

# Effect of inhomogeneties and substrate on the dynamics of the metal-insulator transition in VO<sub>2</sub> thin films

M. Rodriguez-Vega,<sup>1</sup> M. T. Simons,<sup>1</sup> E. Radue,<sup>1</sup> S. Kittiwatanakul,<sup>2</sup>  
J. Lu,<sup>2</sup> S. A. Wolf,<sup>3,2</sup> R. A. Lukaszew,<sup>1</sup> I. Novikova,<sup>1</sup> and E. Rossi<sup>1</sup>

<sup>1</sup>*Department of Physics College of William and Mary Williamsburg VA, 23187*

<sup>2</sup>*Department of Material Science University of Virginia Charlottesville VA, 22904*

<sup>3</sup>*Department of Physics University of Virginia Charlottesville VA, 22904*

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We study the thermal relaxation dynamics of VO<sub>2</sub> films after the ultrafast photo-induced metal-insulator transition for two VO<sub>2</sub> film samples grown on Al<sub>2</sub>O<sub>3</sub> and TiO<sub>2</sub> substrates. We find two orders of magnitude difference in the recovery time (a few ns for the VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> sample vs. hundreds of ns for the VO<sub>2</sub>/TiO<sub>2</sub> sample). We present a theoretical model that accurately describes the MIT thermal properties and interpret the experimental measurements. We obtain quantitative results that show how the microstructure of the VO<sub>2</sub> film and the thermal conductivity of the interface between the VO<sub>2</sub> film and the substrate affect long time-scale recovery dynamics. We also obtain a simple analytic relationship between the recovery time-scale and some of the film parameters.

## I. INTRODUCTION

Vanadium dioxide (VO<sub>2</sub>) undergoes a metal-insulator transition (MIT) around room temperature<sup>1-3</sup>, enabling a wide range of potential applications. It has recently been demonstrated that the MIT can also be induced electrically and optically. In fact, great progress has been made in this direction in recent years: experiments have shown that is possible to photo-induce the MIT in VO<sub>2</sub> in the sub-picosecond timescale<sup>4-10</sup>. This finding makes VO<sub>2</sub> a material of great interest for electronic and photonic applications, such as ultra-fast switches or transistors. The realization of VO<sub>2</sub>-based switches requires the ability to control the VO<sub>2</sub> MIT dynamics using external fields, as well as a better understanding of the recovery mechanisms after the external field is turned off and the material returns to its normal state. The mechanism by which the photo-induced MIT takes place in VO<sub>2</sub> is still not fully understood due to the complexity of the electronic behavior of VO<sub>2</sub> arising from the presence of strong electron-lattice coupling and electron-electron interactions. As a result, VO<sub>2</sub> is a unique material of great fundamental and practical interest.

At low temperatures ( $T \lesssim 340$  K) the VO<sub>2</sub> lattice has a monoclinic structure, whereas at high temperatures ( $T \gtrsim 340$  K) it has a tetragonal structure. This difference in lattice structure is reflected in the band structure: VO<sub>2</sub> is an insulator in the monoclinic phase and a metal in the tetragonal phase. This simple picture is complicated by the fact that in VO<sub>2</sub> electron-electron correlations are very strong and can provide an important contribution to the localization of the electronic states via the Mott mechanism<sup>11-13</sup>. It appears that a full account of the MIT must take into account the interplay of the lattice dynamics and the electron dynamics driven by strong electron-electron correlations. This is a fascinating and extremely challenging problem that in addition is complicated by the unavoidable presence of inhomogeneities<sup>14</sup>, and the short timescale dynamics have been investigated

in previous reports<sup>15-23</sup>.

In this work, using a combined theoretical and experimental approach, we focus on the role played by inhomogeneities and the substrate in determining the relaxation dynamics of the photo-induced MIT in VO<sub>2</sub> films. Using a pump-probe experimental arrangement, we investigated the time evolution of the optical properties after a photo-induced MIT in VO<sub>2</sub> films grown under identical conditions on two different substrates, sapphire (Al<sub>2</sub>O<sub>3</sub>) and rutile (TiO<sub>2</sub>). We found more than two orders of magnitude difference in the thermal relaxation rate, as shown in Fig. 1. We show that such a significant difference cannot be explained solely by the differences in the substrate thermal conductivity, and requires taking into account both structural differences in the two films and the thermal properties of the interface between the film and the substrate.

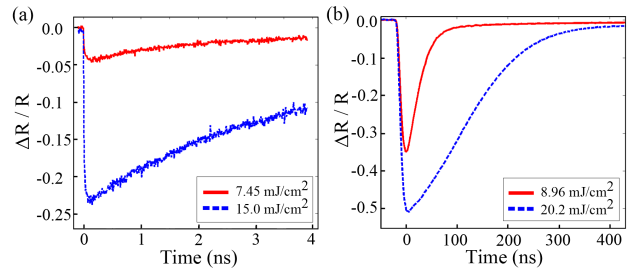


FIG. 1. Relative change in reflectivity ( $\Delta R/R$ ) for the VO<sub>2</sub> film on (a) Al<sub>2</sub>O<sub>3</sub> substrate and (b) TiO<sub>2</sub> substrate as a function of time after the MIT is induced at time  $t = 0$  by a strong ultrafast pump pulse. The values of the pump fluence are shown in the legend, and the sample temperature is set to 311 K in (a) and 280 K in (b), which correspond to approximately 30 K below the critical temperature  $T_c$  for thermally-induced MIT for each sample.

We present a theoretical model that illustrates that both spatial inhomogeneities in the VO<sub>2</sub> films and the thermal properties of the film-substrate interface play an

important role in the thermal relaxation dynamics. We also demonstrate that such model is capable of describing the experimental results with good accuracy in a wide range of the pump laser fluences and sample temperatures. Thus, our experimental and theoretical findings are directly relevant to understanding the effect of inhomogeneities and different substrates and interfaces in the physics of the VO<sub>2</sub> MIT. In addition, our work is also relevant to the more general problem of how spatial inhomogeneities affect a first order phase transition. The ability of our treatment to contribute to this general problem relies on the fact that in VO<sub>2</sub> the two phases across the first-order transition have very different electronic properties (metallic vs. insulating behavior) that allows us to get an accurate phase mapping, via optical reflectivity measurement, of the time evolution of the metallic (insulating) fraction and, indirectly, of the spatial inhomogeneities present during the transition.

The work is organized as follows. Sec. II describes the experimental arrangements to measure the optical reflectivity time-evolution. The details of the theoretical model that we use to characterize the distribution of the films's inhomogeneities using thermally induced MIT and the long-time dynamics of the reflectivity after a photo-induced MIT are presented in Secs. III and IV, respectively. In Sec. V we demonstrate how the variations in statistical properties of the two films result in significant difference in the relaxation timescales, and in Sec. VI we provide our conclusions.

## II. EXPERIMENTAL SETUP

In our experiments we studied two VO<sub>2</sub> thin-film samples, both of which were produced using reactive-bias target ion beam deposition (RBTIBD)<sup>24</sup>. One sample was grown on 0.5 mm thick c-Al<sub>2</sub>O<sub>3</sub>, and the thickness of the VO<sub>2</sub> film was 80 nm. The other sample was grown on a 0.5 mm thick TiO<sub>2</sub> (011) substrate, and was measured to be 110 nm thick. XRD evaluation of both films showed them to be crystalline, and detailed characterization information is available in previous reports<sup>25,26</sup>.

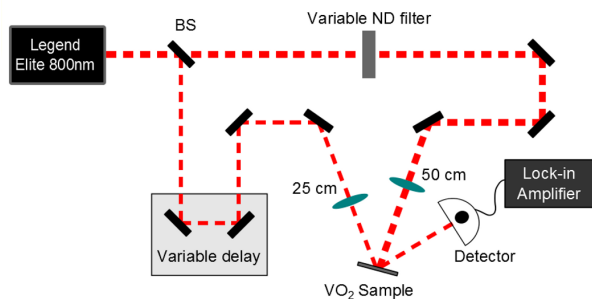


FIG. 2. Schematic of the ultrafast pump-probe setup. BS is an 80/20 beam splitter.

For the photoinduced MIT experiments we used an

ultrafast laser system (Coherent Mantis oscillator and Legend Elite regenerative amplifier) with approximately 100 fs pulses with a central wavelength at 800 nm and a repetition rate of 1 kHz. The properly attenuated output of the laser was split into strong pump pulses and weaker probe pulses using a beam splitter in a standard pump-probe configuration, shown in Fig. 2. The more powerful pump beam, focused to a 180  $\mu\text{m}$  diameter spot on the surface of the sample, was used to induce the MIT, and its fluence was controlled using a variable neutral-density filter (VF). The fluence of the probe beam was further attenuated to a value well below the MIT threshold ( $\phi_{\text{probe}} \leq 100 \mu\text{J}/\text{cm}^2$ ), and we used its reflectivity from the sample to monitor the instantaneous optical properties of the VO<sub>2</sub> film. The probe pulses were directed along a variable delay stage to accurately control the relative timing between the pump and probe pulses by up to 4 ns with a few fs precision. The probe beam was focused on the sample at the same spot as the pump beam, using a shorter focal length lens. When tuned to the center of the pump beam focal spot, the smaller probe beam diameter (90  $\mu\text{m}$ ) ensured to probe a region of uniform pump intensity.

The reflected probe power was measured using a silicon photodetector, and further analyzed using a lock-in amplifier. To minimize the effects of probe pulse instabilities, as well as long-term drifts due to environmental changes, we report the relative change in probe reflection  $\Delta R/R$  with the pump beam on or off.

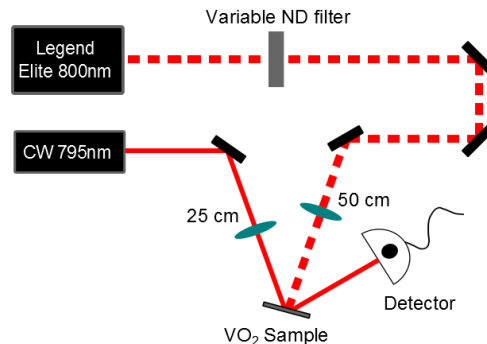


FIG. 3. Schematic of the experimental setup using a continuous-wave probe laser.

Notably the MIT relaxation of the VO<sub>2</sub>/TiO<sub>2</sub> sample was not measurable with the femtosecond probe, as its characteristic decay time exceeded the 4 ns maximum pulse separation, determined by the length of the delay stage. To measure the relaxation of the metallic VO<sub>2</sub> grown on the rutile sample we modified our experimental setup by replacing the femtosecond probe pulses with a continuous-wave (CW) diode laser operating at 785 nm and a fast photodiode (measured response time of approximately 10 ns), as shown in Fig. 3. This detection method allowed us to measure changes in reflectivity for times longer than  $\approx 20$  ns after MIT, that were inacces-

sible with the femtosecond probe arrangement.

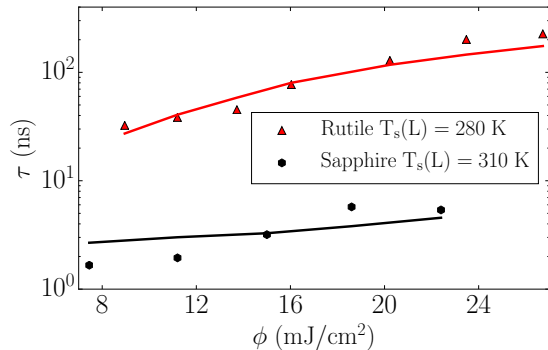


FIG. 4. Dependence of metal state decay constant  $\tau$  on the laser pump fluence and substrate temperature. Dots represent experimental data, and lines corresponds to the results of the theoretical calculations. The initial temperature  $T_s$  for both sample substrates was approximately 30 K below their respective MIT critical temperatures.

Figure 1 shows sample measurements of both the  $\text{VO}_2/\text{Al}_2\text{O}_3$  and  $\text{VO}_2/\text{TiO}_2$  films, using the femtosecond and CW probe arrangements respectively. The analysis of the relative reflectivity for both  $\text{VO}_2$  samples demonstrate that after the initial rapid change during the ultrafast MIT, its time evolution during relaxation is well fitted by a single exponential function with a recovery time constant  $\tau$ :

$$R_{\text{fit}}(t) = A(1 - e^{-t/\tau}). \quad (1)$$

The results of such measurements are shown in Fig. 4: for  $\text{VO}_2/\text{Al}_2\text{O}_3$  films we obtained values of  $\tau$  of the order of few nanoseconds, whereas it took the  $\text{VO}_2/\text{TiO}_2$  sample a few hundred nanoseconds to relax back to the insulating state. This two orders of magnitude difference in the recovery times was even more surprising considering that the characteristic times for the transition itself were quite similar, as demonstrated in previous studies<sup>15</sup>. In the discussion below we demonstrate that the relaxation dynamics strongly depend on the microstructure of the  $\text{VO}_2$  films which in turn is strongly influenced by the properties of the substrate and their interface.

Figure 4 also reveals that the rate of thermal relaxation for both samples increases with higher pump power. This is an intuitively expected trend: since there is a significant light absorption in the  $\text{VO}_2$  thin films, more energy must be dissipated for the case of a more powerful pump pulse. Moreover, for the higher pump powers we observed deviation of the measured reflectivity from a simple exponential recovery. The observed “flattening” of the curves is due to the pump pulse heating the sample to a temperature above the threshold value for the thermally-induced MIT. In this case reflectivity stays unchanged at the level corresponding to a fully metallic phase until the sample cools down to the transition temperature. For all experimental curves only the later exponential part of the

measured reflectivity was included into the fitting thermal relaxation time analysis.

### III. THEORETICAL MODELING OF NON-HOMOGENEITY IN A $\text{VO}_2$ THIN FILM

In order to take into account the effect of the inhomogeneities on the MIT dynamics the first step it to characterize them. Theoretical and experimental evidence<sup>27</sup> indicates that for thin films the probability distribution  $P(D)$  of the grain size  $D$  typically follows a log-normal distribution

$$P(D) = \frac{1}{\sqrt{2\pi}\sigma D} \exp \left[ -\frac{[\ln D/\hat{D}]^2}{2\sigma^2} \right]. \quad (2)$$

In Eq. (2)  $D$  is the effective diameter of a grain assumed spherical,  $\hat{D}$  is the grain size (diameter) such that  $\ln \hat{D} = \langle \ln D \rangle$ , where angle brackets denote averaged quantities, and  $\sigma$  is the standard deviation of  $\ln(D)$ . We note that we have also considered the case of non-spherical grains, like ellipsoids or cylinder; these cases, however, require introduction of additional fitting parameters that are not well constrained experimentally and therefore weaken the strength of the theoretical model.

Direct surface imaging experiments of the  $\text{VO}_2$  films provides an order of magnitude of the grain size but cannot be used to quantitatively determine the parameters  $\hat{D}$  and  $\sigma$  since they do not yield information related to variation of grain sizes across the thickness of the film. For this reason we use instead the reflectivity profile as a function of temperature through a thermally induced MIT on the same thin film sample to indirectly assess the non-homogeneity distribution function  $P(D)$ , as shown in Figure 5.

The temperature driven MIT in  $\text{VO}_2$  is a first-order transition. In the ideal case its properties, such as electrical resistivity or optical reflectivity, are expected to exhibit a finite, step-like change at the critical temperature  $T_c$ , at which the sample goes from a low-temperature insulating state to a high-temperature metallic state. In thin films, however, the optical reflectivity smoothly changes from the value corresponding to the insulating phase ( $R_{\text{ins}}$ ) to the value characteristic to the metallic phase  $R_{\text{m}}$  as the temperature increases, as Fig. 5 illustrates. The fact that the MIT takes place over a range of temperatures implies that different regions of the sample have different  $T_c$  values. Notice that this is different from the statement, obvious for a first order transition, that metallic regions coexist with insulating regions. One natural explanation of the fact that different regions have different values of  $T_c$  is that the film is comprised of grains of different size,  $D$ , and therefore with different  $T_c$ <sup>28,29</sup>. The simplest, physically justified, relation between  $D$  and

$T_c$ <sup>30,31</sup> is

$$T_c = T_c^{(\text{bulk})} \left( 1 - \frac{1}{D/D_0} \right), \quad (3)$$

where  $T_c^{(\text{bulk})}$  is the bulk transition temperature and  $D_0$  is the grain diameter below which the metallic (rutile) and insulating (monoclinic) phases are indistinguishable for a given probe<sup>29</sup>. In general, the  $T_c$  of a grain will be affected also by the local strain experienced by the grain<sup>32</sup>. We note that for the purpose of calculating the time evolution of the reflectivity after the photo-induced MIT it is sufficient to know the probability distribution of  $T_c$ . In this respect Eqs. (3), (2), can be interpreted simply as effective, physically justified, equations to obtain  $P(T_c)$ :

$$P(T_c) = P(D(T_c)) \frac{dD}{dT_c}. \quad (4)$$

In the remainder we take  $D_0$  to be the diameter of a sphere with a volume equal to the volume of 8 VO<sub>2</sub> unit cells in the rutile phase, and we use the effective critical temperature  $T_c = 355$  K, which is the temperature at which the VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> sample is completely metallic. This value is somewhat different from the bulk VO<sub>2</sub>,  $T_c^{(\text{bulk})} = 340$  K due to film strain<sup>28,33</sup>. The knowledge of  $P(T_c)$  allows us to calculate the insulating fraction of the film ( $\eta_I$ ) for any value of the temperature  $T$ , and therefore the metallic fraction is  $\eta_M = 1 - \eta_I$ :

$$\eta_I(T) = \int_T^\infty P(T_c) dT_c. \quad (5)$$

To obtain the variation of the film reflectivity  $R$  with temperature across the MIT, we use an effective medium theory (EMT)<sup>34-37</sup>. In EMT an inhomogeneous system is replaced by an effective homogeneous medium having the same, bulk, electric properties. Let  $\epsilon_M$ ,  $\epsilon_I$  be the dielectric constants (at the probing frequency) of VO<sub>2</sub> in the metallic and insulating phase respectively. Then, the dielectric constant of the effective medium  $\epsilon_{EMT}$  is given by the following equation:

$$\frac{\eta_I(\epsilon_I - \epsilon_{EMT})}{\epsilon_{EMT} + g(\epsilon_I - \epsilon_{EMT})} + \frac{\eta_M(\epsilon_M - \epsilon_{EMT})}{\epsilon_{EMT} + g(\epsilon_M - \epsilon_{EMT})} = 0. \quad (6)$$

In Eq. (6)  $g$  is a factor that depends on the shape of the grain. Approximating the inhomogeneous film as an effective 3D composite with compact spherical inclusions we used  $g = 1/3$ . Let  $n$  and  $k$  be the real and imaginary parts respectively of the index of refraction, so that for the effective medium  $n + ik = \sqrt{\epsilon_{EMT}}$  and therefore

$$R = \left| \frac{\cos \theta_0 - \sqrt{(n + ik)^2 - \sin^2 \theta_0}}{\cos \theta_0 + \sqrt{(n + ik)^2 - \sin^2 \theta_0}} \right|^2, \quad (7)$$

where  $\theta_0$  corresponds to the probe incidence angle. Using Eqs. (2)-(7), knowing  $\epsilon_M$  and  $\epsilon_I$  we can obtain the

evolution of the  $R(T)$  across the thermally induced MIT. By fitting the theoretically obtained profiles of  $R(T)$  to the experimental ones we can then identify the values of the parameters  $\hat{D}$ , and  $\sigma_D$  that characterize the inhomogeneous film microstructure.

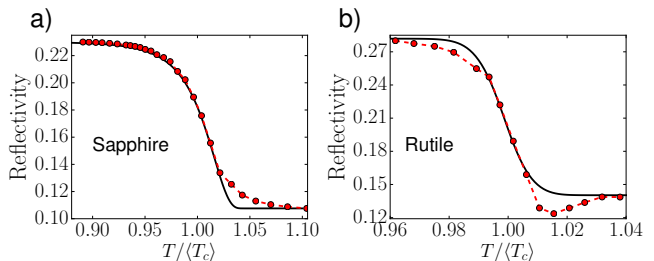


FIG. 5. Evolution of the reflectivity across thermally induced MIT for case of sapphire and rutile substrate normalized to the average critical transition temperature. For rutile substrate  $\langle T_c \rangle = 314.0$  K, and for the sapphire substrate  $\langle T_c \rangle = 340.1$  K.

Figures 5 (a), (b) show the evolution as a function of temperature of the reflectivity across the thermally induced MIT for a VO<sub>2</sub> film grown on sapphire and TiO<sub>2</sub>, respectively. Dots connected by dashed lines show the experimental measurements, while the solid lines show the results obtained using the theoretical approach described above.

To determine the parameters entering the theory we first set the value of the imaginary part of the complex index of refraction  $k_M$ ,  $k_I$  for the metallic and insulating VO<sub>2</sub> films to the measured values, consistent with the values reported in the literature<sup>38,39</sup>. Then we use Eq. (7), that relates the complex index of refraction, and optical reflectivity  $R$ , and the measured experimental value of  $R_I$  ( $R_M$ ) at the insulating (metallic) phase to fix the corresponding values of the real part of the complex index of refraction  $n_I$  ( $n_M$ ), and therefore obtain the dielectric constants  $\epsilon_I$ ,  $\epsilon_M$  entering Eq. (6). Using Eq. (5),  $\eta_M = (1 - \eta_I)$ , and Eq. (6) we can then obtain  $\epsilon_{EMT}$ , and therefore the value of the effective medium reflectivity using Eq. (7), across the thermally induced MIT. By fitting the profile of  $R(T)$  to the experimental data for the case of VO<sub>2</sub> on sapphire we obtain that the average grain size, and the standard deviation, that determine the probability distribution  $P(D)$  are  $\langle D \rangle = 64.7$  nm and  $\sigma_D = 38.5$  nm, respectively. For the case of VO<sub>2</sub> on TiO<sub>2</sub> we obtain  $\langle D \rangle = 17.4$  nm and  $\sigma_D = 1.1$  nm. These results show that the average grain size is of the same order in the two type of VO<sub>2</sub> films, VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> and VO<sub>2</sub>/TiO<sub>2</sub>. On the other hand, the standard deviation of the grain size is almost an order of magnitude different: VO<sub>2</sub>/TiO<sub>2</sub> samples appear to have a much smaller variation in the grain size than VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> samples. This is also reflected in the distribution of  $T_c$ . Figures 6 (a), (b) show the probability distributions  $P(D)$  corresponding to the values of  $\langle D \rangle$  and  $\sigma_D$  used to obtain the good theoretical fit to the evolution of  $R(T)$  shown in Fig. 5.

Figures 6 (c), (d) show the corresponding probability distribution for  $T_c$ . With knowledge of  $P(D)$  the theoretical

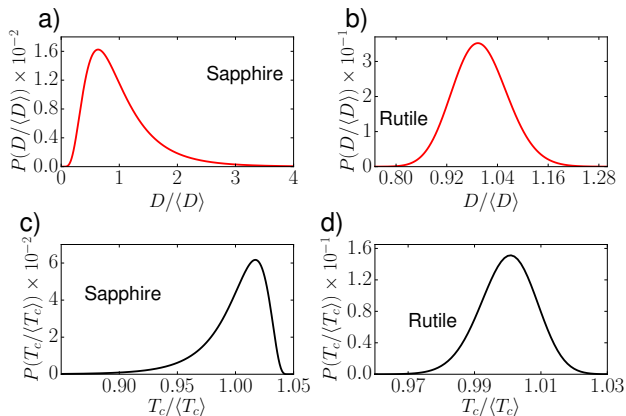


FIG. 6. Panels (a) and (b) show the grain size probability distributions normalized to the average grain size for sapphire ( $\langle D \rangle = 64.7$  nm) and rutile ( $\langle D \rangle = 17.4$  nm) substrate respectively. Panels (c) and (d) show the critical temperature probability distribution normalized to the average critical temperature for sapphire ( $\langle T_c \rangle = 340.1$  K) and rutile ( $\langle T_c \rangle = 314.0$  K) respectively. The bulk critical temperature is taken to be  $T_c^{(bulk)} = 355$  K.

model also allows us to obtain the temperature evolution of the insulating and metallic fraction of the  $\text{VO}_2$  film across the thermally induced MIT. The results for the case of  $\text{VO}_2$  on sapphire and  $\text{TiO}_2$  are shown in Figures 7 (a), (b) respectively. The much smaller value of  $\sigma_D$  for  $\text{VO}_2/\text{TiO}_2$  compared to  $\text{VO}_2/\text{Al}_2\text{O}_3$  is reflected in a much steeper variation of  $\eta_I$ , and therefore of  $R(T)$ , across the MIT.

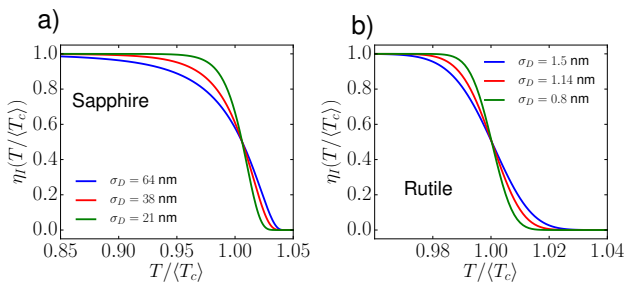


FIG. 7. Evolution of the insulating partial volume  $\eta_I$  across the thermally induced MIT for case of (a) sapphire and (b) rutile substrates. For rutile,  $\langle T_c \rangle = 314.0$  K, and for sapphire  $\langle T_c \rangle = 340.1$  K.

#### IV. THEORETICAL MODELING OF THE RELAXATION DYNAMICS OF THE MIT

In this section we show that the proposed treatment of the inhomogeneous  $\text{VO}_2$  films can also accurately describe their thermal relaxation at long-time scales after

the photo-induced MIT, when the dynamics of the  $\text{VO}_2$  film can be assumed to be adiabatic. In this regime the film cools by transferring heat to the substrate. In the process of cooling more and more metallic regions become insulating and therefore cause a significant change in reflectivity.

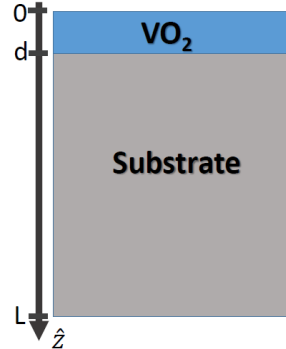


FIG. 8. Sketch of the heterostructure considered in this work. It is composed of a vanadium dioxide ( $\text{VO}_2$ ) thin-film deposited on top of a substrate. The substrates considered in this work are titanium dioxide ( $\text{TiO}_2$ ), and aluminum oxide ( $\text{Al}_2\text{O}_3$ ). For  $\text{VO}_2/\text{TiO}_2$   $d = 110$  nm while for  $\text{VO}_2/\text{Al}_2\text{O}_3$   $d = 80$  nm. For both substrates,  $L = 0.5$  mm.

In our experiment the  $\text{VO}_2$  films have a thickness  $d$  equal to or smaller than 110 nm (see Fig. 8), which is comparable with the laser  $1/e$  penetration depth  $\delta \simeq 110 - 130$  nm<sup>15</sup>. Thus, we can assume that the pump pulse heats the film uniformly throughout its thickness. To describe the heat transfer process between the film and the substrate, we assume the temperature to be uniform throughout the film for all times. Effectively, given this conditions, the heat transfer problem becomes a one-dimensional problem, and the equation for the rate of change of the heat ( $Q$ ) in the film takes the form:

$$\frac{dQ}{dt} = A \times d \times (\rho_I C_I \eta_I(T_f) + \rho_M C_M \eta_M(T_f) + L(T_f) \tilde{P}(T_f) \rho_{av}) \frac{\partial T_f}{\partial t}, \quad (8)$$

where  $T_f$  is the film temperature,  $A$  is the area of the film,  $\rho_I$  ( $\rho_M$ ) is the density in the insulating (metallic) phase,  $\rho_{av} \equiv (\rho_I + \rho_M)/2$ ,  $C_I$  ( $C_M$ ) is the heat capacity in the insulating (metallic) phase,  $L$  is the specific heat and  $P(T_f)dT_f$  is the fraction of the sample undergoing the MIT in the time interval  $dt$  during which the film temperature is in the interval  $[T_f, T_f + dT_f]$ . In analogy to  $T_c$ , we assume

$$L = L^{(bulk)} \left( 1 - \frac{1}{D/D_0} \right), \quad (9)$$

where  $L^{(bulk)}$  is the value of the specific heat for bulk  $\text{VO}_2$ . For a given  $T_f$ , the values of  $\eta_M(T_f)$  and  $\eta_I(T_f)$  are obtained using the method previously described, having



fixed the parameters characterizing  $P(D)$  via the results for the thermally induced MIT.

The rate of change of heat in the film given by Eq. (8) must be equal to the heat current ( $J_Q$ ) across the interface between the film and the substrate:

$$J_Q = -\sigma_K A(T_f - T_s(d)) \quad (10)$$

where  $\sigma_K$  is the Kapitza constant characterizing the thermal conductivity of the interfaces<sup>40-43</sup>, and  $T_s(d)$  is the temperature of the substrate at the surface facing the VO<sub>2</sub> film. Combining Eq. (8) and Eq. (10), for  $T_f$  we obtain the equation:

$$\left[ \rho_I C_I \eta_I(T_f) + \rho_M C_M \eta_M(T_f) + L(T_f) \tilde{P}(T_f) \rho_{av} \right] \frac{\partial T_f}{\partial t} = -\frac{\sigma_K}{d} (T_f - T_s(d)). \quad (11)$$

Notice that equation (11) is highly nonlinear due to the  $T_f$ -dependence of the terms within the square brackets on the left hand side.

The temperature distribution within the substrate,  $T_s(z, t)$ , satisfies the diffusion equation:

$$\frac{\partial T_s(z, t)}{\partial t} = \frac{k_s}{C_s \rho_s} \frac{\partial^2 T_s(z, t)}{\partial z^2} \quad (12)$$

where  $k_s$ ,  $C_s$ ,  $\rho_s$  are the thermal conductivity, heat capacity, and mass density, respectively, of the substrate. The bottom of the substrate, for which  $z = L$  (see Fig. 8), is kept at a fixed temperature  $T_s(L)$ . At the film/substrate interface the heat transferred from the film must be equal to the heat current  $k_s \partial T_s / \partial z|_{z=d}$ . We then have that the boundary conditions completing Eq. (12) are:

$$T_s(z = L, t) = T_s^{(B)}; \quad (13)$$

$$k_s \left. \frac{\partial T_s(z, t)}{\partial z} \right|_{z=d} = -\sigma_K (T_f(t) - T_s(z = d, t)). \quad (14)$$

Equations (11)-(14), combined with knowledge of the probability distribution  $P(D)$ , and relations (3), (9), completely define the temperature evolution of the VO<sub>2</sub> film. While these equations can in general be solved only numerically, some qualitative understanding of the decay time  $\tau$  can be gained if we make some simplifications. Let us assume first that the critical temperature probability distribution is well approximated by a Gaussian profile  $\tilde{P}(T_c) = 1/(\sqrt{2\pi}\sigma_{T_c}) \exp\{-(T_c - \langle T_c \rangle)^2 / (2\sigma_{T_c}^2)\}$ , as it is the case for the VO<sub>2</sub> sample grown on rutile (see Fig. (6d)). In this case, the insulating volume fraction is simply given by  $\eta_I(T) = \frac{1}{2} [1 - \text{erf}((T - \langle T_c \rangle) / (\sqrt{2}\sigma_{T_c}))]$ . Then assuming that the pump pulse is strong enough to drive the entire film into a fully metallic state at  $t = 0$ , the time-dependence of the insulating volume fraction can be approximated by a simple exponential form  $\eta_I(t) = 1 - Ae^{-t/\tau}$ . In this case, an expression for the temperature can be obtained through the relationship  $\eta_I(T(t)) = \eta_I(t)$ . Furthermore, assuming that the substrate temperature  $T_s$

does not change with time, and that the latent heat  $L$  is temperature-independent, we can calculate the decay constant  $\tau$ :

$$\tau = Cd \frac{\sigma_{T_c}}{\sigma_K} \frac{(C_M \rho_M + L \rho_{av} \tilde{P}(T_0))}{T_0 - T_s} + \tau_0, \quad (15)$$

where the constant  $C > 0$ , and  $\tau_0$  can only be determined by solving the full system of equations (11)-(13)

It is interesting to note that despite its many limitations, Eq.(15) captures many important qualitative traits of the actual relaxation process. For example, Figure (9) shows a plot of the decay constant as a function of  $\sigma_{T_c}$  obtained solving the full system of equations (11)-(13) for two different values of the average critical temperature, and same initial temperature  $T_0 = 360$  K. It is easy to see that the decay time  $\tau$  follows the linear trend predicted by Eq. (15) in the limit  $(T_0 - \langle T_c \rangle) \gg \sqrt{2}\sigma_{T_c}$ . Similarly, an exact solution shows the inverse dependence of  $\tau$  on the Kapitza constant,  $\tau \propto \sigma_K^{-1}$ , as shown in Figure 10.

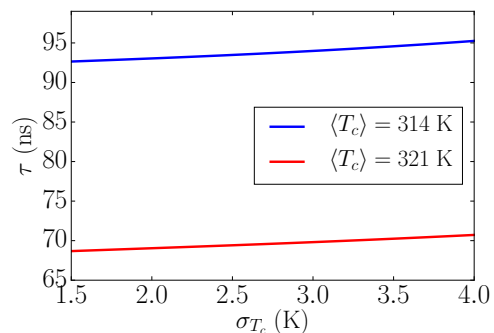


FIG. 9. Full numerical calculation of the dependence of metal state decay constant  $\tau$  on  $\sigma_{T_c}$  for two different values of the sample average critical temperature  $\langle T_c \rangle$ , and  $T_s(L) = 280$  K. The initial temperature  $T_0 = 360$  K is such that the sample is initially fully metallic, and  $(T_0 - \langle T_c \rangle) / (\sqrt{2}\sigma_{T_c}) \approx 9$ .

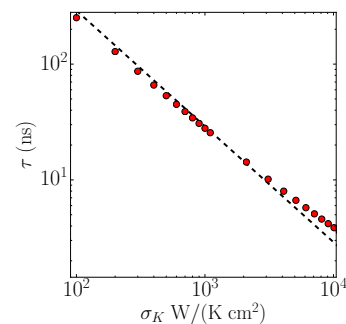


FIG. 10. VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> metal state decay time  $\tau$  dependence on the Kapitza constant  $\sigma_K$  for  $\langle D \rangle = 64.7$  nm,  $\sigma_D = 38.5$  nm, substrate temperature  $T_s(L) = 310$  K, and fluence  $\phi = 8$  mJ/cm<sup>2</sup>. The red dots correspond to numerical calculations, and the dashed line is given by  $\tau \propto \sigma_K^{-1}$ .

VO <sub>2</sub> heat capacity insulating phase $C_I^{44}$	0.656 J/(g K)
heat capacity metallic phase $C_M^{44}$	0.78 J/(g K)
density insulating phase $\rho_I^{43}$	4.57 g/cm <sup>3</sup>
density metallic phase $\rho_M^{43}$	4.65 g/cm <sup>3</sup>
thermal conductivity insulating phase $\kappa_I^{45}$	3.5 W/(m K)
thermal conductivity metallic phase $\kappa_M^{45}$	6 W/(m K)
bulk latent heat $L^{(Bulk)^{44}}$	51.8 J/g
TiO <sub>2</sub> heat capacity $C_s^{46}$	0.686 J/(g K)
density $\rho_s^{43}$	4.25 g/cm <sup>3</sup>
thermal conductivity $\kappa_s^{47}$	8 W/(m K)
Al <sub>2</sub> O <sub>3</sub> heat capacity $C_s^{48}$	0.779 J/(g K)
density $\rho_s^{48}$	3.98 g/cm <sup>3</sup>
thermal conductivity $\kappa_s^{48}$	30 W/(m K)
VO <sub>2</sub> /TiO <sub>2</sub> absorption coefficient at 800 nm $\alpha^{15}$	0.01 nm <sup>-1</sup>
VO <sub>2</sub> /Al <sub>2</sub> O <sub>3</sub> absorption coefficient at 800 nm $\alpha^{15}$	0.0076 nm <sup>-1</sup>

TABLE I. Parameters of VO<sub>2</sub> and substrates.

	VO <sub>2</sub> /TiO <sub>2</sub>	VO <sub>2</sub> /Al <sub>2</sub> O <sub>3</sub>
$\langle D \rangle$	17.4 nm	64.7 nm
$\sigma_D$	1.1 nm	38.5 nm
$\langle T_c \rangle$	314.0 K	340.1 K
$\sigma_{T_c}$	2.6 K	8.8 K
$n_M + ik_M$	1.53 + $i0.8$	1.49 + $i0.65$
$R_M$	0.14	0.11
$n_I + ik_I$	3.03 + $i0.57$	2.60 + $i0.60$
$R_I$	0.28	0.23

TABLE II. Comparative table between VO<sub>2</sub>/TiO<sub>2</sub>, and VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> sample parameters.

## V. EFFECT OF INHOMOGENEITIES ON THE RELAXATION DYNAMICS OF THE PHOTO-INDUCED MIT

Using the theoretical approach described in Sec. IV we can obtain the time evolution of the optical reflectivity  $R$  through the MIT, as well as explain the significant difference in relaxation timescales between the two VO<sub>2</sub> samples considered. In all the numerical calculations we assume  $C_I$ ,  $\rho_I$ ,  $C_M$ ,  $\rho_M$  to be equal to the bulk value for insulating and metallic VO<sub>2</sub>, see Table I. The initial film temperature is fixed by the pump fluence taking into account the gaussian profile of the pulse and the fact that some of the heat is lost by the film during the time interval  $[0, t_0]$  for which our analysis does not apply,  $t = 0$  is time at which the pump pulse hits the VO<sub>2</sub> film and  $t_0 = 10$  ns for VO<sub>2</sub>/TiO<sub>2</sub> films and  $t_0 = 0.5$  ns for VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> films.

Considering that we have identified the parameters in both samples that determine  $P(D)$  using the profiles of  $R(T)$  for the thermally induced MIT, the only undeter-

mined parameter entering the theory is the Kapitza constant  $\sigma_K$ . This last step can be done by analyzing the experimentally measured time evolution of the optical reflectivity for different laser fluences and substrate temperatures, shown in Fig. 11 for the VO<sub>2</sub>/TiO<sub>2</sub> sample. It is easy to see that all experimental curves are well approximated assuming the same value for the Kapitza constant  $\sigma_K = 1100$  W/(K cm<sup>2</sup>).

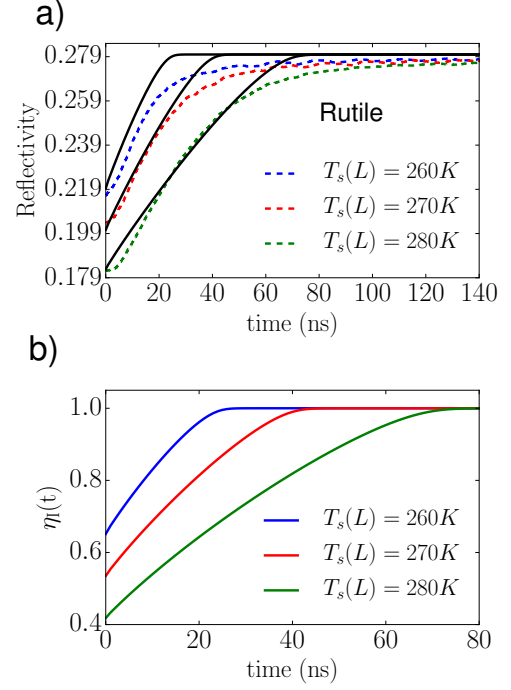


FIG. 11. (a) Time evolution of reflectivity after the photo-induced MIT for VO<sub>2</sub>/TiO<sub>2</sub> for three different  $T_s(L)$  and  $\phi = 9$  mJ/cm<sup>2</sup>. The solid curves correspond to the theoretical results, and the dashed curves correspond to the experimental results. For the three theory curves we use  $\sigma_K = 1100$  W/(K cm<sup>2</sup>). Panel (b) shows the corresponding insulating fraction time evolution.

To understand the origin of the large difference between the recovery time scale VO<sub>2</sub>/TiO<sub>2</sub> films and VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> films, we first consider the effect of the average and standard deviation of the grain size. The theoretical calculations predict that inhomogeneities in the film can play an important role. For example, Fig. 12 shows the calculated dependence of the decay constants  $\tau$  on the grain size standard deviation  $\sigma_D$  for two values of  $\langle \ln D \rangle$ , as defined in Eq. (2). Broader distributions correspond to larger time scales. At the same time differences in  $\langle D \rangle$  and  $\sigma_D$  between VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> and VO<sub>2</sub>/TiO<sub>2</sub> films alone cannot explain the observed differences in the recovery times of  $R(t)$  for the two films. To illustrate this, we model the time evolution of  $R(t)$  after the photo-induced MIT in the VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> film using the grain distribution parameters  $\langle D \rangle$  and  $\sigma_D$  obtained by fitting the corresponding  $R(T)$  curve, but using the value of the Kapitza constant, obtained for the

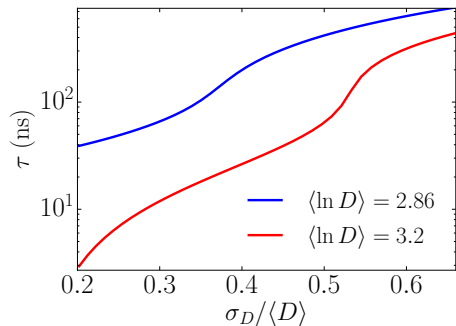


FIG. 12. Dependence of the VO<sub>2</sub>/TiO<sub>2</sub> metal state decay time constant  $\tau$  on  $\sigma_D$  for two values of  $\langle \ln D \rangle$ , as defined in Eq. (2), Kapitza constant  $\sigma_K = 1100$  W/(K cm<sup>2</sup>), substrate temperature  $T_s(L) = 280$  K, and initial fluence  $\phi = 9$  mJ/cm<sup>2</sup>.

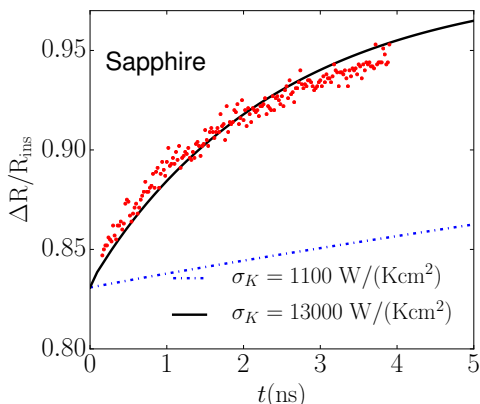


FIG. 13. VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> reflectivity time evolution after photo-induced MIT for  $\phi = 7.5$  mJ/cm<sup>2</sup>. The red dots correspond to the experimental result. The dotted curve correspond to the theory with  $\sigma_K = 1100$  W/(K cm<sup>2</sup>), and the solid curve corresponds to  $\sigma_K = 13000$  W/(K cm<sup>2</sup>).

VO<sub>2</sub>/TiO<sub>2</sub> sample  $\sigma_K = 1100$  W/(K cm<sup>2</sup>). The results of these calculations are shown as a dashed curve in Fig. 13. It is evident that the large difference in the values of  $\tau$  must be due to a significant difference in  $\sigma_K$  between the two films. Indeed, the experimental curves  $R(t)$  for VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> are well approximated by the theoretical results assuming  $\sigma_K = 13,000$  W/(K cm<sup>2</sup>), shown as a solid curve in Fig. 13.

Figure 14 shows the time evolution of the VO<sub>2</sub> film and substrate temperatures (close to the interface) for the VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> film, panel (a), and for the VO<sub>2</sub>/TiO<sub>2</sub> film, panel (b), using the parameter values summarized in Table I. It helps to qualitatively understand the differences in thermal relaxation between two samples. Due to lower values of Kapitza constant and thermal conductivity, thermal energy stays more concentrated near the VO<sub>2</sub>-TiO<sub>2</sub> interface, keeping the temperature of the VO<sub>2</sub> film above  $T_c$  longer.

To investigate the temperature dependence of the

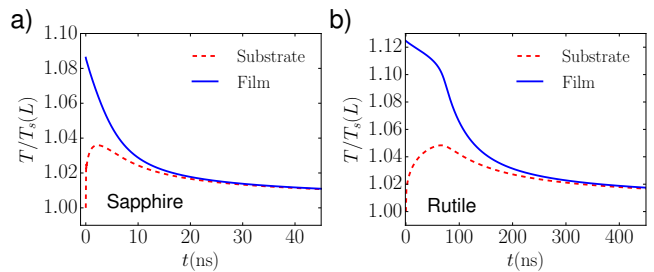


FIG. 14. Film and substrate temperature time evolution. For sapphire (a),  $T_s(L) = 310$  K, and for rutile (b),  $T_s(L) = 280$  K.

thermal relaxation we repeated the measurements while changing the base substrate temperature of the VO<sub>2</sub>/TiO<sub>2</sub> sample. For these measurements the sample was placed inside a cryostat, and cooled down to temperatures  $T_s(L)$  between 260 K and 298 K. Since the VO<sub>2</sub> relaxation is due to the thermal exchange with the substrate, we expect that lowering the substrate temperature should lead to faster relaxation. The results of these measurements, along with the theoretical calculations, are shown in Figure 15. We again observe a good semiquantitative agreement between theoretical and experimental results. Also, note that the simple expression for the decay constant  $\tau$  Eq. (15) captures the overall decay rate drop at lower substrate temperatures  $T_s(L)$ . It is worth pointing out that all the theoretical curves are obtained using the fixed set of parameters shown in Table I.

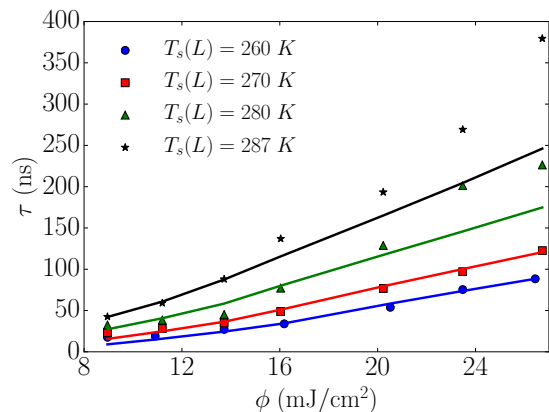


FIG. 15. Dependence of metal state decay constant  $\tau$  on fluence and substrate temperature for VO<sub>2</sub>/TiO<sub>2</sub>.

An important advantage of the presented theoretical model is that it allows an accurate description of the relaxation dynamics of different VO<sub>2</sub> films under a wide range of experimental conditions using a single set of parameters describing the inhomogenous distribution of grains in each film and the thermodynamic constants for the film-substrate interface. For example, Fig. 4 shows an excellent agreement between the experimental mea-



surements and theoretical calculations across the entire range of pump laser fluences, limited on the lower end by our ability to reliably detect the variation in the probe reflectivity, and on the upper end by the damage threshold of our sample (pump fluence  $> 40$  mJ/cm<sup>2</sup>).

## VI. CONCLUSIONS

In conclusion, we have presented a combined experimental and theoretical study of the long time-scale recovery dynamics of VO<sub>2</sub> films following an ultrafast photo-induced metal-insulator transition. We have presented a theoretical approach to describe the time evolution of the dielectric constant of VO<sub>2</sub> films through the MIT, and therefore of the reflectivity, over long-time scales after a photo-induced metal insulator transition. We also have shown how the parameters characterizing the grain size distribution and the thermal conductivity of the interface between the film and substrate affect such evolution. The theory presented can be generalized to describe the dynamics (in the adiabatic limit) across a first order phase transition of other inhomogeneous systems. By compar-

ing theoretical and experimental results we have been able to determine the relevant parameters characterizing the VO<sub>2</sub> films and the thermal conductivity of the interface and attribute the large difference in the recovery time scale between VO<sub>2</sub>/TiO<sub>2</sub> and VO<sub>2</sub>/Al<sub>2</sub>O<sub>3</sub> films to a large difference in the thermal conductivity of the interface. These results might be important for technological application of VO<sub>2</sub> based on its ability to change phase at temperatures close to room temperature.

## VII. ACKNOWLEDGMENTS

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