Physics 786, Spring 2023
Problem Set 6, Due Friday, March 24.

Final Paper and Presentation - Complete draft due Friday, April 21; Final draft due Friday, May 5; You should prepare a 15-minute presentation on the topic of your paper to be presented during the week of May 1.

For your final paper, you should identify and describe either a theoretical aspect of or extension of general relativity, or an experiment or application which either makes use of general relativity or provides a test of general relativity. You should explain in detail those aspects of general relativity which are relevant. Example topics include gravity in extra dimensions, torsion, curvature perturbations during inflation, the cosmic microwave background, Gravity Probe B, LIGO, Planck, BICEP.... As a guideline, aim for ten double-spaced pages.

1. *Curvature of the Two-Sphere*

Consider the two-sphere of radius a, with metric

$$ds^2 = a^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right).$$

a) Calculate the Christoffel symbols of the two-sphere of radius a.

b) In 2D, $R_{\lambda\mu\nu\rho} = \frac{1}{2}R(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu})$.

Calculate all the components of the Ricci tensor $R_{\mu\nu}$ and the Gaussian curvature, $K = -R/2$, of the two-sphere.

2. *Rindler Space*

Consider the 2D spacetime whose metric is given by

$$ds^2 = d\rho^2 - \rho^2 \, d\eta^2.$$

a) Calculate the components of the curvature tensor $R_{\mu\nu\lambda\sigma}$.

b) What can you infer about this spacetime from your results of part (a)?

3. *Weyl tensor*

Show that the Weyl tensor, which in four dimensions is given by

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}\left(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}\right) + \frac{1}{6}R\left(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}\right),$$

is traceless, *i.e.* vanishes upon contraction of any pair of its indices.