Physics 786, Spring 2023
Problem Set 8, Due Friday, April 7, 2023.

Final Paper

You should have already read some literature about your paper topic, and have started writing. Don't leave the paper to the last minute. Aim for 10 double-spaced pages, but there is no page count requirement. A complete draft is due April 21, and I am happy to look at portions of it beforehand if you have questions.

1. *Schwarzschild trajectories*

a) A massive test particle is released from $r = R > 2GM$ in the Schwarzschild geometry (in standard coordinates), and falls radially inward. Show that the following correctly parametrizes the trajectory:

$$r = \frac{R}{2}(1 + \cos\eta)$$
$$\tau = \frac{R}{2}\left(\frac{R}{2GM}\right)^{1/2}(\eta + \sin\eta).$$

b) Show that the proper time elapsed when the particle reaches $r = 2GM$ is finite.

2. *Abandon hope, all ye who enter here.*

A massive test particle is at $r = r_0 < 2GM$ at $t = 0$. Assume that the metric inside the horizon takes the same Schwarzschild form as outside the horizon.

a) Show that

$$\left|\frac{dr}{d\tau}\right| \geq \sqrt{\frac{2GM}{r} - 1}.$$

b) Show that the test particle necessarily reaches $r = 0$ in finite proper time.