experimental data using the Landauer-Büttiker formalism, generalized to allow for Andreev reflection processes. Our analysis is consistent with a lower-bound for the averaged Andreev conversion of about 15%. We attribute the high efficiency of Andreev conversion in our devices to the large transparency of the InAs/NbTiN interface and the consequent strong hybridization of the QH edge modes with the states in the superconductor.

Anyons with non-Abelian statistics are of great fundamental interest [1] and can be used to realize topologically protected, and therefore intrinsically fault-tolerant, qubits [2–4]. Non-Abelian anyons are expected to be realized in few fractional quantum Hall (QH) states [5–9] such as the QH states with filling factor  $\nu = \frac{5}{2}$  [10–12], and, possibly,  $\nu = \frac{12}{5}$  [13]. However, so far, no unambiguous experimental confirmation exists of the presence of non-Abelian anyons in such QH states. An alternative route to realize non-Abelian anyons relies on inducing superconducting pairing between counter-propagating edge modes of QH states that, intrinsically, support only Abelian anyons [14–17]. These theoretical proposals build on an earlier proposal for creating Majorana zero modes, the anyons with the simplest non-Abelian statistics, using 1D modes at the edge of a 2D topological insulator (TI) in contact with a superconductor (SC) [18]. In contrast to TIs, in two-dimensional electron gases (2DEGs) in the QH regime, by varying filling factor  $\nu$ , states can be realized with a variety of topological orders. This allows access to more exotic edge states needed for engineering anyons with richer non-Abelian statistics. Key in all these theoretical proposals is the ability to induce superconducting pairing, via the proximity effect, between the QH edge modes.

The strength of the superconducting correlations that can be induced in a QH-SC heterojunction can be evaluated by obtaining the amplitude of the Andreev reflection of QH edge modes. The early search for Andreev reflection in QH-SC systems focused on InAs and InGaAs semiconductors magneto-resistance oscillations at relatively low magnetic fields [19] followed later by reports of induced su-

perconductivity in QH states [20]. More recently there has been reports on observation of induced superconductivity [21, 22], cross Andreev conversion [23, 24], edge state mediated supercurrent [25] and interference of chiral Andreev edge states [26, 27] in graphene. To make further progress toward the realization of fault-tolerant quantum bits based on QH states it is essential to demonstrate the ability to induce reliably robust superconducting correlations into the edge modes of a QH state, to understand and quantify, in realistic conditions, the microscopic processes at the interface of a QH state and a superconductor, and to maximize the strength of the superconducting correlations that can be induced into the QH edge modes. In this work we show that in high quality InAs/NbTiN heterostructures grown epitaxially very strong superconducting correlations can be induced in the edge modes of integer QH states realized in the InAs-based quantum wells (QWs). We analyze the experimental data in conjunction with a microscopic model to extract the details of the processes determining the transport properties of the QH-SC interface.

Fig.1(a) shows a cross sectional schematic of the epitaxially grown, via molecular beam epitaxy (MBE), structure used in this work. The QW is formed by a 4 nm layer of  $\text{In}_{0.81}\text{Ga}_{0.19}\text{As}$  layer, a 7 nm layer of InAs, and a 10 nm top layer of  $\text{In}_{0.81}\text{Ga}_{0.19}\text{As}$ . The QW is grown on  $\text{In}_x\text{Al}_{1-x}\text{As}$  buffer where the indium content is step-graded from  $x = 0.52$  to 0.81. A delta-doped Si layer with electron doping  $n \sim 1 \times 10^{12} \text{ cm}^{-2}$  is placed 6 nm below the QW. This epitaxial structure has been used in previous studies on mesoscopic superconductivity [28–31], in the development of tunable qubits [32],

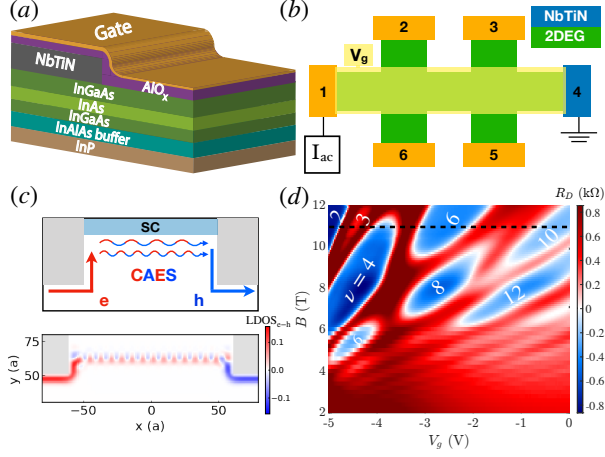


FIG. 1. (a) Schematic of gated NbTiN/InAs hybrid device structure. (b) Device pin-out configuration. Contacts 1,2,3,5, and 6 are normal, contact 4 is superconducting. Contacts 1 and 4 are used as the source and drain, respectively. (c) Andreev conversion via CAES interference along the QH-SC interface (top) and a supporting tight binding calculation of the difference between the electron and hole LDOS ( $LDOS_{e-h}$ ) (bottom). (d) Measured  $R_D$  as a function of  $V_g$  and  $B$ . IQHSs are labeled from complementary  $R_{xy}$  data. The dashed line shows the position of the cut shown in Fig. 2 (a).

and in studies aimed at realizing and detecting topological superconducting states [33–35]. A Hall bar, Fig.1(b), is fabricated by electron beam lithography. In order to study the 2DEG/SC interface, a 90-nm-thick layer of NbTiN (with an upper critical field of  $\sim 28$  T and a critical temperature of  $\sim 13.6$  K) was sputtered as one of the contacts with a  $150\mu m$ -long interface. A metallic top gate is created by depositing a layer of Al oxide followed by an Al layer, to control the QW electron density [36]. The mobility of the QW is determined to be  $\mu \sim 12,000$   $cm^2/Vs$  at  $n \sim 8.51 \times 10^{11} cm^{-2}$  corresponding to an electron mean free path of  $l_e \sim 180$  nm. All data reported here were taken at  $T \sim 30$  mK. We have provided more information on transport properties of the sample in SI.

When the sample is placed in magnetic field, in the classical picture, electron and hole will alternate their skipping orbits across the interface of superconductor and 2DEG [37]. In the full quantum-mechanical analysis the electron and hole edge states hybridize due to the proximity of the SC and form a coherent chiral Andreev edge state (CAES) extended along the QH-SC interface [27, 38, 39]. A schematic of CAES propagation along the QH-SC interface is shown in Fig. 1 (c). In this picture, if more holes

than electrons reach the normal lead downstream from the superconducting electrode (lead 5), then a negative potential difference ( $V_5 - V_4$ ) develops. In Fig. 1 (c) we also show the local density of states of a CAES obtained with a tight binding (TB) calculation performed using the package Kwant [40]. In the TB model the presence of the magnetic field is taken into account via a Peierls phase, and the superconductivity of the QW proximitized by NbTiN via a mean field Bogoliubov de-Gennes (BdG) Hamiltonian with an s-wave pairing term of strength  $\hat{\Delta}$ . The details of the TB model can be found in the SI.

Figure 1(d) shows the results for the downstream resistance,  $R_D$ , measured between the voltage contacts 5 and 4 shown in Fig. 1(b), as a function of gate voltage ( $V_g$ ) and magnetic field ( $B$ ). Hall resistance data measured between contacts 2 and 6 of Fig. 1(b) allow us to determine the filling factor of the different regions of Fig. 1(d). Fig. 2(a) shows the horizontal cut at  $B = 11$  T of Fig. 1(d) and the corresponding longitudinal resistance  $R_{xx}$ . From the  $R_{xx}$  measurements we see that we have well developed integer QH states (IQHS). From Figs. 1(d) and 2(a) we clearly observe that  $|R_D|$  is negative for IQHS, a fact that strongly suggests the presence of Andreev

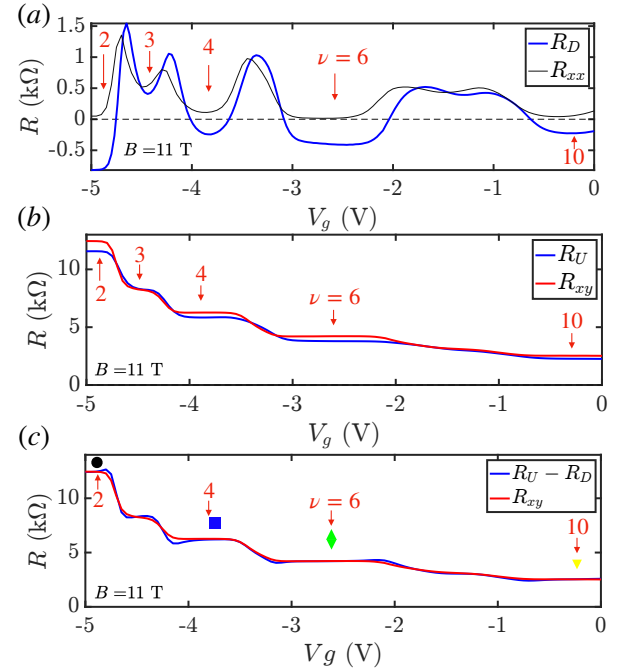


FIG. 2. (a)  $R_D$  and  $R_{xx}$ , (b)  $R_U$  and  $R_{xy}$ , (c)  $R_U - R_D$  and  $R_{xy}$  shown as a function of  $V_g$ . All traces taken at  $B = 11$  T. IQHSs are labeled and markers are shown for the states used in Fig.3(c).

processes at the QH-SC interface for these IQHS.

Remarkably, the upstream resistance  $R_U$  (measured between contacts 3 and 4 of Fig. 1 (a)) exhibits plateaus in correspondence of the  $R_{xy}$  plateaus in magnetic field but with resistance values lower than  $R_{xy}$ . Moreover,  $R_U - R_D$  recovers the quantized Hall value,  $R_{xy}$ , as shown in Fig. 2 (c). Note that this difference does not necessarily match the  $R_{xy}$  data outside the QH regime.

To understand the results shown in Fig. 1 (d) and Fig. 2 we have developed a Landauer-Büttiker (LB) theory of edge state transport taking into account the presence of the superconducting lead by allowing for electron-hole conversion. We start by the six-terminal setup shown in Fig. 1(b). Assuming thermal equilibrium, floating leads (2, 3, 5 and 6) will be in equilibrium with the edge states and therefore there are only four distinct edge state chemical potentials:  $V_1, V_3 = V_2, V_5 = V_6$ , and  $V_4$ ; allowing us to describe the Hall bar as a four-terminal system. The LB relation between the chemical potentials and the current of the 4 distinct leads, for the IQHS with filling factor  $\nu$ , is

$$\begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix} = \frac{\nu}{R_H} \begin{pmatrix} 1 & 0 & 0 & -1 \\ -\alpha & \alpha & 0 & 0 \\ \alpha - 1 & -\alpha & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} V_3 \\ V_4 \\ V_5 \\ V_1 \end{pmatrix}. \quad (1)$$

The details of the derivation of Eq. (1) are given in the SI. In Eq. (1)  $R_H = h/e^2$  and  $\alpha = 2A + T$  with  $A$  the probability of electron-hole conversion due to Andreev processes and  $T$  the probability for a single electron to tunnel into the SC per mode. Quasiparticle conservation requires  $A + T + T_{\text{edge}} = 1$ , with  $T_{\text{edge}}$  the probability of an incident electron to scatter to the downstream edge. It is important to note that due to the lack of quasiparticle charge conservation at the QH-SC interface the measurement of the currents and voltages that appear in the LB Eq. (1) does not allow the determination of all the transport coefficients. Therefore, for the systems considered, the experimental transport measurements, combined with LB theory, only allow us to place constraints on the values of the transport coefficients, such as  $A$ .

In our experiment, we have  $I_1 = I$  and  $I_4 = -I$ . In this case, from Eq. (1), we obtain

$$R_U = \frac{V_3 - V_4}{I} = \frac{R_H}{\nu} \frac{1}{\alpha} \quad (2a)$$

$$R_D = \frac{V_5 - V_4}{I} = \frac{R_H}{\nu} \left( \frac{1}{\alpha} - 1 \right) \quad (2b)$$

Figure 3 (a) shows the scaling of  $R_D$  with respect to  $1/\nu$  for different values of  $B$ . From the slope of the fits to the experimental data shown in Fig. 3 (a) we obtain the value of  $\alpha$ . Knowing  $\alpha$  we can obtain the lower bound  $A_\ell$  for  $A$ . This is due to the fact that we can write  $A = \alpha - 1 + T_{\text{edge}}$  and that the smallest value that  $T_{\text{edge}}$  can take is zero.  $A_\ell$  extracted from the measurements of  $R_D$  and  $R_U$  is shown in Fig. 3 (b) as a function of  $B$ . We see that  $A_\ell$  is close to 15% and weakly dependent on  $B$ . This is a remarkable result as it shows that in our devices, even in the worst case scenario, very large superconducting correlations are induced in the QH edge modes. Figure 3 (c) shows the consistency of the measured values of  $R_D$  and  $R_U$  with the LB predictions by plotting the ratio  $(R_U - R_D)/R_{xy}$  as function of  $B$  that according to Eqs. (2a-2b) is expected to be equal to 1.

An estimate of  $A$  could be obtained using a microscopic model. We can calculate the dispersion of the CAES modes, shown in Fig. 4 (a), using a TB model. This helps us finding the velocity,  $v_d$ , and Fermi wave vector  $k_F$ , of such modes, by taking into account the renormalization due to the proximity of the SC. The knowledge of  $v_d$  and  $k_F$  allows

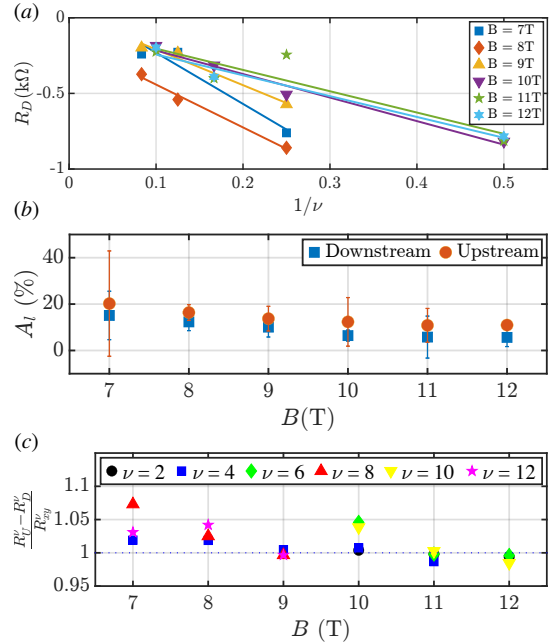


FIG. 3. (a)  $R_D$  vs  $1/\nu$  for different IQHSs and values of  $B$ , and linear fits corresponding to each magnetic field. (b)  $A_\ell$  obtained from the slope of linear fits to  $R_D$  and  $R_U$  data vs  $1/\nu$  with their corresponding error bars. (c)  $(R_U - R_D)/R_{xy}$  for different  $\nu$ s and values of  $B$ .

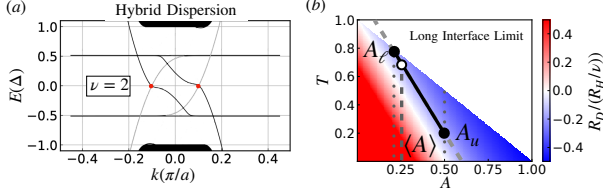


FIG. 4. (a) BdG spectrum for a hybrid QH-SC structure showing CAES (black) and vacuum edge states (gray). Trivial states corresponding to higher LL's have been removed from the spectrum for clarity. (b) Colormap of  $R_D$  calculated from Eq. (2b) illustrating the upper and lower bounds of  $A$  for  $\alpha = 1.2$ .

us to build a one-dimensional (1D) effective BdG Hamiltonian  $H_{BdG}$  for the 1D chiral edge modes in the presence of superconducting pairing. We can assume the edge modes to be spin-degenerate so that  $H_{BdG} = \int dx \psi^\dagger(x)\mathcal{H}(x)\psi(x)$  with  $\mathcal{H}(x) = v_d(-i\partial_x)\tau_0 - v_d k_F \tau_z + \tilde{\Delta}\tau_x$  where  $\tau_i$  are Pauli matrices in Nambu space, and  $\psi = (c_{x\uparrow}, c_{x\downarrow}^\dagger)$  is the spinor formed by the annihilation (creation) operator for a fermion at position  $x$  and spin up (down).

Using  $H_{BdG}$  we obtain the transfer matrix describing the propagation of the CAES [38, 41] and then the expression for the electron-hole conversion probability:  $A = \sin^2(\delta\phi)/(1 + (v_d k_F/\tilde{\Delta})^2)$  where  $\delta\phi = \frac{L_{sc}}{v_d} \sqrt{\tilde{\Delta}^2 + (v_d k_F)^2}$  is the phase difference between coupled electron-like and hole-like CAES's accumulated along the QH-SC interface of length  $L_{sc}$ . This result shows that the value of  $A$  oscillates with  $L_{sc}$ . This is a consequence of the fact that no decoherence processes are taken into account. Imperfections of the QH-SC along the long interface, and the presence of vortices, can cause decoherence processes that result in stochastic modulations of the phase  $\delta\phi$ . The effect of decoherence can be taken into account by calculating the *interface-averaged* electron-hole conversion probability:  $\langle A \rangle_{L_{sc}} = v_d(1 - \text{sinc}^2(2\delta\phi))/2L_{sc}\delta\phi^2$ . The effect of the stochastic modulations in the phase accumulation can be taken into account by calculating the *interface-averaged* electron-hole conversion probability:  $\langle A \rangle_{L_{sc}} = v_d(1 - \text{sinc}^2(2\delta\phi))/2L_{sc}\delta\phi^2$ . From this equation we observe that the upper bound for  $\langle A \rangle$  is  $1/2$ . Considering that in our device  $L_{sc} = 150\mu\text{m}$ , we can take the limit in which  $L_{sc}$  is very large for which  $\langle A \rangle = 1/[2(1 + (v_d k_F/\tilde{\Delta})^2)]$ . From the results of the TB calculation we have that  $v_d k_F/\tilde{\Delta} \approx 1$ . We then estimate that in our device  $\langle A \rangle \approx 1/4$ .

Figure 4 (b) show the value of  $\nu R_D/R_H$  as function of  $A$  and  $T$  as obtained from the LB theory. The negative diagonal connecting the (0,1) and (1,0) points corresponds to the  $T_{\text{edge}} = 0$  boundary: points to the right of this diagonal correspond to negative values of  $T_{\text{edge}}$  and are therefore not physical. The solid line shows the segment corresponding to the value of  $\alpha$  extracted from our measurements. The dotted lines identify the lower and upper bound for  $A$ . The dashed line shows the estimate of  $\langle A \rangle$  obtained from the microscopic model. The intersection of this line with the solid line allows one to obtain the corresponding value of  $T$ . At  $B = 7$  T,  $\alpha \approx 1.2$  which implies  $T = 0.7$  and  $T_{\text{edge}} = 0.05$  if  $\langle A \rangle = 1/4$ .

In conclusion, we have fabricated a QH-SC epitaxial heterostructure based on InAs and NbTiN and characterized the transport properties of its QH edge modes propagating along a superconducting interface. We have obtained negative values for the downstream resistance  $R_D$  between a normal lead and the superconducting lead. The negative values of  $R_D$  are an unambiguous sign that at the QH-SC interface there is a very large electron-hole conversion probability,  $A$ . Using only a Landauer-Büttiker analysis we were able to show that our transport measurement imply a lower bound for  $A$  of the order of 15%, weakly dependent on the value of the magnetic field ( $A \approx 20\%$  for  $B = 7$  T,  $A \approx 10\%$  for  $B = 12$  T). Using a microscopic model, and taking into account the presence of processes causing the loss of decoherence along the long QH-SC interface, the value of  $A$  to be  $\sim 25\%$ . Even the lower bound values of  $A$  that we measure are remarkable, larger than any published results for QH-SC devices. This shows that in our InAs devices very strong superconducting correlations can be induced into the QH edge modes, an essential prerequisite to use QH-SC heterojunctions to realize non-Abelian anyons and topologically protected qubits and quantum gates based on such unusual quantum states.

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